BOOK OF ABSTRACTS

Second Italian Meeting

on

Probability and Mathematical Statistics

June 17 - 20, 2019, Vietri sul Mare (SA), Italy a cura di

A. Buonocore e A. Di Crescenzo

Associazione di volontariato PrismaCampania
Fisciano (SA)

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Dipartimento di Matematica, Politecnico di Milano
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MIUR - PRIN 2017, project "Stochastic Models for Complex Systems"

Cooperating Institutions
Comune di Salerno
Comune di Vietri sul Mare
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FOREWORD

This International Meeting is the sequel to the First Italian Meeting on Probability and Mathematical Statistics, that was held successfully in Torino in June 19-22, 2017. Like the previous event, the conference promotes the scientific exchange between Italian mathematicians working in Probability and Mathematical Statistics in Italy or abroad, foreign researchers working in Italy, and any interested scientist.
The program includes four plenary talks, fifty oral sessions, a poster session, and round table discussions. Topics range over the most current developments in the field of probability, mathematical statistics and their applications.
We are pleased that more than two hundred scientists will join the conference. The program will be quite intense, but we encourage all the attendees to discuss with other participants and to take advantage of such an opportunity to start new collaborations.

Vietri sul Mare
June 2019

The Organizing Committee

Monday 17th June

8:00 - 9:30	Registration

9:30 - 10:15	Opening

10:15 - 11:10 **Plenary Talk** (Room F)
Intrinsic volumes of convex bodies and cones: concentration, limit theorems and sparse recovery
Giovanni Peccati, Luxembourg University
Chair: D. Marinucci, University of Roma Tor Vergata

11:10 - 11:45	Coffee break

Session S17F1 *New applications of the Stein-Malliavin method for Gaussian approximation* (Room F)
Organizer/Chair: A. Vidotto, University of Roma Tor Vergata

11:45 - 12:10 *Stein-Malliavin techniques and Poisson based U-statistics: asymptotics*
C. Durastanti, Sapienza University, Roma
12:10 - 12:35 *Stein-Malliavin approximation for local geometric functionals of random spherical harmonics*
A.P. Todino, Ruhr University Bochum
12:35 - 13:00 *Stein-Malliavin techniques for spherical functional autoregressions*
A. Caponera, Sapienza University, Roma

Session S17E1 *Conditional expectations and Bayesian nonparametric problems* (Room E)
Chair: F. Spizzichino, Sapienza University, Roma

11:45 - 12:10 *A Bayes nonparametric prior for semi-Markov processes*
P. Muliere, Bocconi University
12:10 - 12:35 *Can one define conditional expectations for probability charges?*
C. Sempi, University of Salento
12:35 - 13:00 *Clustering structure for species sampling sequences with general base measure*
F. Bassetti, Polytechnic University of Milano

Session S17D1 *Stochastic processes and applications in finance* (Room D)
Organizers/Chairs: E. Di Nardo, University of Torino
P. Semeraro, Polytechnic University of Torino

11:45 - 12:10 *Insurance capacity*
E. Luciano, University of Torino
12:10 - 12:35 *Asymptotic results for the Fourier estimator of the integrated quarticity*
M.E. Mancino, University of Firenze
12:35 - 13:00 *On the combinatorics of cumulants for multivariate subordinated Lévy processes*
E. Di Nardo, University of Torino

Session S17C1 *Optimal control of random systems* (Room C)
Organizer/Chair: G. Zanco, LUISS Guido Carli

11:45 - 12:10 *Optimal control of a stochastic phase-field model for tumor growth*
L. Scarpa, University of Vienna
12:10 - 12:35 *Some optimal control problems for non-local random systems*
C. Orrieri, University of Trento
12:35 - 13:00 *Path dependent HJB equations via BSDEs*
F. Masiero, University of Milano Bicocca

Monday 17th June

13:00 – 14:30 Lunch

Session S17F2 *Information geometry* (Room F)
Organizers/Chairs: P. Siri, Polytechnic University of Torino, Italy
B. Trivellato, Polytechnic University of Torino, Italy

14:30 - 14:55 *Information geometry of the Gaussian space*
G. Pistone, Collegio Carlo Alberto, Torino, Italy
14:55 - 15:20 *Derivative-free optimization by Wasserstein natural gradient*
L. Malagò, Romanian Institute of Science and Technology, Romania
15:20 - 15:45 *Non-geometric rough paths on manifolds*
E. Rossi Ferrucci, Imperial College London, United Kingdom

Session S17E2 *Advances in stochastic processes* (Room E)
Chair: C. Ceci, University of Chieti-Pescara, Italy

14:30 - 14:55 *On the existence of continuous processes with given one-dimensional distributions*
P. Rigo, University of Pavia, Italy
14:55 - 15:20 *Quasi-infinitely divisible processes and measures*
R. Passeggeri, Imperial College London, United Kingdom
15:20 - 15:45 *Intermediate and small scale limiting theorems for random fields*
R. Maffucci, University of Oxford, United Kingdom

Session S17D2 *Dependence modeling* (Room D)
Organizer/Chair: F. Durante, University of Salento, Italy

14:30 - 14:55 *Nonparametric Bayesian estimation of the extremal dependence*
S. Padoan, Bocconi University, Italy
14:55 - 15:20 *Measuring linear correlation between random vectors*
G. Puccetti, University of Milano, Italy
15:20 - 15:45 *Representation of multivariate Bernoulli distributions with a given set of specified moments*
P. Semeraro, Polytechnic University of Torino, Italy

Session S17C2 *Advanced control problems, stochastic simulation and estimates for diffusions* (Room C)
Chair: C. Macci, University of Roma Tor Vergata, Italy

14:30 - 14:55 *Multi-level Monte-Carlo methods and upper/lower bounds in nested risk computations*
S. De Marco, Ecole Polytechnique, France
14:55 - 15:20 *Estimating functions for discretely observed diffusion processes conditioned to nonabsorption*
R. Sirovich, University of Torino, Italy
15:20 - 15:45 *An algorithm to construct subsolutions of convex optimal control problems*
G. Bet, University of Firenze, Italy

Monday 17th June

15:45 – 16:15	Coffee break

Session S17F3 *Recent developments in stochastic geometry* (Room F)
Organizer/Chair: C. Durastanti, Sapienza University, Roma, Italy

16:15 - 16:40 *Two point function for critical points of a random plane wave*
V. Cammarota, Sapienza University, Roma, Italy
16:40 - 17:05 *Nodal lengths of random spherical harmonics*
M. Rossi, University of Pisa, Italy
17:05 - 17:30 *The isotropic constant of random polytopes*
N. Turchi, University of Luxembourg, Luxembourg
17:30 - 17:55 *Fourth moment theorems on the Poisson space in any dimension*
A. Vidotto, University of Roma Tor Vergata, Italy

Session S17E3 *Stochastic quantization, invariant measures and mean-field limits* (Room E)
Organizer/Chair: G. Pistone, Collegio Carlo Alberto, Torino, Italy

16:15 - 16:40 *Fluctuations of point vortices and 2D Euler invariant measures*
F. Grotto, Scuola Normale Superiore of Pisa, Italy
16:40 - 17:05 *Strong Kac's chaos in the mean-field Bose-Einstein Condensation*
S. Ugolini, University of Milano, Italy
17:05 - 17:30 *Elliptic stochastic quantization*
F. De Vecchi, University of Bonn, Germany

Session S17D3 *Finitely additive probabilities and some of their applications* (Room D)
Organizer/Chair: P. Rigo, University of Pavia, Italy

16:15 - 16:40 *Probability and uncertainty in decision theory*
S. Cerreia-Vioglio, Bocconi University, Italy
16:40 - 17:05 *Decisions under different scenarios in a finitely additive framework*
D. Petturiti, University of Perugia, Italy
17:05 - 17:30 *Finitely additive set-valued measures and applications in economic theory*
N. Urbinati, University of Napoli Federico II, Italy

Session S17C3 *Random interfaces and universality* (Room C)
Organizer/Chair: E. Bisi, University College Dublin, Ireland

16:15 - 16:40 *Corner growth model, symplectic characters, and KPZ universality*
E. Bisi, University College Dublin, Ireland
16:40 - 17:05 *A new universality class for (1+1)-dimensional random interfaces: the Brownian castle*
G. Cannizzaro, University of Warwick, United Kingdom
17:05 - 17:30 *Entropic repulsion for the Gaussian free field conditioned on disconnection by level-sets*
A. Chiarini, ETH Zurich, Switzerland

Tuesday 18th June

<u>Session S18F1</u> *<u>Results on stochastic geometry and growth-fragmentation equations</u>* (Room F)
Chair: B. Martinucci, University of Salerno

9:00 – 9:25 *The generalized moment method for parameters estimate in stochastic fibre processes*
A. Micheletti, University of Milano
9:25 – 9:50 *A probabilistic approach to the asymptotic behaviour of the growth-fragmentation equations*
B. Cavalli, University of Zürich
9:50 – 10:15 *On the estimation of the mean density of lower dimensional germ-grain models in* R^d
E. Villa, University of Milano

<u>Session S18E1</u> *<u>Advances in Bayesian modelling</u>* (Room E)
Organizer: I. Pruenster, Bocconi University
Chair: F. Bassetti, Polytechnic University of Milano, Italy

9:00 – 9:25 *Quantifying the dependence structure in Bayesian nonparametric models*
M. Catalano, Bocconi University
9:25 – 9:50 *Hybrid nonparametric priors for clustering*
G. Rebaudo, Bocconi University
9:50 – 10:15 *Closed form Bayesian filtering for multivariate binary time series*
A. Fasano, Bocconi University

<u>Session S18D1</u> *<u>Recent results in insurance and market dynamics</u>* (Room D)
Chair: F. Pellerey, Polytechnic University of Torino

9:00 – 9:25 *Analytical approximation of counterparty value adjustment*
A. Ramponi, University of Roma Tor Vergata
9:25 – 9:50 *A new approach to forecast market interest rates*
R.M. Mininni, University of Bari
9:50 – 10:15 *Joint life insurance pricing using extended Marshall-Olkin models*
S. Mulinacci, University of Bologna

<u>Session S18C1</u> *<u>Probabilistic algorithms and games on networks</u>* (Room C)
Organizers/Chairs: M. Quattropani, University of Roma Tre
M. Scarsini, LUISS Guido Carli

9:00 – 9:25 *The buck passing game on networks*
M. Quattropani, University of Roma Tre
9:25 – 9:50 *Processing data sets on networks: random forests and other probabilistic tools*
L. Avena, Leiden University
9:50 – 10:15 *On the emergent behavior of the 2-choices dynamics*
E. Cruciani, Gran Sasso Science Institute

Tuesday 18th June

10:15 - 11:10 **Plenary Talk** (Room F)
A brief personal history of stochastic partial differential equations
Lorenzo Zambotti, Sorbonne Université
Chair: F. Flandoli, Scuola Normale Superiore of Pisa

11:10 – 11:45	Coffee break

Session S18F2 *Backward stochastic differential equations and their applications* (Room F)
Organizers/Chairs: K. Colaneri, University of Leeds
E. Issoglio, University of Leeds

11:45 – 12:10 *BSDEs driven by possibly non quasi-left-continuous random measures and optimal control of PDMPs*
E. Bandini, University of Milano Bicocca
12:10 – 12:35 *Optimal switching problems with an infinite set of modes: an approach by randomization and constrained backward SDEs*
M. Fuhrman, University of Milano
12:35 – 13:00 *A Feynman-Kac result via Markov BSDEs with generalized drivers*
E. Issoglio, University of Leeds

Session S18E2 *Analysis in Wiener spaces 1* (Room E)
Organizer/Chair: S. Bonaccorsi, University of Trento

11:45 – 12:10 *Integration by parts formulae on open convex sets in Wiener spaces*
G. Menegatti, University of Ferrara
12:10 – 12:35 *Equivalent characterizations of BV functions on domains of Wiener spaces*
M. Miranda Jr, University of Ferrara
12:35 – 13:00 *Surface measures and integration by parts formula on levels sets induced by functionals of the Brownian motion in* R^n
M. Zanella, LUISS Guido Carli

Session S18D2 *Reliability, stochastic dependence and differential games* (Room D)
Chair: M. Longobardi, University of Napoli Federico II

11:45 – 12:10 *Role of multivariate conditional hazard rates in the analysis of non-transitivity and aggregation/marginalization paradoxes for vectors of non-negative random variables*
F. Spizzichino, Sapienza University
12:10 – 12:35 *ROCOF of higher order for continuous time semi-Markov systems*
G. D'Amico, University of University of Chieti-Pescara
12:35 – 13:00 *Nonzero-sum stochastic differential games between an impulse controller and a stopper*
D. De Santis, London School of Economics

Session S18C2 *KPZ and new universality* (Room C)
Organizer/Chair: A. Occelli, Bonn University

11:45 – 12:10 *Stationary half-space last passage percolation*
A. Occelli, Bonn University
12:10 – 12:35 *The finite temperature Plancherel measure and process*
D. Betea, Bonn University
12:35 – 13:00 *Hard shocks in (T)ASEP*
P. Nejjar, Institute of Science and Technology, Klosterneuburg

Tuesday 18th June

13:00 – 14:30 Lunch

Session S18F3 *Methods for stochastic filtering and optimal control of processes with jumps* (Room F)
Organizers/Chairs: E. Bandini, University of Milano Bicocca
A. Calvia, University of Milano Bicocca

14:30 – 14:55 *Stochastic filtering of a pure jump process with jump-diffusion observation and path-dependent local characteristics*
K. Colaneri, University of Leeds
14:55 – 15:20 *Optimal control of stochastic processes with jumps: a backward stochastic differential equations approach*
F. Confortola, Polytechnic University of Milano
15:20 – 15:45 *Optimal reduction of public debt under partial observation of the economic growth*
G. Ferrari, Bielefeld University

Session S18E3 *Analysis in Wiener spaces 2* (Room E)
Organizer/Chair: S. Bonaccorsi, University of Trento

14:30 – 14:55 *Analyticity of nonsymmetric Ornstein-Uhlenbeck semigroup with respect to a weighted Gaussian measure*
D. Addona, University of Milano Bicocca
14:55 – 15:20 *Absolute continuity and Fokker-Planck equation for the law of Wong-Zakai approximations of Ito SDEs*
A. Lanconelli, University of Bologna
15:20 – 15:45 *An infinite dimensional Gaussian random matching problem*
D. Trevisan, University of Pisa

Session S18D3 *Random dynamical systems and related problems* (Room D)
Chair: T. Vargiolu, University of Padova

14:30 – 14:55 *Hilbert modules in probability*
M. Skeide, University of Molise
14:55 – 15:20 *Optimal stopping of the exponential of a Brownian bridge*
A. Milazzo, Imperial College London
15:20 – 15:45 *Optimal installation of solar panels: a two-dimensional singular control problem*
T. Vargiolu, University of Padova

Session S18C3 *Chemical reaction networks* (Room C)
Organizer/Chair: E. Bibbona, Polytechnic University of Torino

14:30 – 14:55 *An introduction to chemical reaction network models*
E. Bibbona, Polytechnic University of Torino
14:55 – 15:20 *Stationary distributions for biochemical reaction networks*
D. Cappelletti, ETH Zurich
15:20 – 15:45 *Large deviations for chemical reaction networks*
A. Agazzi, Duke University, Durham

Tuesday 18th June

15:45 – 16:15	Coffee break

Session S18F4 *Stochastic models for opinion dynamics* (Room F)
Organizer/Chair: I.G. Minelli, University of L'Aquila

16:15 – 16:40 *Opinion dynamics with Lotka-Volterra type interactions*
M. Aleandri, LUISS Guido Carli
16:40 – 17:05 *Synchronization in interacting stochastic systems with individual and collective reinforcement*
P.-Y. Louis, University of Poitiers
17:05 – 17:30 *Opinion dynamics in random networks evolving via preferential attachment*
I.G. Minelli, University of L'Aquila

Session S18C4 *Spunti didattici e formativi di probabilità e statistica per le scuole secondarie* (Room C)
Organizer/Chair: A. Buonocore, University of Napoli Federico II

16:15 – 16:40 *Il laboratorio di calcolo combinatorio e probabilità nell'ambito del Piano Lauree Scientifiche*
A. Buonocore, University of Napoli Federico II
16:40 – 17:05 *Il laboratorio di statistica per l'Alternanza Scuola-Lavoro e per il Piano Lauree Scientifiche*
A. Di Crescenzo, University of Salerno
17:05 – 17:30 *Approccio soggettivista alla probabilità per la formazione degli insegnanti*
M. Mellone, University of Napoli Federico II

16:15 – 17:30 **Poster Session** (Room E)

- G. Albano	*Inferring time non-homogeneous Ornstein Uhlenbeck type stochastic process*
- G. Ascione	*On the exit time from open sets of some semi-Markov processes*
- F. Buono	*Generalized reversed aging intensity functions*
- C. Calì	*Distorted representations and comparison results for inactivity times of systems under double monitoring*
- N. Cangiotti	*Notes on the Ogawa integrability and a condition for convergence in the multidimensional case*
- D. Conte	*On theta-methods for stochastic Volterra integral equations*
- B. Martinucci	*On the elastic telegraph process*
- A. Meoli	*Finite velocity random motions with jumps governed by an alternating fractional Poisson process*
- L. Paolillo	*Residual varentropy of random lifetimes*
- P. Paraggio	*Birth-death and diffusion processes to model the logistic growth: analysis and comparisons*
- E. Pirozzi	*On a fractional Ornstein-Uhlenbeck process with stochastic forcing and its applications*
- P. Siri	*Minimization of the Kullback-Leibler divergence over a log-normal exponential arc*
- S. Spina	*Random denials in rumor spreading models*
- F. Torres-Ruiz	*A diffusion process related to a Gompertz curve with multiple inflection points*
- F. Travaglino	*Brownian motion governed by the telegraph process in stochastic modeling of the inflation and deflation episodes of Campi Flegrei*
- J. van Oostrum	*Wasserstein geometry on Gaussian densities with trace one covariance matrix*
- A. Zass	*Existence of Gibbians fields via entropy methods*

19:30 – 22:30	***Networking Event***

Wednesday 19th June

<u>Session S19F1</u> *<u>Stochastic processes with interaction: random environment and particle systems</u>* (Room F)
Organizer/Chair: L. Andreis, Weierstrass Institute, Berlin

9:00 – 9:25 *Random walk in a non-integrable random scenery time*
A. Bianchi, University of Padova
9:25 – 9:50 *Hydrodynamics and duality in dynamic random environment*
F. Sau, Delft University of Technology
9:50 – 10:15 *Interacting particle systems from a duality point of view*
C. Franceschini, IST - Universidade de Lisboa

<u>Session S19E1</u> *<u>Stochastic games and their applications: N-player games</u>* (Room E)
Organizers/Chairs: L. Campi, London School of Economics
T. De Angelis, University of Leeds
G. Ferrari, Bielefeld University

9:00 – 9:25 *Market manipulation of a producer: a game-theoretic perspective*
L. Campi, London School of Economics
9:25 – 9:50 *Dynkin games with incomplete and asymmetric information*
T. De Angelis, University of Leeds
9:50 – 10:15 *Nonzero-sum submodular monotone-follower games: Existence and approximation of Nash equilibria*
J. Dianetti, Bielefeld University

<u>Session S19D1</u> *<u>Probabilistic models in non-equilibrium statistical mechanics and applications</u>* (Room D)
Organizer/Chair: G. Bet, University of Firenze

9:00 – 9:25 *Hitting time asymptotics for hard-core interactions on bipartite graphs*
F. R. Nardi, University of Firenze
9:25 – 9:50 *Queue-based activation protocols for random-access wireless networks with bipartite interference graphs*
M. Sfragara, Leiden University
9:50 – 10:15 *Gaussian mean-field lattice gas*
A. Troiani, University of Padova

<u>Session S19C1</u> *<u>First-passage times and stochastic Langevin equations</u>* (Room C)
Chair: A. Pascucci, University of Bologna

9:00 – 9:25 *Asymptotic Results for first-passage times of some exponential processes*
C. Macci, University of Roma Tor Vergata
9:25 – 9:50 *Joint distribution of first-passage time and first-passage area of certain Lévy processes*
M. Abundo, University of Roma Tor Vergata
9:50 – 10:15 *On stochastic Langevin and Fokker-Planck equations*
A. Pesce, University of Bologna

Wednesday 19th June

10:15 - 11:10 **Plenary Talk** (Room F)
Bootstrap percolation and kinetically constrained particle systems: critical time scales
Cristina Toninelli, Université Paris Dauphine
Chair: P. Dai Pra, University of Padova

11:10 - 11:45	Coffee break

Session S19F2 *Phase transition and particle systems* (Room F)
Organizer/Chair: F. Sau, Delft University of Technology

11:45 - 12:10 *Coagulating particles and gelation phase transition: a large-deviation approach*
L. Andreis, Weierstrass Institute, Berlin
12:10 - 12:35 *Emergence of periodic behavior in complex systems*
M. Formentin, University of Padova
12:35 - 13:00 *Sticky Brownian motion as scaling limit of the inclusion process*
C. Giardinà, University of Modena and Reggio Emilia

Session S19E2 *Stochastic games and their applications: mean-field games* (Room E)
Organizers/Chairs: L. Campi, London School of Economics
T. De Angelis, University of Leeds
G. Ferrari, Bielefeld University

11:45 - 12:10 *Nonzero-sum stochastic games with impulse controls*
M. Basei, University of California, Berkeley
12:10 - 12:35 *On the convergence problem in mean field games: a two state model without uniqueness*
A. Cecchin, Université Nice Sophia Antipolis,
12:35 - 13:00 *N-player games and mean-field games with smooth dependence on past absorptions*
M. Ghio, Scuola Normale Superiore of Pisa

Session S19D2 *Stochastic models for complex systems: non-Markovian dynamics and limit theorems* (Room D)
Organizer/Chair: B. Toaldo, University of Torino

11:45 - 12:10 *Piecewise linear processes with Poisson-modulated switching times and market models*
N. Ratanov, Universidad del Rosario
12:10 - 12:35 *Delayed and rushed motions*
M. D'Ovidio, Sapienza University, Roma
12:35 - 13:00 *Limit theorems for the non-homogeneous fractional Poisson process*
E. Scalas, University of Sussex

Session S19C2 *Optimal transport methods for empirical processes and Bayesian stability* (Room C)
Organizers/Chairs: E. Mainini, University of Genova
G. Conforti, École Polytechnique, Palaiseau
E. Dolera, University of Pavia

11:45 - 12:10 *Ergodic results for a mean field Schrödinger problem*
G. Conforti, École Polytechnique, Palaiseau
12:10 - 12:35 *Optimal rates of mean Glivenko-Cantelli convergence*
E. Dolera, University of Pavia
12:35 - 13:00 *Lipschitz continuity of probability kernels and applications to Bayesian inference*
E. Mainini, University of Genova

Wednesday 19th June

13:00 - 14:30	Lunch

Session S19F3 *Interacting random walks in statistical mechanics* (Room F)
Organizer/Chair: N. Torri, Université Paris-Est Créteil, Créteil

14:30 – 14:55 *Localization for directed polymers in random (heavy-tail) environment*
Q. Berger, Sorbonne Université, Paris
14:55 - 15:20 *Sub-ballistic random walks among biased random conductances in one dimension: a story of wells and walls*
M. Salvi, Ecole Polytechnique, France
15:20 - 15:45 *'Essential enhancements' for activated random walks*
L. Taggi, Weierstrass Institute, Berlin

Session S19E3 *Advances in stochastic control and optimal stopping with applications* (Room E)
Chair: T. De Angelis, University of Leeds

14:30 - 14:55 *On a class of infinite-dimensional singular stochastic control problems*
S. Federico, University of Siena
14:55 - 15:20 *Tail optimality and preferences consistency for stochastic optimal control problems*
E. Vigna, University of Torino
15:20 - 15:45 *Analytical valuation of surrender options in life insurance contracts with minimum guaranteed*
G. Stabile, Sapienza University, Roma

Session S19D3 *Probability and non-local operators: anomalous diffusive dynamics* (Room D)
Organizer/Chair: F. Polito, University of Torino

14:30 - 14:55 *Probabilistic representations of the Barenblatt-type solutions, diffusion equations and fractional operators*
A. De Gregorio, Sapienza University, Roma
14:55 - 15:20 *Fractional derivatives of a function with respect to another function: applications to Dodson and relativistic diffusions*
R. Garra, Sapienza University, Roma
15:20 - 15:45 *Tempered fractional derivatives and related drifted Brownian motions*
F. Iafrate, Sapienza University, Roma

Session S19C3 *Theoretical aspects of probability and applications* (Room C)
Chair: C. Sempi, University of Salento

14:30 - 14:55 *Logical operations among conditional events: theretical aspects and applications*
G. Sanfilippo, University of Palermo
14:55 - 15:20 *Coherent upper conditional expectation defined by Hausdorff outer measure*
S. Doria, University of Chieti-Pescara
15:20 - 15:45 *How to interpret probability using a purely mathematical approach*
P. Rocchi, IBM and LUISS Guido Carli

Wednesday 19th June

15:45 - 16:15	Coffee break

Session S19F4 *Stochastic systems with interaction* (Room F)
Organizers/Chairs: C. Orrieri, University of Trento
L. Scarpa, University of Vienna

16:15 - 16:40 *McKean-Vlasov stochastic control and Hamilton-Jacobi-Bellman equations on Wasserstein space*
A. Cosso, University of Bologna
16:40 - 17:05 *The convergence problem for finite state mean field games*
G. Pelino, University of Padova
17:05 - 17:30 *A mean-field model with discontinuous coefficients and spatial interaction*
G. Zanco, LUISS Guido Carli

Session S19E4 *Quantum probability and applications* (Room E)
Organizer: Y.G. Lu, University of Bari, Italy
Chair: E. Sasso, University of Genova, Italy

16:15 - 16:40 *Dephasing, decoherence and classical stochastic processes arising in quantum theory*
F. Fagnola, Polytechnic University of Milano
16:40 - 17:05 *Uniquely ergodic C*-dynamical systems for the noncommutative 2-torus and uniform convergence of Cesaro averages*
F. Fidaleo, University of Roma Tor Vergata
17:05 - 17:30 *The role of the atomic decoherence-free subalgebra in the study of Quantum Markov Semigroups*
V. Umanità, University of Genova

Session S19D4 *Probability and non-local operators: non-Markovian and time-changed processes* (Room D)
Organizer/Chair: L. Beghin, Sapienza University, Roma

16:15 - 16:40 *Subordinated fractional Poisson processes*
A. Maheshwari, Indian Institute of Management Indore
16:40 - 17:05 *From linear superposition of Langevin-driven Brownian particles to the fractional Brownian motion*
G. Pagnini, Basque Center for Applied Mathematics
17:05 - 17:30 *On discrete-time semi-Markov processes*
C. Ricciuti, Sapienza University, Roma

Session S19C4 *Enlargement of filtrations and financial applications* (Room C)
Organizers/Chairs: B. D'Auria, Madrid University Carlos III
C. Fontana, University of Padova

16:15 - 16:40 *Progressive enlargement of filtrations by the reference filtration of a general semi-martingale: results and applications*
B. Torti, University of Roma Tor Vergata
16:40 - 17:05 *Risk measures and progressive enlargement of filtrations: a BSDE approach*
A. Calvia, University of Milano Bicocca
17:05 - 17:30 *Optimal liquidation time of a stock in presence of insider information*
B. D'Auria, Madrid University Carlos III

18:00 - 18:45	**Meeting "Towards the future"**	(Room F)

Thursday 20th June

Session S20F1 *Advances in stochastic analysis* (Room F)
Chair: E.Priola, University of Pavia

9:00 - 9:25 *On the two-dimensional KPZ and Stochastic Heat Equation*
F. Caravenna, University of Milano Bicocca
9:25 - 9:50 *Quasi-tensor algebra and rough path theory*
C. Bellingeri, Sorbonne Université, Paris
9:50 - 10:15 *On the Onsager-Machlup functional for Brownian motion on the Heisenberg group*
M. Carfagnini, University of Connecticut, United States

Session S20E1 *Approximate Bayesian Computation (ABC)* (Room E)
Organizer/Chair: M. Tamborrino, Johannes Kepler University, Linz

9:00 - 9:25 *Spectral density-based and measure-preserving ABC for partially observed diffusion processes*
M. Tamborrino, Johannes Kepler University, Linz
9:25 - 9:50 *Approximate Bayesian conditional copula*
C. Grazian, University of Chieti-Pescara
9:50 - 10:15 *Variance reduction for fast ABC using resampling*
U. Picchini, Chalmers University of Technology and the University of Gothenburg

Session S20D1 *Stochastic processes with applications to statistical mechanics* (Room D)
Organizer/Chair: M. Gianfelice, Calabria University

9:00 - 9:25 *Stochastic Ising model with temperature fast decreasing to zero*
E. De Santis, Sapienza University, Roma
9:25 - 9:50 *Uniform bound of the entanglement for the ground state of the quantum Ising model with large transverse magnetic field*
M. Gianfelice, Calabria University
9:50 - 10:15 *Ornstein-Zernike behaviour for the correlation functions of the ground state of the quantum Ising model with transverse magnetic field*
M. Campanino, University of Bologna

Session S20C1 *Stochastic methods in neuroscience* (Room C)
Organizer/Chair: G. D'Onofrio, University of Torino

9:00 - 9:25 *Examining whether brain types typical of males are typical of women, and vice versa*
I. Meilijson, Tel Aviv University
9:25 - 9:50 *Spatio-temporal spike pattern detection in experimental parallel spike trains using SPADE*
A. Stella, Jülich Research Center, Germany
9:50 - 10:15 *The first-passage time properties of diffusion neuronal models with multiplicative noise*
G. D'Onofrio, University of Torino

Thursday 20th June

10:15 - 11:10 **Plenary Talk** (Room F)
On regularized estimation for stochastic differential equations
Stefano M. Iacus, University of Milano
Chair: L. Sacerdote, University of Torino

11:10 - 11:45	Coffee break

Session S20F2 *Stochastic differential equations* (Room F)
Chair: F. Caravenna, University of Milano Bicocca

11:45 - 12:10 *On SDEs with additive noise driven by stable processes*
E. Priola, University of Pavia
12:10 - 12:35 *On the Itô-Alekseev-Gröbner formula for stochastic differential equations*
S. Mazzonetto, University of Potsdam
12:35 - 13:00 *Weak well-posedness for some degenerate SDEs driven by stable processes*
L. Marino, University of Évry Val d'Essonne, Évry

Session S20E2 *Diffusions and their first passage times* (Room E)
Organizer/Chair: C. Zucca, University of Torino

11:45 - 12:10 *Exact simulation of the first passage time of diffusions*
S. Herrmann, Institut de Mathématiques de Bourgogne, Dijon
12:10 - 12:35 *A stochastic algorithm based on the approximation of hitting times for the initial-boundary value problem for the heat equation*
M. Deaconu, INRIA Nancy & IECL, France
12:35 - 13:00 *Inverse first passage time for some two-dimensional diffusion processes*
C. Zucca, University of Torino

Session S20D2 *Fractional stochastic models* (Room D)
Organizer/Chair: E. Pirozzi, University of Napoli Federico II

11:45 - 12:10 *Fractional Pearson diffusions and continuous time random walks*
N. Leonenko, Cardiff University, Wales
12:10 - 12:35 *The non local diffusion equation and the aggregation of Brownian motion*
B. Toaldo, University of Torino
12:35 - 13:00 *SPDEs with fractional noise in space: continuity in law with respect to the Hurst index*
L.M. Giordano, University of Milano

Session S20C2 *Stochastic fluid dynamics* (Room C)
Organizer/Chair: M. Zanella, LUISS Guido Carli

11:45 - 12:10 *Mean field limit of interacting filaments for 3D Euler equations*
M. Coghi, Weierstrass Institute, Berlin
12:10 - 12:35 *The Vlasov-Fokker-Planck-Navier-Stokes system as a scaling limit of particles in a fluid*
M. Leocata, University of Lyon
12:35 - 13:00 *Existence of nonnegative vortex sheets for 2D stochastic Euler equations*
M. Maurelli, University of Milano

13:00 - 14:30	Lunch
14:30 - 14:45	Closing

Contents

On regularized estimation for stochastic differential equations

PLENARY TALK

S.M. Iacus

Department of Economics, Management and Quantitative Methods, University of Milan, Via Conservatorio 7 - 20123 Milano

In this talk we will review recent results on regularization methods for the parametric estimation for high-dimensional dynamical systems with small noise and also for high-dimensional ergodic diffusions with discrete observations. Regularized estimation is at the same time a dimensionality reduction technique and model selection tool as well as an efficient estimation method especially in the case of high-dimensional systems which admit a sparse representation. Indeed, in such situation, variable selection becomes particularly important when it comes to correctly identify significant predictors that will improve the forecasting performance of the fitted model. To exemplify the idea, the LASSO method [6] for the linear regression model

$$Y = \theta_0 + \theta_1 X_1 + \theta_2 X_2 + \cdots + \theta_k X_k + \epsilon$$

consists in deriving the least squares estimator under L^1 constraints, i.e.

$$\hat{\theta} = \arg\min_{\theta} \left\{ (Y - \theta X)^T (Y - \theta X) + \sum_{i=1}^{k} |\theta_i| \right\}$$

and model selection occurs when some of the θ_i are estimated as zeros. In our talk we will consider regularized estimation from discrete observations of the multi-dimensional ergodic diffusion process

$$dX_t = b(\alpha, X_t)dt + \sigma(\beta, X_t)dW_t$$

with $\alpha = (\alpha_1, ..., \alpha_p)' \in \Theta_p \subset R^p$ and $\beta = (\beta_1, ..., \beta_q)' \in \Theta_q \subset R^q$, $p, q \geq 1$ and adaptive L^γ,$\gamma \geq 1$, penalty (see [2,4,5]). We will also present the case of adaptive L^γ penalties, $\gamma > 0$, for the dynamical system with small noise

$$dX_t = S_t(\theta, X)dt + \epsilon dW_t, \quad \epsilon \to 0,$$

with high-dimensional parameter vector θ (see [1]). For both models we will show the oracle properties and asymptotic efficiency. Numerical analysis will also be presented through the Yuima R package [3].
We will later extend these analyses to the more general case of Adaptive Elastic Net estimation [7,8], which generalises the LASSO method to a least square estimation method with both L^1 and L^2 penalties in the same objective function. We will show the advantages of this approach in the case when the dimension of the parameter space diverges with the sample size. This is an on-going joint work with A. De Gregorio and N. Yoshida.

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Intrinsic volumes of convex bodies and cones: concentration, limit theorems and sparse recovery

PLENARY TALK

G. Peccati

RMATH, Luxembourg University

Intrinsic volumes of convex sets are natural geometric quantities that also play important roles in applications, such as linear inverse problems with convex constraints, and constrained statistical inference. It is a well-known fact that, given a convex body or a closed convex cone in a d-dimensional Euclidean space, then its (possibly normalized) intrinsic volumes determine a probability measure on the finite set $\{0, 1, ..., d\}$. The aim of this talk is to show how one can use probabilistic and information-theoretic methods to characterise the concentration properties and fluctuations (in the high-dimensional limit) of such a probability measure. In the framework of closed convex cones, our results can be used in order to deduce the Gaussian nature of sharp phase transitions phenomena observed in many regularised linear inverse problems with convex constraints, one prominent example being compressed sensing.

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Bootstrap percolation and kinetically constrained particle systems: critical time scales

PLENARY TALK

C. Toninelli

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Recent years have seen a great deal of progress in understanding the behavior of bootstrap percolation models, a particular class of monotone cellular automata. In the two dimensional lattice there is now a quite complete understanding of their evolution starting from a random initial condition, with a universality picture for their critical behavior. Much less is known for their non-monotone stochastic counterpart, namely kinetically constrained models (KCM). In KCM each vertex is resampled (independently) at rate one by tossing a p-coin iff it can be infected in the next step by the bootstrap model. In particular infection can also heal, hence the non-monotonicity. Besides the connection with bootstrap percolation, KCM have an interest in their own: when $p \to 0$ they display some of the most striking features of the liquid/glass transition, a major and still largely open problem in condensed matter physics.

I will discuss some recent results on the characteristic time scales of KCM as $p \to 0$ and the connection with the critical behavior of the corresponding bootstrap models.

A brief personal history of stochastic partial differential equations

PLENARY TALK

L. Zambotti

Sorbonne Université, Paris

Stochastic partial differential equations were invented in the late 60's and developed over the following decades by a heterogeneous and growing community. Starting from the mid 70's, Italy has been one of the main laboratories of this topic, which mixes probability theory and partial differential equations, and has important motivations in theoretical physics. I want to give a survey of the history of this collective endeavour from a personal point of view, concentrating on a few equations that I know best, from the beginnings of the theory to the more recent spectacular successes.

Joint distribution of first-passage time and first-passage area of certain Lévy processes

M. Abundo and S. Furia

Dipartimento di Matematica, Università "Tor Vergata", Roma

This is a continuation of the papers [1] and [2]; actually, in [1] we studied the distribution of the first-passage area (FPA) $A(x) = \int_0^{\tau(x)} X(t)dt$, swept out by a one-dimensional jump-diffusion process $X(t)$, starting from $x > 0$, till its first-passage time (FPT) $\tau(x)$ below zero, while in [2] we examined the special case (without jumps) when $X(t)$ is Brownian motion (BM) B_t with negative drift $-\mu$, that is, $X(t) = x - \mu t + B_t$, studying in particular the joint distribution of $\tau(x)$ and $A(x)$. Here, we investigate the joint distributions of $\tau(x)$ and $A(x)$, in the case when $X(t)$ is a Lévy process of the form

$$X(t) = x - \mu t + \sigma B_t - N_t, \ x > 0, \tag{1}$$

where $\mu \geq 0, \ \sigma \geq 0, \ B_t$ is standard BM, and N_t is a homogeneous Poisson process with intensity $\theta > 0$, starting from zero, and independent of B_t; thus, $X(t)$ turns out to be the superposition of drifted BM and Poisson process.

Referring to the Lévy process (1), we state and solve differential-difference equations for the Laplace transform of the two-dimensional random variable $(\tau(x), A(x))$. In particular, when $\mu = \sigma = 0$, we obtain the joint moments $E[\tau(x)^m A(x)^n]$ of the FPT and FPA, and we present an algorithm to find them recursively, for any m and n; moreover, we find the expected value of the time average of $X(t)$ till its FPT below zero.

Studying the FPA of a process $X(t)$ such as (1) is peculiar when modeling the evolution of certain random systems described by the superposition of a continuous stochastic process and a jump process (see references in [1]); these arise e.g. in solar physics studies, non-oriented animal movement patterns, and DNA breathing dynamics, as regards systems where the jump component can be absent (see e.g. [5], and references in [6]). Applications can be found in Queueing Theory, in the case without jumps, i.e. $\theta = 0$, if one identifies $X(t)$ with the length of a queue at time t, and $\tau(x)$ with the busy period, that is the time until the queue is first empty; then, $A(x)$ represents the cumulative waiting time experienced by all the "customers" during a busy period; further applications exist in Finance, in the framework of default-at-maturity model, which assumes the exchange rate follows a jump-diffusion process (see e.g. [3], [4]); for other examples from Economics and Biology, see e.g. [1] and references therein.

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Analyticity of nonsymmetric Ornstein–Uhlenbeck semigroup with respect to a weighted Gaussian measure

D. Addona

Università di Milano Bicocca

In my talk I show that the realization in $\mathrm{L}^p(X, \nu_\infty)$ of the nonsymmetric Ornstein-Uhlenbeck operator L is sectorial for any $p \in (1, +\infty)$ and I provide an explicit sector of analyticity. Here $(X, \mu_\infty, H_\infty)$ is an abstract Wiener space, i.e., X is a separable Banach space, μ_∞ is a centred non degenerate Gaussian measure on X and H_∞ is the associated Cameron-Martin space. Further, ν_∞ is a weighted Gaussian measure, that is, $\nu_\infty = e^{-U}\mu_\infty$ where U is a convex function which satisfies some minimal conditions.

Large deviations for chemical reaction networks

A. Agazzi[ab], A. Dembo[b] and J.-P. Eckmann[a]

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[b] *Stanford University, Department of Statistics*

At the microscopic level, the dynamics of networks of chemical reactions can be modeled as jump Markov processes. The rates of these processes are in general neither uniformly Lipschitz continuous nor bounded away from zero, obstructing the straightforward application of large deviation theory to this framework. We bypass these issues by respectively applying tools of Lyapunov stability theory and recent results on interacting particle systems. This way, we characterize a class of processes obeying a LDP in path space, and extend the latter to infinite time intervals through Wentzell-Freidlin (W-F) theory. Finally, we provide natural sufficient topological conditions on the network of reactions for the applicability of our LDP and W-F results. These conditions can be checked algorithmically.

Inferring time non-homogeneous Ornstein Uhlenbeck type stochastic process

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Let $\{X(t),\ t \in [t_0, T]\}$ be a stochastic diffusion process defined in $\mathbb{R}$ and described via the following SDE:

$$dX(t) = [-aX(t) + b(t)]dt + \sigma(t)dB(t), \qquad (2)$$
$$\mathbb{P}[X(0) = x_0] = 1$$

where $a, x_0 \in \mathbb{R}$, $b(\cdot)$ and $\sigma(\cdot)$ are continuous deterministic functions with $\sigma(t) > 0\ \forall t \geq 0$.
Eq. (2) consists in a generalization of the classical Ornstein Uhlenbeck diffusion process in which the infinitesimal moments include the time dependent functions $b(t)$ and $\sigma(t)$.
The paper provides the inference for the process (2) based on d sample paths for the times t_{ij}, with $i = 1, \dots, d$ and $j = 1, \dots, n$ and we assume that $t_{i1} = t_1$ for $i = 1, \dots, d$.
The proposed methodology is essentially based on an iterative procedure in which in each step the classical maximum likelihood (ML) estimation and a generalized method of moments (GMM) are combined. Moreover, a primary interpolation of the suitable tranformations of the data is made. The procedure runs until a fixed precision level is obtained. The novelty of the proposed procedure lies in the fact that the algorithm does not make any assumption on the functional form of the unknown functions, neither informations drawn from other time series such as control groups.
Further, the consistence of the proposed estimator naturally derives from the consistence of the ML and GMM estimators in addition to the uniform convergence of the interpolation method (for example cubic spline interpolation). So, when the number of data points increases the proposed estimate becomes better and better.
Two simulation experiments are performed to illustrate the validity of the proposed procedure. The first experiment considers the homogeneous case, i.e. $b(\cdot)$ and $\sigma(\cdot)$ are both constant, so a simple MLE can be adopted to estimate the parameters of the model. We compare the results, in terms of absolute errors, obtained by using our procedure with those ones obtained by using the MLE implemented in the R-package *sde*. We find that our procedure shows performances of the same order of the MLE, without the a priori assumption of the constancy of the parameteres in (2).
The second simulation experiment is aimed to show the validity of the iterative procedure in the more general time-non-homogeneous case. Several functional forms for $b(\cdot)$ and $\sigma^2(\cdot)$ are considered and the results show very good performances in terms of absolute errors.
Finally, an application to CO_2 emissions in Morocco is presented, corresponding to the period 1980-2005 considered in [4]. Also in this case, the fitted values via the proposed procedure appear to be very closed to the real ones and they satisfactory capture the trend of the CO_2 emissions in Morocco better then MLE.

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Opinion dynamics with Lotka-Volterra type interactions

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According to social scientists (see, e.g., [2], [3], [4]), two fundamental characteristics in opinion formation are social influence, i.e. the tendency of each individual to adjust her opinion to the one of her neighbours, and homophily, i.e. the tendency to interact more frequently with individuals who are more similar. In dichotomic models, opinions are binary and social influence is usually described in terms of an attractive interaction between agents. In this work we study a dichotomic model in which we introduce an inhomogeneity in order to describe homophily; we divide the population into two interacting families of individuals. Each family has an intrinsic mean field ”Voter-like” dynamics [5], which is influenced by interaction with the other family. The interaction parameters describe a cooperative/conformist or competitive/nonconformist attitude of one family with respect to the other.
We prove chaos propagation, i.e., we show that on any time interval $[0, T]$, as the size of the population goes to infinity, each individual behaves independently of the others with transition rates driven by a macroscopic equation. We focus in particular on models with cooperative vs. competitive families, where, although the microscopic system is driven a.s. to *polarization* (that is, as time goes to infinity, the system converges to a configuration where all the individuals in the same family have the same opinion), a periodic behaviour arises in the macroscopic scale.
In order to describe fluctuations between the limiting periodic orbits, analogously to [8], we identify a slow variable in the microscopic system and, through an averaging principle, we find a diffusion which describes the macroscopic dynamics of such variable on a larger time scale. This talk is based on joint work with Ida Germana Minelli [1].

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Coagulating particles and gelation phase transition: a large-deviation approach

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At least since the days of Smoluchowski, there is a desire to understand the behaviour of large particle systems that undergo chemical reactions of coagulation type. One of the phenomena that attracts much attention is the question for the existence of a phase transition of gelation type, i.e., the appearance of a particle of macroscopic size in the system. In this talk, we consider the (non-spatial) coagulating model (sometimes called the Marcus-Lushnikov model), starting with N particles with mass one each, where each two particles coagulate after independent exponentially distributed times that depend on a given coagulation kernel, function of the two masses. In [1] we focus on the case in which the corresponding coagulation kernel is multiplicative in the two masses, hence the process is identified as the multiplicative coagulation process. This case is of particular interest also for its strong relations with the time dependent Erdős-Rényi random graph. We work for fixed time $t > 0$ and derive, for the number N of initial particles going to infinity, a joint large-deviation principle for all relevant quantities in the system (microscopic, mesoscopic and macro- scopic particle sizes) with an explicit rate function. We deduce laws of large numbers and in particular derive from that the well-known phase transition at time $t = 1$, the time at which a macroscopic particle (the so-called gel) appears, as well as the Smoluchowski characterisation of the statistics of the finite-sized particles. We discuss also current ongoing work on extending this approach to include systems with more general coagulation kernels and systems where particles are provided with a spatial position.

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On the exit time from open sets of some Semi-Markov processes

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Here, we will expose some results on exit times from an arbitrary open set for a class of semi-Markov processes obtained as time-changed Markov process. We estimate the asymptotic behaviour of the survival function (for large t) and of the distribution function (for small t) and we provide some conditions for absolute continuity. Moreover, in the particular case of the time-changed Brownian motion with drift via inverse stable subordinator, we deduce a fractional parabolic PDE for the distribution of the first passage time through a constant threshold. Finally, applications of these results will be cited.

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Processing data sets on networks: random forests and other probabilistic tools

L. Avena[a], F. Castell[b], A. Gaudillière[b] and C. Mélot[b]

[a] *Leiden University, Mathematical Institute – Leiden, The Netherlands*
[b] *Aix-Marseille Université, CNRS, Centrale Marseille – Marseille, France.*

Inspired by combinatorial problems from statistical physics, some years ago, with Alexandre Gaudilliere we characterized properties of certain random spanning forests on arbitrary weighted finite graphs. These objects are related to fundamental algebraic and probabilistic structures of the given weighted graph (or of the associated adjacency matrix). Further more, such random spanning forests can be efficiently sampled by running proper (loop-erased) random walk explorations of the graph.
These studies branched into questions of different nature and led to a number of applications within the analysis of real-world networks. The core of this talk is on three main such applications obtained recently with Gaudilliere, Fabienne Castell and Clothilde Melot:

1. a procedure for downsampling well-distributed nodes/vertices,
2. coarse-graining or renormalization schemes for graphs and processes,
3. pyramidal wavelets-like algorithms to process and compress signals (real-valued functions) defined on the vertex set of a graph.

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BSDEs driven by general random measures and optimal control for piecewise deterministic Markov processes

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We consider an optimal control problem for piecewise deterministic Markov processes (PDMP) on a bounded state space. Here a pair of controls acts continuously on the deterministic flow and on the transition measure describing the jump dynamics of the process. For this class of control problems, the value function can be characterized as the unique viscosity solution to the corresponding integro-differential Hamilton-Jacobi-Bellman equation with a non-local type boundary condition. We are able to provide a probabilistic representation for the value function in terms of a suitable backward stochastic differential equation, known as nonlinear Feynman-Kac formula. The jump mechanism from the boundary entails the presence of predictable jumps in the PDMP dynamics, so that the associated BSDE turns out to be driven by a random measure with predictable jumps. Existence and uniqueness results for such a class of equations are non-trivial and are related to recent works on well-posedness for BSDEs driven by non quasi-left-continuous random measures.

Nonzero-sum stochastic games with impulse controls

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We consider a general class of nonzero-sum N-player stochastic games with impulse controls, where players control the underlying dynamics with discrete interventions. We adopt a verification approach and provide sufficient conditions for the Nash equilibria (NEs) of the game. We then consider the limit situation of $N \to \infty$, that is, a suitable mean-field game (MFG) with impulse controls. We show that under appropriate technical conditions, the MFG is an ϵ-NE approximation to the N-player game, with $\epsilon = O\left(\frac{1}{\sqrt{N}}\right)$. As an example, we analyze in details a class of stochastic games which extends the classical cash management problem to the game setting. In particular, we characterize the NEs for its two-player case and compare the results to the single-player case, showing the impact of competition on the player's optimal strategy, with sensitivity analysis of the model parameters.

Clustering structure for species sampling sequences with general base measure

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Species sampling sequences with general base measure are exchangeable sequences $(\xi_n)_n$ with directing measure given by

$$P = \sum_{j\geq 1} p_j \delta_{Z_j},$$

where $(Z_j)_{j\geq 1}$ are i.i.d. random variables taking values in a Polish space with common distribution H (not necessarily diffuse) and $(p_j)_{j\geq 1}$ are random positive weights in [0, 1] independent from $(Z_j)_{j\geq 1}$. The clustering properties of these sequences are interesting for Bayesian nonparametrics applications, where mixed base measures are used, for example, to accommodate sharp hypotheses in regression problems and provide sparsity.

Motivated by the recent interest in species sampling models with spike and slab base measure, we discuss some relevant properties of random partitions induced by species sampling sequences with general base measure, thus generalizing both [3] and [5].

We prove a stochastic representation for $(\xi_n)_n$ with a general base measure in terms of a latent exchangeable random partition Π. When H is diffuse Π is the partition induced by $(\xi_n)_n$, see [4], while if H has atoms, Π is strictly finer than the partition induced by $(\xi_n)_n$.

We provide explicit expression of the *Exchangeable Partition Probability Function* (EPPF) of the partition generated by by $(\xi_n)_n$ in terms of the EPPF of Π. Finally, we investigate the asymptotic behaviour of the total number of blocks and of the number of blocks with fixed cardinality of the partition generated by $(\xi_n)_n$.

Some applications to hierarchical species sampling models [2] will be also presented.

The talk is based on the paper [1].

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Quasi-tensor algebra and rough path theory

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Starting from the classical example of the iterated integrals of a d-dimensional brownian motion with respect to the Itô and the Stratonovich integration, we will consider the two algebraic operations of shuffle and quasi-shuffle product to introduce the notion of quasi-geometric rough path. Quasi-geometric rough path have approximately the same properties of the geometric rough path but differently from the geometric setting, it is possible to write a non-trivial change of variable formula on them.

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Localization for directed polymers in random (heavy-tail) environment

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The directed polymer model has been introduced more than 30 years ago and has been intensively studied since then. It can be used to describe a polymer (a directed random walk) interacting with the impurities of a heterogeneous medium. The model is known to exhibit a localization phenomenon, the polymer "stretching" to reach more favorable regions of the environment – we refer to reference 3 for a comprehensive overview of the model. However, describing the localized trajectories (super-diffusivity exponent, scaling limits, etc.) is still mostly open. I will present the case of an environment with heavy-tail distribution, where some of these results are at reach.
When the random environment has no second moment, we find explicitly the super-diffusive transversal fluctuations of the polymer: it depends on the tail decay of the environment distribution and on the strength of the coupling between the polymer and the environment. Additionally, we are able to describe the scaling limits of the polymer trajectories, which are expressed as an energy/entropy variational problem.

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An algorithm to construct subsolutions of convex optimal control problems

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We propose an algorithm that produces a non-decreasing sequence of subsolutions for a class of optimal control problems distinguished by the property that the associated Bellman operators preserve convexity. In addition to a theoretical discussion and proofs of convergence, numerical experiments are presented to illustrate the feasibility of the method.

The finite temperature Plancherel measure and process

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The Plancherel measure on partitions, counting standard Young tableaux, has remarkable scaling properties. In the large size limit and upon poissonization, it exhibits (discrete) sine kernel asymptotics in the bulk and Tracy–Widom GUE fluctuations at the edge, both behaviours seen before and considered universal in (continuum) random matrix theory. It is also the one-point distribution of the Hammersley process (polynuclear growth—PNG—in probability parlance) coming from longest increasing subsequences of random permutations.

In this talk, we present a generalization first discussed by Borodin—the so-called "cylindric" or "finite temperature" Plancherel measure. It comes from counting standard Young tableaux of skew shape in the same way the original measure comes from counting tableaux of non-skew shape, and can be viewed as a Hammersley-type process on a cylinder (alternatively, as a Poisson limit of an appropriate cylindric last passage percolation model). Using the theory of Schur measures and fermions in finite temperature, we analyze the edge behavior of this measure and obtain in the fluctuation limit—for the first time in a discrete system—the finite temperature Tracy–Widom GUE distribution. The latter was first obtained by Johansson in (continuum) finite temperature random matrix theory (the so-called Moshe–Neuberger–Shapiro matrix model) and provides an interpolation between two classical extreme statistics distributions: the Tracy–Widom GUE distribution of the Kardar–Parisi–Zhang (KPZ) universality class and the Gumbel distribution of the Edwards–Wilkinson universality class. It also appears in the scaling of finite time "solutions" of the KPZ equation and the scaling of the O'Connell–Yor polymer partition function. It is tempting to conjecture it is universal for systems in finite-temperature much like the Tracy–Widom distribution is universal in zero temperature. The result, joint with Jérémie Bouttier [1], could be viewed as a finite temperature analogue of the Baik–Deift–Johansson theorem on the edge scaling of poissonized Plancherel random partitions and the longest increasing subsequence of random permutations. If time permits, we shall discuss the associated stationary stochastic process, as well as recent progress on other (pfaffian/free boundary) variants, associated limit shapes and bulk limits, and possible representation theoretic connections.

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Random walk in a non-integrable random scenery time

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In this talk we consider a one-dimensional process in random environment, also known in the physical literature as Levy-Lorentz gas. The environment is provided by a renewal point process that can be seen as a set of randomly arranged targets, while the process roughly describes the displacement of a particle moving on the line at constant velocity, and changing direction at the targets position with assigned probability. We investigate the annealed behavior of this process in the case of inter-distances between targets having infinite mean, and establish, under suitable scaling, a functional limit theorem for the process. In particular we show that, contrary to the finite mean case, the behavior of the motion is super-diffusive with explicit scaling limit related to the Kesten-Spitzer process. The key element of the proof is indeed a representation of the consecutive *hitting times on the set of targets* as a suitable random walk in random scenery.

An introduction to chemical reaction network models

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Chemical Reaction Networks are mathematical models of several phenomena in cell biology, epidemiology and other applied fields, where identical individuals (molecules) that belong to different groups (chemical species) interact according to given rules (reactions). We will introduce the basic language of this modeling paradigm together with some toy example. Deterministic and stochastic (Markov chain, MC) model do exist. The relation among them will be discussed. A set of typical mathematical problems that arise in this field will be briefly reviewed, including

- explosions, positive recurrence, extinctions, stationary distributions of the MC, also in relation with the deterministic counterpart
- the approximation of MC with simpler models (diffusion processes or variants, multiscale approximations, . . .)
- large deviations
- statistical inference.

A recent result (joint work with J. Kim, UC, Irvine and C. Wiuf, U. Copenhagen) on the stationary distribution of the so called Togashi–Kaneko model will be introduced if time permits.

Corner growth model, symplectic characters, and KPZ universality

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We present a discrete random interface known as *corner growth model*, considering flat initial conditions. We reformulate it in terms of an interacting particles system (*totally asymmetric simple exclusion process* with alternating initial configuration) and a lattice path model (point-to-line *last passage percolation*). We then derive an exact formula for these equivalent models in terms of representation theoretic functions called *symplectic characters*. Thanks to such a rich algebraic structure, in the large N limit we obtain fluctuations of order cube root of N and the GOE Tracy-Widom distribution from random matrix theory. This central limit theorem (very different from the classical Gaussian one!) permits setting our models in the framework of the *KPZ universality class*.

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Generalized reversed aging intensity functions

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Let X be an absolutely continuous random variable that takes values in $(0, +\infty)$ and let F be its distribution function. The reversed aging intensity function $L^*(x)$ is defined, for $x > 0$, as follows

$$L^*(x) = \frac{-xf(x)}{F(x)\log F(x)} = \frac{-xq(x)}{\log F(x)},$$

where $f(x)$ indicates the density function of X and $q(x)$ is the reversed hazard rate of X. The reversed aging intensity function is defined as the ratio of the instantaneous reversed hazard rate to the average value that it will assume in the future. It analyzes the aging property quantitatively: the smaller the reversed aging intensity, the stronger the tendency of aging. Here, a family of generalized reversed aging intensity functions is introduced and studied. Those functions characterize the distribution functions of univariate positive absolutely continuous random variables.

Keywords: Generalized reversed aging intensity, Reversed hazard rate, Inverse two-parameter Weibull distribution.

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Il laboratorio di calcolo combinatorio e probabilità nell'ambito del Piano Lauree Scientifiche

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Le attività nell'ambito del Piano Lauree Scientifiche hanno una duplice finalità: (a) orientare gli studenti del secondo biennio e, maggiormente, quelli del quinto anno ad effettuare una scelta più consapevole del proprio percorso formativo a carattere universitario; (b) costituire un'iniziativa di formazione per i loro insegnanti.
Per questa comunicazione si è scelto di descrivere l'analisi (con le relative conclusioni) che è stata fatta per la seconda di queste finalità considerando anche il fatto che le attività del *Laboratorio di Calcolo Combinatorio e Probabilità* nella sede di Napoli (Federico II) sono rivolte agli studenti del secondo biennio.
Nell'economia complessiva del rapporto insegnamento/apprendimento risulta necessario tenere conto di un "tacito patto": l'insieme dei comportamenti dell'allievo che sono attesi dall'insegnante e, ovviamente, il suo duale. La sua considerazione, infatti, può permettere all'insegnante di avere alcune chiavi interpretative riguardo alle difficoltà degli studenti. Alcuni esempi, ovviamente non esaustivi, sono i seguenti.

1) Gli studenti, di fronte all'enunciato di un problema, non sono abituati a mettere in discussione la validità delle richieste dell'insegnante perché ripongono fiducia in lui e di conseguenza sono portati a pensare che ogni problema ha una sua soluzione che si può ricavare proprio utilizzando i dati del problema stesso.

2) Il tentativo ostinato, nella risoluzione di un problema, di ricordare degli schemi risolutivi quando si tratterebbe invece di ragionare ex novo.

3) Il tentativo (in verità, assai meno frequente del precedente) di costruire un ragionamento risolutivo originale laddove basterebbe applicare una formula opportuna.

Tali esempi manifestano chiaramente una difficoltà nella concettualizzazione, ovvero una frattura tra i processi cognitivi degli studenti e la capacità di astrazione tipica della matematica. Non è il formalismo la vera difficoltà, dal momento che una formula, un enunciato possono essere ripetuti a memoria, piuttosto l'articolazione di un pensiero fatto di concetti e relazioni astratte. [1] *Il problema matematico, essendo inaccessibile nel processo, diviene allora accessibile nel prodotto, attraverso la formula risolutiva che lo studente, in seguito al tacito patto, si aspetta dal professore.* La richiesta pressante di soluzioni-per-trovare-risultati si accoppia dunque alle difficoltà nella rappresentazione-comunicazione-argomentazione.
Dunque, quali strategie di azioni adottare per costruire le condizioni per un lavoro di classe basato maggiormente sui processi e un po' meno sui prodotti? La costruzione dei concetti matematici potrebbe essere strettamente dipendente dalla capacità:

4) di saper usare più registri di rappresentazioni semiotiche;

5) di scegliere i tratti distintivi del concetto da rappresentare e rappresentarli in un dato registro;

6) di trattare tali rappresentazioni all'interno di uno stesso registro;

7) di convertire tali rappresentazioni da un dato registro ad un altro.

Così, se da un lato è sembrato naturale e opportuno utilizzare il problema del contare come un grande contenitore di nozioni, di simboli e di linguaggi oltre che di stimoli per l'incremento delle capacità logiche-deduttive (ad esempio, [2] in relazione alle proprietà del triangolo dei coefficienti binomiali), dall'altro la probabilità condizionata offre una grande varietà di problemi (o che sono stati notevoli traguardi culturali durante l'evoluzione del calcolo delle probabilità oppure di recente ricerca in didattica della matematica [3]) che possono permettere all'insegnante di costruire percorsi didattici stimolanti e ricchi di collegamenti tra i vari temi delle indicazioni nazionali.

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Distorted representations and comparison results for inactivity times of systems under double monitoring

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1. Introduction
Let T be the system lifetime, the inactivity time of the system at time t is $(t-T|T<t)$. In real life situation the monitoring of a system can be scheduled at different times, for example at time t_1 and t_2. Under these double inspection, the information about the system can be different and can be affected by the condition of the components of the system at the inspection points. In the literature some authors paid attention to the problem of double monitoring, obtaining results on coherent systems with IID (independent and identically distributed) components (see, e.g., Poursaeed and Nematollahi 2010). We extend the results described above to more general coherent systems studying the inactivity time of a coherent system formed by components with possibly dependent lifetimes under periodical inspections, considering different information available at the inspection times.

2. Distorted representations
We use distortion functions to obtain representations for the reliability of inactivity times, $(t_2-T|t_1<T<t_2)$, under double inspection, analysing in particular three different cases:

- we know that the system was working at a time t_1 and that it is broken at another time t_2, with $0 \leq t_1 < t_2$;
- we know which components are working (and which have failed) at the inspection times t_1 and t_2, with $0 \leq t_1 < t_2$, assuming that the system has failed in the interval (t_1, t_2);
- we know that t_1 is the first component failure time, but this failure does not imply the system failure; at t_2 time we know that some components have failed (and the other are working) causing the system failure.

3. Comparison results
We compare inactivity times of systems under different assumptions by using the representations obtained in the preceding section through distortion functions. We also give some illustrative examples and counterexamples. In particular, we analyse in detail series and parallel systems. We also show some results about general coherent systems.

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Risk measures and progressive enlargement of filtrations: a BSDE approach

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From the beginning of the 21st century, connections between dynamic risk measures and Backward Stochastic Differential Equations (or BSDEs, for short) have been studied in the literature. BSDEs are well established tools in mathematical finance (see, e.g., [2]) and, as is known, one can induce dynamic risk measures from their solutions. The theory of g-expectations, developed by S. Peng (see, e.g., [4]), paved the way for this connection, that has been thoroughly studied when the noise driving BSDEs is either a brownian motion (see, e.g., [1,6]) or a brownian motion and an independent Poisson random measure (as in [5]).
Here we consider a class of BSDEs with jumps (BSDEJ) introduced by I. Kharroubi and T. Lim, whose driving noise is given by a brownian motion and a marked point process. Starting from the existence and uniqueness results of the solution (Y, Z, U) of the BSDEJ with fixed terminal time $T > 0$ provided in [3], we define the dynamic risk measure $\rho = (\rho_t)_{t\in[0,T]}$ given by $\rho_t(\xi) = Y_t$, for any essentially bounded terminal condition ξ.
From a financial perspective, such BSDEJs and, consequently, the induced dynamic risk measures, can be adopted to evaluate the riskiness of a future financial position ξ when there are possible default events, described by the marked point process driving the BSDEJ. Another important feature is that the information available to financial agents is progressively updated as these random events occur. This feature is mathematically encoded in the progressive enlargement of a brownian *reference filtration*. It is proved in [3] that under such a framework it is possible to provide a decomposition of the solution (Y, Z, U) into processes that are solution, between each pair of consecutive random times, of BSDEs driven only by the brownian motion.
The aim of this paper is to show, in the single jump case to ease the notation, that a similar decomposition holds also for the induced dynamic risk measure ρ: we obtain two risk measures, acting respectively before and after the default time. Furthermore, we prove that properties of the driver of the BSDEJ are reflected into desirable properties of ρ, such as monotonicity, convexity, homogeneity, etc. . . Finally, we show that ρ is time consistent, focus on its dual representation and provide some examples.

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Two point function for critical points of a random plane wave

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Random plane wave is conjectured to be a universal model for high-energy eigenfunctions of the Laplace operator on generic compact Riemannian manifolds. This is known to be true on average. In the present paper we discuss one of important geometric observable: critical points. We first compute one-point function for the critical point process, in particular we compute the expected number of critical points inside any open set. After that we compute the short-range asymptotic behaviour of the two-point function. This gives an unexpected result that the second factorial moment of the number of critical points in a small disc scales as the fourth power of the radius. Joint work with Dmitry Beliaev and Igor Wigman.

Ornstein-Zernike behaviour for the correlation functions of the ground state of the quantum Ising model with transverse magnetic field

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We study the asymptotic behaviour of the correlation functions of the quantum Ising model with tranverse magnetic field above the critical point in Z^d. Using the stochastic representation of the model we show that the exact power law correction to the exponential decay is given by $r^{-d/2}$.

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Market manipulation of a producer: a game-theoretic perspective

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We propose three different models for a producer who maximises her expected profit coming from production as well as from a short position in a derivative, whose underlying is the very commodity she produces. Within the production company, a financial division takes care of hedging the derivative using the risk-neutral approach.

In the first two models, the producer controls the production rate and can affect the price volatility. Since the derivative price is increasing in the volatility, whereas having a small volatility is good for the production side, the producer faces a non-trivial trade-off. We prove in particular that the stochastic optimization problems of the producer are well-defined over a small time horizon, so preventing arbitrage opportunities, and we solve them both semi-explicitly. Further, the maximal time horizon can be precisely quantified in the first model.

In the third model, we study a competition between the producer, acting on her production rate, and a trader (long in the derivative) who controls the market price volatility. This can be modelled as a nonzero-sum stochastic differential game, where we show that a Nash equilibrium exists provided the time horizon is short enough. Moreover, such an equilibrium can be semi-explicitly characterized. Finally, through numerical experiments we illustrate qualitatively our results and compare the three models.

Notes on the Ogawa integrability and a condition for convergence in the multidimensional case

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After the introduction of stochastic integral in the 1940s due to Kiyoshi Itō and the developments of the Itō calculus in the succeeding years, particular interest has been devoted to the hypothesis of causality, which are fundamental in stochastic integration theory. In fact, the Itō calculus relies upon concepts as adapted processes, filtration, martingale, conditions that seems to be consistent with a sort of principle of causality in physics. Hence, for many years, the stochastic problems arising in physical modelling (e.g. the phenomenon of diffusion) could be effectively formulated using Itō calculus. Furthermore, the theory of martingales underlying in the Itō calculus provides a powerful tool. However, at the end of 1960s, the interest to construct a new stochastic theory independently from causality conditions began to take hold. In this context, many approaches have been developed. In particular Anatoliy Skorokhod defined, in 1970s, the so-called Skorokhod integral and introduced the *anticipative calculus*. A few years later, in 1979, Shigeyoshi Ogawa independently introduced the so-called Ogawa integral and the corresponding *noncausal calculus*. We focused on the latter with the aim to generalize the conditions for Ogawa integrability to the multidimensional case.
There are many approaches to the noncausal stochastic calculus. The Ogawa integral was extensively studied also in relation with the Skorohod integral and the Stratonovich integral. Its definition has been extended even to the case of random fields; however a detailed study of the case where the integrand function is d-dimensional (with $d \geq 2$) is still lacking. Here we are going to investigate the condition of universal Ogawa integrability in the multidimensional by exploiting Ramer's functional. In the framework of abstract Wiener spaces, we prove that it cannot hold in general without the introduction of a "renormalization term".

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A new Universality class for $(1+1)$-dimensional random interfaces: the Brownian Castle

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In the context of randomly fluctuating interfaces in (1+1)-dimensions two Universality Classes have generally been considered, the Kardar-Parisi-Zhang (KPZ) and the Edwards-Wilkinson (EW). Models within these classes exhibit universal fluctuations under 1:2:3 and 1:2:4 scaling respectively. Starting from a modification of the classical Ballistic Deposition model we will show that this picture is not exhaustive and another Universality Class, whose scaling exponents are 1:1:2, has to be taken into account. We will describe how it arises, briefly discuss its connections to KPZ and EW and introduce a new stochastic process, the Brownian Castle, deeply connected to the Brownian Web, which should capture the large-scale behaviour of models within this Class. This is joint work with M. Hairer.

Stein-Malliavin techniques for spherical functional autoregressions

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We present a class of space-time processes, which can be viewed as functional autoregressions taking values in the space of square integrable functions on the sphere. We exploit some natural isotropy requirements to obtain a neat expression for the autoregressive functionals, which are then estimated by a form of frequency-domain least squares. For our estimators, we are able to show consistency and limiting distributions. We prove indeed a quantitative version of the central limit theorem, thus deriving explicit bounds (in Wasserstein metric) for the rate of convergence to the limiting Gaussian distribution; to this aim we exploit the rich machinery of Stein-Malliavin methods. Our results are then illustrated by numerical simulations.

Stationary distributions for biochemical reaction networks

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Chemical reaction networks are mathematical models ubiquitously used in biochemistry. Typically, if few molecules are present in the system, the time evolution of the molecule counts are modeled through a continuous time Markov chain. In this setting, information on the form or even on the existence of a stationary distribution is important to perform model simplifications, to infer qualitative properties of the model, and to design synthetic biochemical circuits. However, such information is typically not available, except for restricted scenarios. I will give a brief overview of the known results. I will then present a novel and fast convex programming technique that checks if sufficient conditions for the existence of a piece-wise linear Foster-Lyapunov function hold. If this is the case, the existence of a stationary distribution follows the seminal work of Meyn and Tweedie.

On the two-dimensional KPZ and Stochastic Heat Equation

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We consider the Kardar-Parisi-Zhang equation (KPZ) and the Stochastic Heat Equation (SHE) with *multiplicative* space-time white noise, in two space dimensions. These are singular stochastic PDEs which lack a robust solution theory, so it is standard to consider a regularized version of these equations – e.g. by convolving the noise with a smooth mollifier – and then to investigate the behavior of the regularized solution in the limit when the regularization is removed.
Remarkably, the regularized solution can be interpreted as the partition function of a directed polymer in random environment, a much studied model in statistical mechanics. Building on this representation, we will show that a *phase transition* emerges, with explicit critical point, as one varies the disorder strength on a logarithmic scale. In the sub-critical regime, the regularized solution – suitably rescaled – converges to the solution of the SHE with *additive* noise (these are so-called Edwards-Wilkinson fluctuations). We will also present recent progresses in the critical regime.

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On the Onsager-Machlup functional for Brownian motion on the Heisenberg group

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Given a stochastic process, finding the corresponding Onsager-Machlup functional is a well known problem in probability theory that has been intensively studied in the last fifty years. For a stochastic process taking values in a Riemannian manifold and whose infinitesimal generator is an elliptic operator, the Onsager-Machlup functional is already known. In this talk, we present a way to compute the Onsager-Machlup functional associated to the Brownian motion on the Heisenberg group. This takes in consideration a hypoelliptic operator instead of an elliptic one. The geometric setting changes as well. Indeed, a Riemannian manifold is replaced by the Heisenberg group, which is the simplest example of sub-Riemannian manifold. It plays the role of the flat space in sub Riemannian geometry, as the Euclidean space does in Riemannian geometry. Tools like the exponential map and the parallel transport fail in this new setting, since the exponential map is not smooth anymore and the metric becomes degenerate. Indeed, new techniques are used in the sub-Riemannian case. Despite the proof being purely probabilistic, we will see interesting consequences in analysis and geometry. In particular, this approach may open the way to a definition of scalar curvature in the sub-Riemannian setting.

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Quantifying the dependence structure in Bayesian nonparametric models

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We analyse the dependence structure of vectors of random measures with independent increments by relying on the Wasserstein distance. A compound Poisson approximation is used to achieve bounds for the Wasserstein distance in terms of the underlying Lévy measures. Depending on the support of the Lévy measure, we develop different techniques for the explicit evaluation of such bounds. These are then specialized to noteworthy examples in the Bayesian literature, where vectors of random measures with independent increments represent a common tool to induce dependence among nonparametric priors.

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A probabilistic approach to the asymptotic behaviour of the growth-fragmentation equations

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To describe the growth and division of particles over time, one of the key equations in the field of structured population dynamics is the so-called *growth-fragmentation equation*. This equation arises in models of cell division, protein polymerization and even telecommunications protocols and has been extensively studied over many years. For all these applications, the common point is that the particles under concern (which can be cells, polymers, dusts, windows, etc.) are well-characterized by their "size", *i.e.*, a one-dimensional quantity which grows over time at a certain rate depending on the size and which is distributed among the offspring when the particle divides, in a way in which the total mass is conserved. The population is thus described by the concentration of particles of size x, for all $x > 0$, at time $t \geq 0$, denoted by $u_t(x)$, whose evolution is governed by the growth-fragmentation equation.
An important question in this framework concerns the large time asymptotic of its solutions. Typically, one wishes to find a constant $\lambda \in \mathbb{R}$, called the *Malthus exponent*, for which $e^{-\lambda t} u_t$ converges, in some suitable space, to a so-called *asymptotic profile* ν. In the literature, the above convergence is often known as *Malthusian behaviour*. When it holds, it is furthermore important to estimate the speed of convergence.
These questions have traditionally been studied using analytic techniques such as entropy methods or splitting of operators. In this talk, we present a probabilistic approach, which relies on a Feynman-Kac representation of the solutions of the growth-fragmentation equation in terms of an instrumental Markov process. This representation enables to express the fundamental quantities of the analysis, such as the Malthus exponent and the asymptotic profile, in terms of such process.
In particular, we focus on the so-called critical case, in which growth is a linear function of the mass and fragmentations are homogeneous. Doumic and Escobedo (2016) observed that in this case, the so-called Malthusian behaviour fails. We go further in the analysis of the critical case considering a piecewise-linear growth, where the small particles grow faster than the bigger ones. Using the probabilistic approach outlined above, we provide necessary and sufficient conditions on the coefficients that ensure the Malthusian behaviour with exponential speed of convergence to an asymptotic profile. Moreover, we provide an explicit expression of the latter. Our approach relies crucially on properties of so-called refracted Lévy processes that arise naturally in this setting.
To conclude, we show the relationship between the growth-fragmentation equation and a particular class of branching particle systems, called growth-fragmentation processes. It can be shown, in fact, that the intensity measure of such processes is a solution of the growth-fragmentation equation. Thus, establishing the Malthusian behaviour for its solutions gives important informations on the average behaviour of the system. In this context, a further step consists in proving a stronger notion of Malthusian behaviour, concerning the evolution of the system as a whole, rather than only its average.

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On the convergence problem in mean field games: a two state model without uniqueness

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1. Finite state mean field game

Mean Field Games were introduced as limit models for symmetric non-zero-sum non-cooperative N-player dynamic games when the number N of players tends to infinity. Here we focus on finite time horizon problems with continuous time dynamics, where the position of each agent belongs to $\{-1, +1\}$. The players control their transition rate in order to minimized a cost functional, assumed to be symmetric. The limiting dynamics is given by a finite state Mean Field Game (MFG) system made of two coupled forward-backward 1d ODEs: its solution (z, m) reprent the optimal switching rate and the mean of the optimal process, respectively. We study the problem of convergence of the feedback Nash equilibria for the N-player game to solutions of the Mean Field Game, when the number of players goes to infinity. The feedback Nash equilibrium for the N-player game is unique, symmetric, and is provided by the value functions V^N, which are the solutions of a system of $2N$ ODEs, the Nash system.

2. Convergence under monotonicity

In [2] we assumed the monotonicity assumptions of Lasry and Lions, which ensure that the MFG system admits a unique solution for any time horizon T. Applying the idea developed in [1] for the continuous state space case, we proved (for general finite state space $\{1, \dots, d\}$) convergence of the Nash system to the unique classical solution of the Master Equation, a first order PDE stated in the symplex probability measures, whose characteristic curves are given by the Mean Field Game system. Such convergence provided also a propagation of chaos property for the N-player optimal trajectories, as well as a Central Limit Theorem and a Large Deviation Principle for the associated empirical measures.

3. The two state model

In the literature, there are no works about the convergence problem outside the monotonicity regime. We study here an example with $\{-1, +1\}$ as state space and anti-monotonous costs, meaning that players prefer to aggregate, rather than to spread.

We show that the mean field game has exactly three solutions (for T large) and, consequently, there are no classical solutions to the master equation. Our main result states that the N-player value functions still converge, the limit being given by the entropy solution to the master equation, which in this case can be written as a scalar conservation law in one space dimension. The optimal trajectories also admit a limit: they select one mean field game solution if the intial average is not zero, so there is propagation of chaos. The main ingredient for proving these results is a qualitative characterization of the Nash equilibrium for the N player game.

Notably, solutions of the MFG system, whether selected by the limit of N-player Nash equilibria or not, always yield approximate Nash equilibria in decentralized symmetric feedback strategies; but two of them give a completely different behaviur with respect to the true (unique) Nash equilibrium.

Moreover, viewing the mean field game system as the necessary conditions for optimality of a deterministic control problem, we show that the N-player game selects the optimizer of this problem, when it is unique.

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Probability and uncertainty in decision theory

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Everyday actions involve an amount of uncertainty in the final outcome they will deliver. Using Knight's view and terminology, such uncertainty can be separated into two kinds. Some of this uncertainty is "measurable" (henceforth, we will call it "Risk") while some of it is "not measurable" (henceforth, "Ambiguity"). The first refers to the case in which a known objective probability is governing the likelihood of the random events, while the second one refers to the case in which an objective probability either does not exist or it is not known.

In Economics, understanding agents' behavior in such conditions of uncertainty is of fundamental importance. For many years, and in both contexts, the standard model of decision making has been the Expected Utility (briefly, EU) model. Its success rested on two main factors. First, its axiomatization seemed extremely appealing from a normative and, at first sight, also descriptive viewpoint. Second, in applications, it turned out to be a rather flexible and easy to handle model, which allowed researchers to carry meaningful comparative statics exercises. Nevertheless, after the famous thought experiments of Allais (1953) and Ellsberg (1961), the EU paradigm came into question.

In order to address the two different critiques of the EU model, and the mounting experimental evidence against it, a number of decision making models were then produced (see, for a survey, Starmer (2000) and Gilboa and Marinacci (2013)). In looking at this literature, a few but fundamental observations come to mind which the talk will cover:

1. Many departures from the EU model have been considered, but almost all of them feature nonadditive probabilities or sets of finitely additive probabilities.

2. A similar observation holds for price functionals which end up not to be linear and with representations featuring sets of finitely additive probabilities or nonadditive probabilities.

3. One typical assumption in Economics is that agents make choices following a stable preference relation. This seems to be in contrast with the experimental evidence. One way to reconcile this evidence with the idea that an agent makes choices according to a stable preference relation is by resorting to Machina (1985). In Machina's view, agents randomize their choices because they have an inherent preference for randomization. This interpretation is rather fascinating, particularly, from a theoretical perspective. In fact, preference for randomization has often been a key feature of models addressing Allais' and Ellsberg's critique of EU.

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Entropic repulsion for the Gaussian free field conditioned on disconnection by level-sets

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We investigate level-set percolation of the discrete Gaussian free field on $\mathbb{Z}^d$, $d \geq 3$, in the strongly percolative regime. We consider the event that the level set of the Gaussian free field below a level α disconnects the discrete blow-up of a compact set $A \subseteq \mathbb{R}^d$ from the boundary of an enclosing box. We derive asymptotic large deviation upper bounds on the probability that the local averages of the Gaussian free field deviate from a specific multiple of the harmonic potential of A, when disconnection occurs. If certain critical levels coincide, which is plausible but open at the moment, these bounds imply that conditionally on disconnection, the Gaussian free field experiences an entropic push down proportional to the harmonic potential of the set A. In particular, due to the slow decay of correlations, the disconnection event affects the field on the whole lattice. (Joint work with M. Nitzschner)

Mean field limit of interacting filaments for 3D Euler equations

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Families of N interacting curves are considered, with long range, mean field type, interaction. A family of curves defines a 1-current, concentrated on the curves, analog of the empirical measure of interacting point particles. This current is proved to converge, as N goes to infinity, to a mean field current, solution of a nonlinear, vector valued, partial differential equation. In the limit, each curve interacts with the mean field current and two different curves have an independence property if they are independent at time zero. All these results are based on a careful analysis of a nonlinear flow equation for 1-currents, its relation with the vector valued PDE and the continuous dependence on the initial conditions. Finally the 3D Euler equations, precisely local smooth solutions of class H^s with $s > 5/2$ are obtained as a mean field limit of finite families of interacting curves.

Stochastic filtering of a pure jump process with jump diffusion observation and path-dependent local characteristics

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We will study the filtering problem for a system of partially observable processes (X, Y), where X is a non-Markovian pure jump process and Y is a general jump diffusion. Our model covers the case where both the compensator of X and the jump measure of Y are not necessarily quasi left-continuous but they may have predictable jumps. We define the process X via the canonical construction. In order to characterize the optional projection of the signal with respect to the observation filtration generated by Y we introduce the *Markovian version* of the signal. This allows for the computation of the filter via the innovation approach and a suitable bijective transformation.

Ergodic results for a mean field Schrödinger problem

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We introduce the problem of finding the most likely evolution of the empirical distribution of weakly interacting Brownian particles between two prescribed configurations. Using classical results of large deviations theory, we formulate it as a McKean- Vlasov control problem and provide ergodic results for the long-time behavior of the particle system. It turns out that this amounts to studying the equilibration properties of an atypical system of PDEs of mean field type. As a consequence of the ergodic results we obtain some generalizations of well known functional inequalites such as Talagrands inequality and the HWI inequality. The proof strategy is based on a combination of probabilistic techniques and ideas coming from optimal transport, establishing some interesting connections between pathwise Itô calculus (FBSDEs) and Otto calculus. Joint work with J. Backhoff, I. Gentil, and C. Léonard.

Optimal control of stochastic processes with jumps with a Backward Stochastic Differential Equations approach

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The topic of optimal control of jump processes has been treated in many papers due to the many applications in queueing theory and other areas in engineering. An increasing number of economic applications has been recently considered.
Several approaches have been proposed to solve optimal control problems for jump processes. In the case of a Markov jump process, the optimality conditions can be derived via the corresponding infinitesimal generator, [11]. For a general jump process that is not necessarily Markovian, the so called martingale approach can be used to derive the optimality conditions by exploiting the Doob-Meyer decomposition of martingales, ([4],[5]).
We present an alternative systematic approach, based on backward stochastic differential equations (BS-DEs).
The main idea of this method is to relate the value function of the optimal control problem to an appropriate stochastic differential equation of backward type, driven by the random measure associated to the process itself. This approach is exhaustively used in the context of classical optimal control for diffusion processes, constructed as solutions to stochastic differential equations of Ito type driven by Browian motion.
To perform the synthesis of the optimal control problem, as preliminary result, we need to solve a suitable class of BSDEs driven by the random measure associated to the controlled jump process. We develop first of all the L^2-theory of this type of equations in Markovian [8] and non-Markovian framework [7], then L^p-theory with $p > 1$ [6]. The case L^1 requires a specific treatment [9].
With this approach we study optimal control problems of jump processes under different integrability assumptions on the data; we show that they have a solution and that the value function and the optimal control can be represented by means of the solution to the BSDE.
We note that in all these results the laws of the corresponding controlled processes are all absolutely continuous with respect to the law of a given, uncontrolled process, so that they form a dominated model.
Using the BSDEs approach we are able to address optimal control problems for jump processes also in more general non dominated models. To be more precise, the stochastic optimal control problem is studied by means of the so-called randomization method. This latter consists in randomizing the control process, by replacing it with an uncontrolled pure jump process associated with a Poisson random measure. This probabilistic methodology allows to prove again that the value admits a representation formula in terms of a suitable BSDE, related to a so called dual problem.
This procedure has been previously applied to a stochastic control problem in finite dimension for diffusive processes (without jumps) ([10],[1]). It has been used to treat optimal control problems for continuous-time pure jump Markov processes [3].
We apply this technique to solve optimal control problems for infinite dimensional jump-diffusions, where the state process lives in a real separable Hilbert space and is driven by a cylindrical Brownian motion and a Poisson random measure in a non Markovian setting; the coefficients are also allowed to be path-dependent and the diffusion coefficient can be degenerate [2].
The talk is based on joint works with Elena Bandini, Andrea Cosso and Marco Fuhrman.

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On ϑ-methods for stochastic Volterra integral equations

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This contribution regards the numerical solution of stochastic Volterra integral equations

$$X_t = X_0 + \int_0^t a(t, s, X_s)ds + \int_0^t b(t, s, X_s)dW_s, \quad t \in [0, T], \tag{3}$$

deriving, for example, from the modeling of economic problems [3]. As regards the right-hand side of (3), we assume that the second integral is an Itô integral taken with respect to the Brownian motion W_s. We present stochastic ϑ-methods for (3), having the form [1,2]

$$Y_n = Y_0 + h\sum_{i=0}^{n-1} (\vartheta a(t_n, t_{i+1}, Y_{i+1}) + (1-\vartheta)a(t_n, t_i, Y_i)) + \sqrt{h}\sum_{i=0}^{n-1} b(t_n, t_i, Y_i)V_i, \tag{4}$$

where $Y_0 = X_0$, $h = t_{n+1} - t_n$, $n = 0, 1, \ldots, N$ and V_i is a standard Gaussian random variable, i.e., it is $\mathcal{N}(0, 1)$-distributed. Under suitable regularity assumptions on the coefficients a and b of (3), the stochastic ϑ-method (4) is convergent of order $1/2$, i.e., there exists a real constant C such that

$$E[(X(t_n) - Y_n)^2] \leq Ch, \tag{5}$$

for any fixed $t_n = nh \in [0, T]$ and sufficiently small values of h. We present stability analysis of ϑ-methods with respect to suitable test equations construct revised ϑ-methods in order to improve their stability properties, by inheriting the mean-square stability properties of the corresponding ϑ-methods for stochastic differential equations.

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McKean-Vlasov stochastic control and Hamilton-Jacobi-Bellman equations on Wasserstein space

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The first part of the talk is devoted to motivate and formulate rigorously the so-called McKean-Vlasov stochastic optimal control problem, which turns out to be related to a N-player game with mean-field interaction and to the notion of cooperative/Pareto equilibrium. We explain how the dynamic programming method can be extended in order to deal with the McKean-Vlasov stochastic control problem. We exploit Lions' differential calculus on Wasserstein space to relate the value function to an Hamilton-Jacobi-Bellman partial differential equation on the space of probability measures with finite second-order moment. Finally, we discuss the notion of viscosity solution for such an equation and, in particular, the corresponding comparison principle. This is based on joint works in progress with F. Gozzi, I. Kharroubi, H. Pham, M. Rosestolato.

On the emergent behavior of the 2-choices dynamics

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We present two recent results [1, 2] that analyze the emergent behavior of a simple Markov process on two classes of networks. The process, known as the 2-Choices dynamics, can be exploited as an efficient distributed algorithm for solving global tasks such as consensus and community detection.
The first class of graphs is known as *Core-Periphery* and is characterized by a small densely connected group of nodes, the *Core*, and of other many other loosely connected nodes, the *Periphery*. We observe that a phase transition (*consensus* vs *metastability*) occurs whenever one considers an initial state where Core and Periphery have different states; we analyze how this depends on the strength of the connection between the two groups of nodes.
The second class includes graphs with two clusters connected by a sparse cut. We prove that, when the states of the nodes are randomly initialized, there is a constant probability that system rapidly and stably converges to a configuration in which the communities maintain internal consensus on different states. We show how the nodes of the network can exploit this phenomenon to identify, with high probability, the community to which they belong to.

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ROCOF of higher order for continuous time semi-Markov systems

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In this paper we study the rate of occurrence of failures (ROCOF) of continuous time semi-Markov systems. The ROCOF is an important reliability indicator which has been considered by Shi (1985) for finite Markov processes (MP). Lam (1997) determined a formula for the ROCOF of a MP or of a higher-dimensional MP admitting the possibility to work with a denumerable state space. A further extension to semi-Markov processes (SMP) was advanced by Ouhbi and Limnios (2002) where a formula for computing the ROCOF was derived and a statistical estimator was studied by assessing its asymptotic properties.
In general, the ROCOF gives information whether there are a lot of failures or only a few within a time interval. In the study of failures of a system, it is also interesting the study of the relative positioning of pairs of failures and more in general of tuples of failures. Consequently, an extension of the ROCOF, called ROCOF of higher order, was calculated for MP in D'Amico (2015). Here we consider SMP and a mixed probability distribution for the initial law of the system taking into account the possible random starting from any state of the system with any duration. Furthermore, we determine a formula for the ROCOF of higher order for SMP and we recover as particular cases the results contained in the above quoted papers. An application demonstrates how to apply the results to real life problems.

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Optimal liquidation time of a stock in presence of insider information

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If an agent is able to obtain access to privileged information about the price of the assets that make her portfolio, she needs to update her stochastic models to take into account of the additional information. For example if she knows the future price of an asset, the semimartingale representation of the price process changes from the one of a geometric Brownian motion, in the original filtration, to the one of a geometric Brownian bridge in the enlarged one.
The additional information is not required to be strictly of insider type. For example, it has been shown that near expiration times of highly traded options one can expect with some probability a pinning effect towards the strike price of these options.
This suggests that it is relevant to analyze optimization problems where the underling process is a stochastic bridge. In this talk we look at a very simple case.
We assume that an agent owns a stock or a derivate of it and that she wants to maximize her gain by selling it. In particular we assume that the model for the stock price process is given by a Brownian bridge. For these settings, we compute the optimal stopping boundary and we show that different models may share the same boundary.
In addition, under some hypotheses on the boundary function, we derive a relation between the expression of the boundary and the parameters of the underlying diffusion bridge.

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Dynkin games with incomplete and asymmetric information

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We study Nash equilibria for a two-player zero-sum optimal stopping game with incomplete and asymmetric information. In our set-up, the drift of the underlying diffusion process is unknown to one player (incomplete information feature), but known to the other one (asymmetric information feature). We formulate the problem and reduce it to a fully Markovian setup where the uninformed player optimises over stopping times and the informed one uses randomised stopping times in order to hide their informational advantage. Then we provide a general verification result which allows us to find Nash equilibria by solving suitable quasi-variational inequalities with some non-standard constraints. Finally, we study an example with linear payoffs, in which an explicit solution of the corresponding quasi-variational inequalities can be obtained.

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Probabilistic representations of the Barenblatt-type solutions, diffusion equations and fractional operators

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We deal with a class of Barenblatt-type density functions containing, for instance, the weak solutions of some nonlinear fractional diffusion equations. We propose alternative probabilistic representations (also in terms of fractional integrals) of these solutions. Furthermore, we provide the connections with some stochastic processes.

Multi-level Monte-Carlo methods and upper/lower bounds in nested risk computations

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The Multi-level Monte-Carlo (MLMC) method developed by Giles [1] has been successfully applied in various fields of stochastic simulation. Quoting Giles [2], MLMC "reduces the computational cost [*with respect to standard Monte-Carlo*] by performing most simulations with low accuracy at a correspondingly low cost, with relatively few simulations being performed at high accuracy and a high cost". A natural application of this method is the evaluation of *nested* expectation of the form $E\left[g(E[f(X,Y)|X])\right]$, where f, g are given functions and (X,Y) a couple of independent random variables. Apart from the pricing of American-type derivatives, such computations arise in a large variety of risk valuations (VaR or CVaR of a portfolio, CVA), or in the assessment of margin costs for centrally cleared portfolios. In this work, we focus on the computation of Initial margins. We analyze the properties and asymptotically optimal choices of MLMC estimators in practical situations of limited regularity of the outer function g (which can have singularities in the first derivative). In parallel, we investigate upper and lower bounds for nested expectations as above, in the spirit of primal/dual algorithms for stochastic control problems.

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Nonzero-sum stochastic differential games between an impulse controller and a stopper

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We study a two-player nonzero-sum stochastic differential game where one player controls the state variable via additive impulses while the other player can stop the game at any time. The main goal of this work is characterize Nash equilibria through a verification theorem, which identifies a new system of quasi-variational inequalities whose solution gives equilibrium payoffs with the correspondent strategies. Moreover, we apply the verification theorem to a game with a one-dimensional state variable, evolving as a scaled Brownian motion, and with linear payoff and costs for both players. Two types of Nash equilibrium are fully characterized, i.e. semi-explicit expressions for the equilibrium strategies and associated payoffs are provided. Both equilibria are of threshold type: in one equilibrium the intervention regions of the players are separated, while in the other one they can overlap producing a situation where the first player induces her competitor to stop the game. Finally, we prove some asymptotic results with respect to the intervention costs.

Stochastic Ising model with temperature fast decreasing to zero

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We present the new concept of a temperature "fast decreasing to zero" for models in statistical mechanics. In this regime, we study the stochastic Ising model following the classification of Gandolfi, Newman, Stein (2000); i.e. a model is of type F if any spin flips finitely many times a.s., it is of type I if any spin flips infinitely many times a.s., or it is of type M otherwise. Conditions to obtain models of type F, I, or M and examples are discussed. In particular, we identify the type when the Ising model is defined on a cubic lattice with interactions that are i.i.d. random variables taking values -1, $+1$.

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Elliptic stochastic quantization

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Stochastic quantization, in the sense of Nelson [1] and Parisi and Wu [2], is based on a relation between invariant solutions to SPDEs and Gibbs measures. A similar relation between the solution to some elliptic SPDEs with additive noise in $d + 2$ dimension and a corresponding Gibbs measure in d dimension was conjectured, with the name of dimensional reduction, in the physics literature by Parisi and Sourlas [3]. We give a proof of this conjecture in the case $d = 0$, extending the work of Klein et al. [4]. Our proof uses in a fundamental way the representation of the law of the SPDE as a supersymmetric quantum field theory and the theory of general transformations of measures in abstract Wiener spaces (see [5]). Even in our $d = 0$ context the arguments are non-trivial and a non-supersymmetric, elementary proof seems only to be available in the Gaussian case.

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A stochastic algorithm based on the approximation of hitting times for the initial-boundary value problem for the heat equation

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In this talk we construct a new algorithm in order to approximate the solution of the Initial-Boundary Value Problem (IBVP) for the heat equation. This algorithm is based on a complex version of the walk on moving spheres algorithm, used for solving classical Dirichlet problems. The main idea is to employ some new results and the associated numerical method to efficiently approximate the hitting time of Bessel processes. The convergence of the algorithm will be presented and some numerical illustrations will be given.
This is a joint work with Samuel Herrmann (University of Bourgogne, France).

Il laboratorio di statistica per l'Alternanza Scuola-Lavoro e per il Piano Lauree Scientifiche

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Sapersi orientare tra le tante informazioni statistiche che giungono da numerose fonti, comprenderne il significato e valutarne criticamente l'attendibilità sta diventando sempre più importante per interpretare la complessità della vita quotidiana. Saper affrontare l'incertezza offre al cittadino attivo uno strumento indispensabile per comprendere i fenomeni (naturali, sociali, economici e politici), per fare scelte consapevoli, per valutarne le conseguenze e quindi assumersene la responsabilità [1].
I percorsi di Alternanza Scuola-Lavoro, ridenominati "Percorsi per le competenze trasversali e per l'orientamento" a partire dall'anno scolastico 2018/2019, si prestano in modo particolare alla diffusione della cultura statistica. Essi, infatti, rappresentano una modalità didattica innovativa che, attraverso l'esperienza pratica, aiuta a consolidare le conoscenze acquisite a scuola e a sviluppare competenze da collocare in un'ottica di orientamento lavorativo e professionale o di studi superiori [2]. Il Piano Nazionale per le Lauree Scientifiche (PLS) condivide, in parte, la stessa finalità, con esclusivo riguardo, però, ai saperi scientifici. Essi sono da considerarsi come occasioni di orientamento attivo che pongano gli studenti delle Scuole secondarie superiori come soggetti attivi di fronte alle discipline scientifiche [3].
Questa relazione mira a descrivere due esperienze di Alternanza Scuola-Lavoro dedicate all'introduzione nella pratica scolastica della statistica e delle sue applicazioni, realizzate in convenzione con il Dipartimento di Matematica dell'Università degli Studi di Salerno. Le attività proposte, inoltre, sono riconoscibili come laboratori PLS.
Entrambe le attività hanno previsto la realizzazione di un'indagine, in tutte le sue fasi. Tuttavia, in un caso la rilevazione dei dati grezzi è avvenuta tramite un questionario opportunamente predisposto, nell'altro i dati rappresentano il risultato di esperimenti di laboratorio. In particolare, sottolineeremo come l'approccio laboratoriale adottato abbia consentito di raggiungere obiettivi di apprendimento che andassero al di là di specifiche conoscenze tecniche, dovendo puntare a sviluppare competenze trasversali per lo sviluppo di ogni persona, con particolare riguardo alla cittadinanza attiva, all'inclusione sociale e all'occupazione.

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On the combinatorics of cumulants for multivariate subordinated Lévy processes

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1. Cumulants

Multivariate cumulants are a powerful tool to handle Lévy processes due to the Lévy-Khintchine characterization. Suppose $\boldsymbol{Y}(t)$ be a $\mathbb{R}^n$-valued Lévy process and let us assume that at each $t \in \mathbb{R}_+$ the process admits moment generating function (mgf) $M_{\boldsymbol{Y}(t)}(\boldsymbol{z}), \boldsymbol{z} = (z_1, \ldots, z_n) \in \mathbb{R}^n$. Thus $M_{\boldsymbol{Y}(t)}(\boldsymbol{z})$ can be written as

$$M_{\boldsymbol{Y}(t)}(\boldsymbol{z}) = \exp K_{\boldsymbol{Y}(t)}(\boldsymbol{z}) = \exp\left(t K_{\boldsymbol{Y}}(\boldsymbol{z})\right) \tag{6}$$

where $K_{\boldsymbol{Y}(t)}(\boldsymbol{z})$ is the cumulant generating function (cgf) of $\boldsymbol{Y}(t)$ and $\boldsymbol{Y} = \boldsymbol{Y}(1)$ is the time one distribution of the Lévy process. In particular, for $t = 1$ equation (6) gives the well-known definition of the cgf of a random vector. In the ring of formal power series $\mathbb{R}[[\boldsymbol{z}]]$, if $K_{\boldsymbol{Y}}(\boldsymbol{z})$ is a delta series, that is $K_{\boldsymbol{Y}}(\boldsymbol{0}) = 0$, then $M_{\boldsymbol{Y}}(\boldsymbol{z}) = \exp K_{\boldsymbol{Y}}(\boldsymbol{z})$ is well defined independently of convergence radius [3]. If

$$M_{\boldsymbol{Y}}(\boldsymbol{z}) = 1 + \sum_{\boldsymbol{i}:|\boldsymbol{i}|\geq 1} \frac{\mathbb{E}[\boldsymbol{Y}^{\boldsymbol{i}}]}{\boldsymbol{i}!} \boldsymbol{z}^{i} \qquad \text{and} \qquad K_{\boldsymbol{Y}}(\boldsymbol{z}) = \sum_{\boldsymbol{i}:|\boldsymbol{i}|\geq 1} \frac{c_{\boldsymbol{i}}(\boldsymbol{Y})}{\boldsymbol{i}!} \boldsymbol{z}^{\boldsymbol{i}} \tag{7}$$

then $\{c_{\boldsymbol{i}}(\boldsymbol{Y})\}$ are said the (formal) joint cumulants of $\{\mathbb{E}[\boldsymbol{Y}^{\boldsymbol{i}}]\}$ of order $\boldsymbol{i}$, with multi-index $\boldsymbol{i} = (i_1, \ldots, i_n) \in \mathbb{N}^n$. Differently from moments, cumulants are a nice tool to manage random vectors due to the following properties [2]:

i) Orthogonality: The joint cumulants of independent random vectors are zero:

$$c_{\boldsymbol{i}}(\boldsymbol{Y}) = 0 \text{ if } \boldsymbol{Y} = (\boldsymbol{Y}_1, \boldsymbol{Y}_2) \text{ with } \boldsymbol{Y}_1 \text{ independent of } \boldsymbol{Y}_2.$$

ii) Additivity for indipendent random vectors $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$:

$$c_{\boldsymbol{i}}(\boldsymbol{Y}_1 + \boldsymbol{Y}_2) = c_{\boldsymbol{i}}(\boldsymbol{Y}_1) + c_{\boldsymbol{i}}(\boldsymbol{Y}_2).$$

iii) Multilinearity: if $A \in \mathbb{R}^n \times \mathbb{R}^n$ then

$$c_{\boldsymbol{i}}(A\boldsymbol{Y}) = \underbrace{(A, \ldots, A)}_{n} \cdot c_{\boldsymbol{i}}(\boldsymbol{Y}) = \textstyle\sum_{j_1,\ldots,j_n} (A)_{i_1}^{j_1} \cdots (A)_{i_n}^{j_n} c_{\boldsymbol{j}}(\boldsymbol{Y}).$$

iv) Semi-invariance: if $A \in \mathbb{R}^n \times \mathbb{R}^n$ and $\boldsymbol{b} \in \mathbb{R}^n$ then

$$c_{\boldsymbol{i}}(A\boldsymbol{Y} + \boldsymbol{b}) = c_{\boldsymbol{i}}(A\boldsymbol{Y}) \quad \text{if } |\boldsymbol{i}| \geq 2.$$

2. Multivariate subordinated Lévy processes

In this contribution, we exploit the combinatorics of cumulants of Lévy processes constructed by multivariate subordination. These processes are quite often employed in finance for modelling multivariate asset return. A univariate subordinator $\{T(t), t \geq 0\}$ is a Lévy process on $\mathbb{R}_+ = [0, \infty)$ with increasing trajectories. The subordination of a $\mathbb{R}^n$-valued Lévy process $\{\boldsymbol{L}(t), t \geq 0\}$ by $T(t)$ defines a new process $\{\boldsymbol{Y}(t), t \geq 0\}$ with $\boldsymbol{Y}(t) := (L_1(T(t)), \ldots, L_n(T(t)))^{\top}$. Then $\{\boldsymbol{Y}(t), t \geq 0\}$ results to be a $\mathbb{R}^n$-valued Lévy process such that

$$K_{\boldsymbol{Y}}(\boldsymbol{z}) = K_T[K_{\boldsymbol{L}}(\boldsymbol{z})] \text{ and } K_{\boldsymbol{L}}(\boldsymbol{z}) \in \mathbb{R}[[\boldsymbol{z}]].$$

The above construction has been further generalized by allowing the introduction of multivariate subordinators [1]. We show how to recover cumulants of a subordinated Lévy process from the cumulants of the subordinand and of the subordinator by using the Faà di Bruno formula [2]. To this aim we use the Faà di Bruno formula for the coefficients of a composition of formal power series in an efficient closed form formula [2], particularly suited to skip the tiresome and recursive computations of the multivariate case as there is no reference to partial derivative operators. The main tool is a combinatorial device which allows us to manage composition of multiparameter formal power series with different numbers of components, which is the case of subordinated $\mathbb{R}^n$-valued Lévy processes with parameter on $\mathbb{R}^d$, with n not necessarily equal to d. For multivariate subordinated Brownian motions, this formula further reduces the number of additive terms by taking advantage of the well-known property that cumulants of Brownian motion are zero when their order is greater than 2. Moreover the formula can be implemented in any open source software, without calling any symbolic packages having partial derivative operators. To show the feasibility of our proposal, we focus our attention on some cases of interest for applications, as the factor based subordinated Brownian motions. In these models the subordinator has the interpretation of stochastic economic time. Economic time models the information flows, which is measured by trade. As a consequence, the subordinator is specified to model cross sectional properties of trade according to the empirical evidence [3]. In particular, the models we introduce assume a factor structure of the subordinator and allow each asset to have its own stochastic time, thus $n = d$. As the sequence of multivariate moments $\{\mathbb{E}[\boldsymbol{Y}(t)^{\boldsymbol{i}}]\}$ is of binomial type in t with coefficients $\{c_{\boldsymbol{i}}(\boldsymbol{Y})\}$, we show how to use these polynomials in a GMM framework. In particular, we estimate the parameters of two multivariate subordinated Brownian motions of normal inverse Gaussian type with different dependence structures. The evolution over time of non-linear dependence is analyzed by using cross cumulants up to order four.

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Nonzero-sum submodular monotone-follower games: Existence and approximation of Nash equilibria

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We consider a class of N-player stochastic games of multi-dimensional singular control, in which each player faces a minimization problem of monotone-follower type with submodular costs. We call these games *monotone-follower games*. In a not necessarily Markovian setting, we establish the existence of Nash equilibria. Moreover, we introduce a sequence of approximating games by restricting, for each $n \in \mathbb{N}$, the players' admissible strategies to the set of Lipschitz processes with Lipschitz constant bounded by n. We prove that, for each $n \in \mathbb{N}$, there exists a Nash equilibrium of the approximating game and that the sequence of Nash equilibria converges, in the Meyer-Zheng sense, to a weak (distributional) Nash equilibrium of the original game of singular control. As a byproduct, such a convergence also provides approximation results of the equilibrium values across the two classes of games. We finally show how our results can be employed to prove existence of open-loop Nash equilibria in an N-player stochastic differential game with singular controls, and we propose an algorithm to determine a Nash equilibrium for the monotone-follower game.

Optimal rates of mean Glivenko-Cantelli convergence

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Let $\{\mathbf{X}_i\}_{i\geq 1}$ be a sequence of i.i.d. d-dimensional random vectors, with common probability distribution μ on $(\mathbb{R}^d, \mathscr{B}(\mathbb{R}^d))$. If $\int_{\mathbb{R}^d} |\mathbf{x}|^p \mu(d\mathbf{x}) < +\infty$ for some $p \geq 1$, it is well-known that $\mathbb{E}\left[\left\{\mathcal{W}_p(\tilde{\mathbf{e}}_n, \mu)\right\}^p\right] \to 0$ as $n \to +\infty$, where $\mathbb{E}$ denotes expectation, $\tilde{\mathbf{e}}_n := \frac{1}{n}\sum_{i=1}^n \delta_{\mathbf{X}_i}$ is the *empirical measure*, and $\mathcal{W}_p$ stands for the *Wasserstein distance* of order $p \geq 1$. Such a result is usually referred to as *mean Glivenko-Cantelli theorem*.
The problem of finding rates of convergence for the mean Glivenko-Cantelli convergence was initiated by Dudley in 1969 (see [3]), who actually considered other weak metrics, and then carried on by researchers such as Talagrand, Ledoux, Bobkov, Massart, Yukich, Gozlan, Fournier, Guillin, Ambrosio, and others. As far as the mean Glivenko-Cantelli convergence with respect to the Wasserstein distance, recent works have displayed explicit upper bounds which, although valid under a sole moment boundedness for μ, are far from being optimal. See, e.g., [4]. For completeness, the optimal rate of convergence amounts to $\mathbb{E}\left[\left\{\mathcal{W}_p(\tilde{\mathbf{e}}_n, \mu)\right\}^p\right] = O(n^{-p/2})$. At the best of our knowledge, such a rate has been obtained only when $d = 1$ by Bobkov and Ledoux [2].
Our main results consist in reaching such an optimal rate of convergence for a class of measures μ of the form $\mu(d\mathbf{x}) = e^{-V(\mathbf{x})}d\mathbf{x}$, for suitable $V : \mathbb{R}^d \to (-\infty, +\infty]$ which are strictly convex. The strategy relies on a dynamical interpretation of the Wasserstein distance due to Benamou and Brenier (see Chapter 8 of [1]). In particular, we use the Fokker-Planck PDE to construct curves in the space of probability measures that connect $\tilde{\mathbf{e}}_n$ with the steady state $e^{-V(\mathbf{x})}d\mathbf{x}$, and we conclude by exploiting well-known techniques from the theory of sums of independent random variables. In the last part, we extend the optimal convergence to the case of i.i.d. random elements taking values in separable Hilbert spaces or in spaces of probability measures.

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The first-passage time properties of diffusion neuronal models with multiplicative noise

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Nonlinear dynamical systems are often affected by different sources of noise and the usual belief is that the presence of noise can hinder or deteriorate the signal transmission in the system. However it has been observed, both in theoretical models and experiments, that random fluctuations can sometimes improve information processing. Mathematical models in neuroscience are one of the most prominent examples of phenomena for which the noise is of primary importance or even a part of the signal itself rather than a source of inefficiency and unpredictability.
Our aim is to contribute to the discussion on the role of noise, studying the effects of a multiplicative noise on the performance of single neuron models using an analytical approach on the related first-passage-time problem.
We consider models where the evolution of the neuronal membrane depolarization between two consecutive spikes is described by a Feller process, an Inhomogeneous Geometric Brownian Motion or a Jacobi process [1]-[3]. We analyze the spiking activity of the neuron under study through the firing rate and the variability of the output related to the first two moments of the random variable first-passage time and we describe the occurrence of a phenomenon of coherence resonance. In particular we present counterintuitive effects due to the presence of the multiplicative noise and its dependence on the input parameters.

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Coherent upper conditional expectation defined by Hausdorff outer measure

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Let Ω be a non empty set and let **B** be a partition of Ω. In the sequel a bounded random variable is a function $X : \Omega \to \Re$ such that $|X| \leq M$ for some real constant M and $L(\Omega)$ is the class of all bounded random variables defined on Ω; for every $B \in \mathbf{B}$ denote by $X|B$ the restriction of X to B and by $\sup(X|B)$ the supremum of values that X assumes on B. Let $L(B)$ be the class of all bounded random variables $X|B$. Denote by I_A the indicator function of any event $A \in \wp(B)$, i.e. $I_A(\omega) = 1$ if $\omega \in A$ and $I_A(\omega) = 0$ if $\omega \in A^c$.
In the subjective approach, coherent upper conditional previsions $\overline{P}(\cdot|B)$ are functionals defined on $L(B)$ (Walley, 1991).

Definition 1. *Coherent upper conditional previsions are functionals $\overline{P}(\cdot|B)$ defined on $L(B)$, such that the following axioms of coherence hold for every X and Y in $L(B)$ and every strictly positive constant λ:*

1) $\overline{P}(X|B) \leq \sup(X|B)$;
2) $\overline{P}(\lambda X|B) = \lambda \overline{P}(X|B)$ *(positive homogeneity);*
3) $\overline{P}(X + Y|B) \leq \overline{P}(X|B) + \overline{P}(Y|B)$ *(subadditivity);*
4) $\overline{P}(I_B|B) = 1$.

Given a partition **B** and a random variable $X \in L(\Omega)$, a coherent upper conditional prevision $\overline{P}(X|\mathbf{B})$ is a random variable on Ω equal to $\overline{P}(X|B)$ if $\omega \in B$.
Suppose that $\overline{P}(X|B)$ is a coherent upper conditional prevision on $L(B)$ then its *conjugate coherent lower conditional prevision* is defined by $\underline{P}(X|B) = -\overline{P}(-X|B)$. Let K be a linear space contained in $L(B)$; if for every X belonging to K we have $P(X|B) = \underline{P}(X|B) = \overline{P}(X|B)$ then $P(X|B)$ is called a coherent *linear* conditional prevision (de Finetti, 1974) and it is a linear, positive and positively homogenous functional on $K \subseteq L(B)$.
The unconditional coherent upper prevision $\overline{P}(\cdot) = \overline{P}(\cdot|\Omega)$ is obtained as a particular case when the conditioning event is Ω. Coherent upper conditional probabilities are obtained when only 0-1 valued random variables are considered. A bounded random variable is called **B**-measurable or measurable with respect to a partition **B** if it is constant on the atoms of the partition (Walley, 1991).
The necessity to propose a new tool to define coherent upper conditional previsions arises because they cannot be obtained as extensions of linear conditional expectations defined, by the Radon-Nikodym derivative, in the axiomatic approach (Billingsley, 1986); it occurs because one of the defining properties of the Radon-Nikodym derivative, that is to be measurable with respect to the σ-field of the conditioning events, contradicts the necessary condition for the coherence (Doria, 2012) $\overline{P}(X|\mathbf{B}) = X$ for every **B**-measurable random variable X and for coherent linear conditional expectations $P(X|\mathbf{B})$.
A model of coherent upper conditional prevision and probability, based on Hausdorff outer measures has been introduced in a metric space (Ω, d) (Doria, 2012) and its applications have been investigated (Doria, 2015).
For every $B \in \mathbf{B}$ denote by s the Hausdorff dimension of B and let h^s be the Hausdorff s-dimensional Hausdorff outer measure associated to the coherent upper conditional expectation. For every bounded random variable X a coherent upper conditional expectation $\overline{P}(X|B)$ is defined by the Choquet integral with respect to its associated Hausdorff outer measure if the conditioning event has positive and finite Hausdorff outer measure in its Hausdorff dimension. Otherwise if the conditioning event has Hausdorff outer measure in its Hausdorff dimension equal to zero or infinity it is defined by a 0-1 valued finitely, but not countably, additive probability.

Theorem 1. *Let (Ω, d) be a metric space and let $\mathbf{B}$ be a partition of Ω. For every $B \in \mathbf{B}$ denote by s the Hausdorff dimension of the conditioning event B and by h^s the Hausdorff s-dimensional outer measure. Let m be a 0-1 valued finitely additive, but not countably additive, probability on $\wp(B)$. Then for each $B \in \mathbf{B}$ the functional $\overline{P}(X|B)$ defined on $L(B)$ by*

$$\overline{P}(X|B) = \begin{cases} \frac{1}{h^s(B)} \int_B X dh^s & if \quad 0 < h^s(B) < +\infty \\ m_B & if \quad h^s(B) \in \{0, +\infty\} \end{cases}$$

is a coherent upper conditional expectation.

If $B \in \mathbf{B}$ is a set with positive and finite Hausdorff outer measure in its Hausdorff dimension s and X is the indicator function of a set A by Theorem 1 we obtain that the fuzzy measure μ_B^* defined for every $A \in \wp(B)$ by $\mu_B^*(A) = \frac{h^s(AB)}{h^s(B)}$ is a coherent upper conditional probability, which is submodular, continuous from below and such that its restriction to the σ-field of all μ_B^*-measurable sets is a Borel regular countably additive probability.

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Delayed and rushed motions

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We consider time-changed processes, that is processes obtained through random time changes given by subordinators and their inverses. Such processes can be considered in order to solve fractional partial differential equations. In this context, very often, we refer to the characterization in terms of subdiffusion, superdiffusion, normal diffusion which is usually given in terms of mean square displacement (instead of velocity correlation!). However, this definition includes a number of very different dynamics. A further characterization of such dynamics can be given by considering the random time as a new clock for the base process. Then, along the trajectory of the base process, we focus on exit times after time changes. In particular, we introduce a definition of delayed and rushed processes and provide some examples which are, in some cases, counter-intuitive. Our analysis shows that, quite surprisingly, inverse processes are not necessarily leading to delayed processes.

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Stein-Malliavin techniques and Poisson based U-statistics: asymptotics

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The aim of the talk is to present quantitative central limit theorems for U-statistics of arbitrary degree computed on a Poisson random field over compact Riemannian manifolds. On the one hand, the U-statistics here discussed are built by means of the so-called needlets, wavelets characterized by strong concentration properties and by an exact reconstruction formula. On the other, the Poisson point processes are defined over the manifold such that the density function associated to its control measure lives in a Besov space. The rates of convergence strongly depending on the degree of regularity of the control measure of the underlying Poisson point process, thus purpose to provide a refined understanding of the connection between regularity and speed of convergence in this framework. This is a joint work with Solesne Bourguin.

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Dephasing, decoherence and classical stochastic processes arising in quantum theory

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The formalism of quantum theory is based on the concept of a density matrix for characterizing parts of a larger system which is typicall described by an entangled quantum state without classical counterparts (i.e. probability measures). The properties of this density matrix explain the random classical behavior of macroscopic objects. In particular, the annoying superpositions of macroscopically different properties can be shown to disappear from these density matrices on an extremely short timescale. This process is called dephasing or decoherence.

In this talk, we develop a notion of dephasing under the action of a quantum Markov semigroup in terms of convergence of operators to a block-diagonal form determined by irreducible invariant subspaces. If the latter are all one-dimensional, we say the dephasing is maximal. We study characterization of a maximally dephasing evolution in terms of unitary dilations with only classical noise. To this end, we make use of a seminal result of Kummerer and Maassen on the class of commutative dilations of quantum Markov semigroups. In particular, we introduce an intrinsic quantity constructed from the generator which quantifies the degree of obstruction to having a classical diffusive noise model.

(Joint work with J.E. Gough, H.I. Nurdin and L. Viola)

Closed form Bayesian filtering for multivariate binary time series

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Non-Gaussian state-space models arise routinely in several applications. Within this framework, the binary time series setting provides a source of constant interest due to its relevance in a variety of studies. However, unlike Gaussian state-space models—where the classical Kalman filter allows to sequentially update the filtering and predictive distributions—binary state-space models require either approximations or sequential Monte Carlo strategies for dynamic uncertainty quantification and prediction. This is due to the apparent absence of conjugacy between the Gaussian random states and the probit or logistic likelihood induced by the observation equation for the binary data. In this work we prove that, when the focus is on flexible Bayesian learning of dynamic probit models monitored at, possibly, infinite times, filtering and predictive distributions belong to the class of unified skew-normal variables and, moreover, the corresponding parameters can be sequentially updated online via tractable expressions. This result allows to develop an exact Kalman filter for online learning of univariate and multivariate binary time series, which provides also methods to draw independent and identically distributed samples from the exact filtering and predictive distributions of the random states, thereby improving Monte Carlo inference. As outlined in an illustrative application, the proposed method improves state-of-the-art strategies routinely used in the literature. A scalable and optimal sequential Monte Carlo, which exploits the unified skew-normal properties, is also developed and additional exact expressions for the smoothing distribution are provided.

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Optimal reduction of public debt under partial observation of the economic growth

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We consider a government that aims at reducing the debt-to-gross domestic product (GDP) ratio of a country. The government observes the level of the debt-to-GDP ratio and an indicator of the state of the economy, but does not directly observe the development of the underlying macroeconomic conditions. The government's criterion is to minimize the sum of the total expected costs of holding debt and of debt's reduction policies. We model this problem as a singular stochastic control problem under partial observation. The contribution of the paper is twofold. Firstly, we provide a general formulation of the model in which the level of debt-to-GDP ratio and the value of the macroeconomic indicator evolve as a diffusion and a jump-diffusion, respectively, with coefficients depending on the regimes of the economy. These are described through a finite-state continuous-time Markov chain. We reduce via filtering techniques the original problem to an equivalent one with full information (the so-called separated problem), and we provide a general verification result in terms of a related optimal stopping problem under full information. Secondly, we specialize to a case study in which the economy faces only two regimes, and the macroeconomic indicator has a suitable diffusive dynamics. In this setting we provide the optimal debt reduction policy. This is given in terms of the continuous free boundary arising in an auxiliary fully two-dimensional optimal stopping problem.

On a class of infinite-dimensional singular stochastic control problems

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We study a class of infinite-dimensional singular stochastic control problems arising in economic theory and finance. The control process linearly affects an abstract evolution equation on a suitable partially-ordered infinite-dimensional space X, it takes values in the positive cone of X, and has right-continuous and nondecreasing paths. We first provide a rigorous formulation of the problem by properly defining the controlled dynamics and integrals with respect to the control process. We then exploit the concave structure of our problem and derive *necessary and sufficient* first-order conditions for optimality. The latter are finally exploited in a specification of the model where we find an explicit expression of the optimal control. The techniques used are those of semigroup theory, vector-valued integration, convex analysis, and general theory of stochastic processes.

Uniquely ergodic C^*-dynamical systems for the noncommutative 2-torus and uniform convergence of Cesaro averages

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Consider a uniquely ergodic C^*-dynamical system $(\mathscr{A}, \Phi)$ based on a identity-preserving $*$-endomorphism Φ of the unital C*-algebra $\mathscr{A}$. We can prove the uniform convergence of Cesaro averages

$$M_{a,\lambda}(n) := \frac{1}{n}\sum_{k=0}^{n-1}\lambda^{-k}\Phi^k(a), \quad a \in \mathscr{A},$$

for all values λ in the unit circle, which are not eigenvalues corresponding to "measurable non-continuous" eigenfunctions. This result generalizes an analogous one, known in commutative ergodic theory, which turns out to be a combination of the Wiener-Wintner theorem and the uniformly convergent ergodic theorem of Krylov and Bogolioubov. We also present counterexamples based on the tensor product construction for which the above average does not converge even in the $*$-weak topology, for some $a \in \mathscr{A}$ and $\lambda \in \mathbb{T}$.
It would however be desirable to produce more general examples than those (perhaps non trivial) based on the tensor product construction, for which the average $M_{a,\lambda}$ corresponding to some peripheral eigenvalue $\lambda \in \mathbb{T}$ fails to converge. It is done as in the classical case, by defining the noncommutative extension of the Anzai skew product on the noncommutative 2-torus $\mathbb{A}_\alpha$ ($2\pi\alpha$ being the deformation angle), and show that, still in these cases, there exist elements $\mathbb{A}_\alpha$ and $\lambda \in \mathbb{T}$ for which the average $M_{a,\lambda}$ does not converge.
The present talk is based on the papers:

(i) F. Fidaleo *Uniform Convergence of Cesaro Averages for Uniquely Ergodic C^*-Dynamical Systems*, Entropy **20** (2018), 987.

(ii) S. Del Vecchio, F. Fidaleo, L. Giorgetti, S. Rossi, in preparation.

Emergence of periodic behavior in complex systems

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An important problem in complex systems is to understand how many interacting components organize to produce a coherent behavior at a macroscopic level. Within this framework we are interested in the emergence of a macroscopic periodic behavior in systems whose microscopic units have no tendency to evolve periodically. In particular we introduce a dissipated microscopic dynamics for the Ising model where the classical reversible Glauber dynamics is perturbed by adding a dissipation term. Dissipation damps the influence of interaction when no spin-flip occurs for a long time. The Ising model with dissipation has a phase transition from a disordered phase, where the magnetization fluctuates closely around zero, to a phase in which displays a macroscopic regular rhythm. We prove the existence of periodic motion of the magnetization in the low temperature regime.

Interacting particle systems from a duality point of view

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Duality theory is a powerful tool to deal with Markov processes: the main simplification arises from the fact that one can infer properties of the initial processes with considerations regarding its dual, usually easier. These two processes are intertwined by a set of so called duality functions. I will focus on some duality examples in the context of interacting particle systems in boundary driven setting such as the well known exclusion process and its inclusion counterpart in contact with particles reservoirs at the boundaries: here a duality relation can be established using algebraic arguments that relies on representation theory of a suitable Lie algebra. If time allows I will conclude presenting some applications of duality: once a duality relation is available, one can hope to derive the two points correlation functions of the initial process, needed in the study of fluctuations out of equilibrium, from consideration regarding the absorption probabilities in the dual world.

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Optimal switching problems with an infinite set of modes: an approach by randomization and constrained backward SDEs

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In classical optimal switching problems, a controller can drive the time evolution of a system choosing among a finite set of possible modes (or regimes) and switching from one mode to another at chosen random times. A set of Hamilton-Jacobi-Bellman equations (or a set of Backward Stochastic Differential Equations - BSDEs) can be associated to this control problem, where each equation is indexed by a mode. Using a different approach introduced by Bouchard, Elie, Kharroubi and others, sometimes called randomization method, one can represent the value of the problem by a single BSDE with a constraint on the martingale part. This makes it possible to extend the BSDE representation to the case of an infinite number of modes, which is natural in many applications.

Fractional derivatives of a function with respect to another function: applications to Dodson and relativistic diffusions

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Fractional derivatives of a function with respect to another function have been considered in the classical monograph by Kilbas et al. [4] (Section 2.5) and recently studied in detail by Almeida in [1]. In this talk we will discuss some new applications of this approach to diffusion problems arising in mathematical-physics and probability. Our first aim is to show the utility of time-fractional derivatives of a function with respect to another function to deal with models with time-varying diffusivity coefficients. In particular we will first consider the generalized Dodson diffusion equation [3] that arises in the context of cooling processes in geology. A second probabilistic application is given by the generalization of the relativistic diffusion equation, widely studied in the literature (see e.g. [2] and [5] and the references therein).

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Uniform bound of the entanglement for the ground state of the quantum Ising model with large transverse magnetic field

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The quantum Ising model with transverse field on the d-dimensional cubic lattice belongs to those quantum statistical machanical models which can be described in terms of interacting two sided markov processes. In turn, these systems admit a description in terms of suitable Gibbs random fields, in the case at hand, on a $d+1$-dimensional lattice. We consider the ground state of the quantum Ising model with tranverse field h in one dimension in a finite volume $\Delta_m := -m, -m+1, \ldots, m+L$. Making use of the representation just introduced, for h sufficiently large, we prove a bound for the entaglement of the interval $\Lambda_L := 0, .., L$ relative to its complement $\Delta_m \backslash \Lambda_L$ which is uniform in m and L. The bound is established by means of a suitable cluster expansion.

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Sticky Brownian motion as scaling limit of the inclusion process

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The *inclusion process* is an interacting particle system, defined in [1], where particles move as independent walkers at rate $\alpha \in (0, \infty)$ plus an attractive interaction that favours particles coagulation. It is the counterpart of the well-known *exclusion process*, where particles have instead a repelling interaction. In the limit where $\alpha \to 0$ the inclusion process exhibits condensation [2], with particles piling up on a single site. We show that in the condensation regime, after a diffusive scaling limit, inclusion walkers converge to Sticky Brownian motions [3].

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SPDEs with fractional noise in space: continuity in law with respect to the Hurst index

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We define a stochastic version of a 1–dimensional non-linear wave and heat equation

$$(Lu^H)(t,x) = b(u^H(t,x)) + \dot{W}^H(t,x), \tag{8}$$

defined for $(t,x) \in [0,T] \times \mathbb{R}$. The operator L is given by:

$$L = \begin{cases} \partial/\partial t - \partial^2/\partial x^2 & \text{for the heat equation,} \\ \partial^2/\partial t^2 - \partial^2/\partial x^2 & \text{for the wave equation.} \end{cases}$$

The function b is assumed to be Lipschitz-continuous, and the noise term $\dot{W}^H$ is white noise in the time variable t, and fractional noise with Hurst parameter $H \in (0,1)$ in the space variable x. We consider deterministic and suitably regular initial conditions for both choices of L.

This type of stochastic partial differential equations (SPDEs) have been thoroughly studied in [3,1], in which it is developed the so-called *martingale measure integration theory* in order to define a suitable framework for the study of the well-posedness of this type of SPDEs. In particular, in [1] a large class of equations of the general type

$$(\tilde{L}u)(t,x) = b(u(t,x)) + \sigma(u(t,x))\dot{F}(t,x)$$

has been studied, with some assumptions on $\tilde{L}, b, \sigma, F$.

In all of these references, as well as in our case, the lack of regularity of the noise term forces to consider a weaker definition of solution for (8), since we cannot expect the solution u^H to be differentiable in the classical sense. We say that u^H is a *mild solution* of (8) if it is an adapted process which satisfies almost surely

$$u^H(t,x) = I(t,x) + \int_0^t \int_{\mathbb{R}} G_{t-s}(x-y) b(u^H(s,y)) dy\, ds + \int_0^t \int_{\mathbb{R}} G_{t-s}(x-y) W^H(ds,dy),$$

where $G_t(x)$ is the fundamental solution associated with the differential operator L, and I is the solution of the deterministic linear equation $Lu^H = 0$ with our initial conditions.

During the talk we will discuss the main result of our paper [2], which is the continuity in law of the mild solution u^H of (8) with respect to the Hurst index $H \in (0,1)$. In detail, we show that for a fixed $H_0 \in (0,1)$

$$u^H \xrightarrow{d} u^{H_0}, \quad \text{whenever} \quad H \to H_0. \tag{9}$$

We denote with $\xrightarrow{d}$ the convergence in law, which is obtained on the space of continuous functions $C([0,T] \times \mathbb{R})$ endowed with the metric of uniform convergence on compact sets. This space is a suitable choice because the paths $u^H(\cdot,\cdot)(\omega) \in C([0,T] \times \mathbb{R})$ for almost every $\omega \in \Omega$.

We will outline the proof of (9). First, we study the linear case $b \equiv 0$. In this case, we prove the tightness of the set of measures $\{u^H, H \in [H_\ell, H_u]\}$, with $0 < H_\ell < H_u < 1$ applying a multidimensional version of Centsov's criterion. The identification of the limit process is done by showing the finite-dimensional convergence, which follows easily from the fact that u^H is a Gaussian process in this case.

Secondly, we study the extension of this result to the general quasi-linear case with b Lipschitz-continuous. We define a deterministic mapping $T : C([0,T] \times \mathbb{R}) \to C([0,T] \times \mathbb{R})$ which maps a path of the solution of the linear equation to a path of the solution of the quasi-linear equation. We show that this mapping, which has an implicit definition, is well-defined and continuous. This implies that the convergence in law holds in the quasi-linear case too.

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N-player games and mean-field games with smooth dependence on past absorptions

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Mean-field games with absorption is a class of games, that have been introduced in Campi and Fischer (2018) and that can be viewed as natural limits of symmetric stochastic differential games with a large number of players who, interacting through a mean-field, leave the game as soon as their private states hit some given boundary.
In this paper, we push the study of such games further, extending their scope along two main directions. First, a direct dependence on past absorptions has been introduced in the drift of players' state dynamics. Second, the boundedness of coefficients and costs has been considerably relaxed including drift and costs with linear growth. Therefore, the mean-field interaction among the players takes place in two ways: via the empirical sub-probability measure of the surviving players and through a process representing the fraction of past absorptions over time. Moreover, relaxing the boundedness of the coefficients allows for more realistic dynamics for players' private states. We prove existence of solutions of the mean-field game in strict as well as relaxed feedback form. Finally, we show that such solutions induce approximate Nash equilibria for the N-player game with vanishing error in the mean-field limit as $N \to \infty$.

Key words and phrases: Nash equilibrium, mean-field game, absorbing boundary, McKean-Vlasov limit, controlled martingale problem, relaxed control.

Approximate Bayesian conditional copula

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Many proposals are now available to model complex data, in particular thanks to the recent advances in computational methodologies and algorithms which allow to work with complicated likelihood function in a reasonable amount of time. However, it is, in general, difficult to analyse data characterized by complicated forms of dependence. Copula models have been introduced as probabilistic tools to describe a multivariate random vector via the marginal distributions and a copula function which captures the dependence structure among the vector components, thanks to the Sklars theorem [1], which states that any d-dimensional absolutely continuous density can be uniquely represented as the product of the marginal distributions and the copula function.
While it is often straightforward to produce reliable estimates for the marginal distributions, making inference on the dependence structure is more difficult. Major areas of application include econometrics, hydrological engineering, biomedical science, signal processing and finance.
In a parametric frequentist approach to copula models, the most popular method is the so called Inference From the Margins (IFM), where the parameters of the marginal distributions are estimated first, and then pseudo-observations are obtained by pluggin-in the estimates of the marginal parameters. Then inference on the copula parameters is performed using the pseudo-observations: this approach obviously does not account for the uncertainty on the estimation of the marginal parameters.
Bayesian alternatives are not yet fully developed, although there are remarkable exceptions ([2, 3], among others). We will present a general method to estimate some specific quantities of interest of a generic copula (such as, for example, tail dependence indices, Spearmans ρ or the Kendall's τ) by adopting an approximate Bayesian approach along the lines of [4]. In particular, we discuss the use of an approximate Bayesian computation algorithm, based on the empirical likelihood approximation of the integrated likelihood of the quantity of interest [5].
Our approach is general, in the sense that it could be adapted both to parametric and nonparametric modelling of the marginal distributions and can be generalised in presence of covariates. It also allow to avoid the definition of the copula function in a setting where it is in general difficult to apply model selection procedures.
The class of algorithms proposed allows the researcher to model the joint distribution of a random vector in two separate steps: first the marginal distributions and, then, a copula function which captures the dependence structure among the vector components.
In particular, the extension which allow to consider covariates is useful, since many available approaches are known to be based on not-consistent estimate of the copula function.

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Fluctuations of point vortices and 2D Euler invariant measures

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The close resemblance between Onsager's point vortices ensembles and Energy-Enstrophy Gaussian invariant measures for the two dimensional Euler flow is known since the works of Kraichnan on two-dimensional turbulence (ref. 7.). We rigorously establish this connection, as we now outline. On a two dimensional domain, which in the following will be the two dimensional torus $\mathbb{T}^2$, Euler equations in vorticity form are given by

$$\begin{cases} \partial_t \omega + u \cdot \nabla \omega = 0 \\ \nabla^{\perp} \cdot u = \omega, \end{cases} \tag{10}$$

The equations have to be complemented with boundary conditions, that is zero space average on $\mathbb{T}^2$. Euler equations are known to be well posed for initial data $\omega_0 \in L^{\infty}(D)$ (ref. 9.), and smooth solutions preserve the first integrals *energy* and *enstrophy*,

$$E = \int_D |u|^2 dx, \qquad S = \int \omega^2 dx. \tag{11}$$

The Gaussian field associated to the quadratic form $\beta E + \gamma S$ on $\mathbb{T}^2$, the *energy-enstrophy measure*, is thus a natural candidate as an invariant measure of the flow. However, the field is only supported on spaces of quite rough distributions -not even measures- so that making sense of Euler equations in this setting is not trivial: this problem has been effectively tackled both by means of Fourier analysis (see for instance refs. 1. and 2.), and approximation by point vortices systems, see refs. 3.-6. The latter ones are defined as systems of N point particles with positions $x_i \in \mathbb{T}^2$ and intensities $\xi_i \in \mathbb{R}$, satisfying the system of ordinary differential equations

$$\dot{x}_{i,t} = \sum_{j \neq i} \xi_j \nabla^{\perp} G(x_{i,t}, x_{j,t}) \tag{12}$$

where the interacting potential is given in terms of the Green function G of the Laplace operator $-\Delta$, and $\nabla^{\perp} = (\partial_2, -\partial_1)$. The vorticity distribution $\omega = \sum \xi_i \delta_{x_i}$ solves Euler equations in weak sense, and the system is Hamiltonian with respect to the conjugate coordinates $(x_{i,1}, \xi_i x_{i,2})$, and Hamiltonian function

$$H(x_1, \ldots, x_n) = \sum_{i<j}^{N} \xi_i \xi_j G(x_i, x_j), \tag{13}$$

that is the interaction energy of the vortices. Notwithstanding the singularity of the interaction potential, the arguments in ref. 9. show that the system is well-posed for almost every initial condition $(x_i, \xi_i)_{i=1,\ldots N}$ with respect to product Lebesgue measure, the latter being invariant. Euler point vortices also preserve the *canonical Gibbs ensemble* at inverse temperature $\beta \geq 0$

$$\nu_{\beta,\gamma,N}(dx_1, \ldots, dx_n) = \frac{1}{Z_{\beta,\gamma,N}} \exp\left(-\beta H(x_1, \ldots, x_n)\right) dx_1, \ldots, dx_n. \tag{14}$$

This measure was first proposed by Onsager in this context, ref. 10. Equilibrium ensembles at high kinetic energy, which exhibit the tendency to cluster vortices of same sign intensities expected in a turbulent regime, were proposed by Onsager allowing negative values of β. Unfortunately, we are not able to treat the case $\beta < 0$ with our arguments.

As our main result, we obtain the Gaussian energy-enstrophy measure as a limit of Gibbsian point vortices ensembles, in a sort of Central Limit Theorem. Namely, we will consider increasingly many vortices sending

$N \to \infty$, while decreasing their intensities $\xi_i = \frac{\sigma_i}{\sqrt{\gamma N}}$, with $\gamma > 0$ and $\sigma_i = \pm 1$, as in the familiar central limit scaling. In fact, our result can be regarded as an investigation of Gaussian fluctuations around the well-known mean-field limit, in the case where the latter vanishes. As usually happens in Statistical Mechanics, the core argument is a fine control on the partition function of the ensemble of vortices, which we prove to converge to the one of the Energy-Enstrophy measure.
Joint Work with Marco Romito.

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Exact simulation of the first passage time of diffusions

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Many biological or physical applications require to simulate random variables with a given probability distribution. Several basic techniques can be used in order to achieve such a goal, but we shall focus our attention in this talk to the rejection sampling which permits quite easily to generate observations with the exact target distribution. This classical method is sometimes time consuming but it remains well-liked since it is based on a simple algorithm. In order to use the acceptance/rejection algorithm, the main condition is to know an explicit expression of the probability density. What should be done if such an information is not available? The aim of our study is to focus on a particular random variable: the first passage time (FPT) of a diffusion process. We introduce (X_t) the unique solution of the following SDE:

$$dX_t = b(X_t)\,dt + \sigma(X_t)\,dB_t, \quad X_0 = x,$$

where (B_t) stands for a one-dimensional Brownian motion and define τ_L the first passage time through the level L. In order to exactly simulate τ_L, we cannot use an explicit expression of its density. The classical way to overcome this difficulty is to use efficient algorithms for the simulation of sample paths, like discretization schemes. Such methods permit to obtain approximations of the first-passage times as a by-product.

For efficiency reasons, it is particularly challenging to simulate directly this hitting time by avoiding to construct the whole paths. The authors introduce a new rejection sampling algorithm which permits to perform an exact simulation of the first-passage time for general one-dimensional diffusion processes. The main ideas are based both on a previous algorithm pointed out by A. Beskos and G.O. Roberts which uses Girsanov's transformation and on properties of Bessel paths. The efficiency of the method is described through theoretical results and numerical examples.

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Tempered fractional derivatives and related drifted Brownian motions

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Tempered fractional derivatives have proved useful to model semi-long range dependence. These operators emerge in the study of transient super-diffusion and relativistic stable subordinators (see e.g. [1], [3]). We present the fractional equations [2] governing the distribution of reflecting drifted Brownian motions, where we examine two different kinds of reflecting barriers. The equations are expressed in terms of tempered Caputo derivatives.

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A Feynman-Kac result via Markov BSDEs with generalized drivers

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In this talk I will investigate BSDEs where the driver contains a distributional term (in the sense of generalised functions) and derive general Feynman-Kac formulae related to these BSDEs. I will introduce an integral operator to give sense to the equation and then show the existence of a strong solution employing results on a related PDE. Due to the irregularity of the driver, the Y-component of a couple (Y, Z) solving the BSDE is not necessarily a semimartingale but a weak Dirichlet process.

Absolute continuity and Fokker-Planck equation for the law of Wong-Zakai approximations of Itô SDEs

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We investigate the regularity of the law of Wong-Zakai-type approximations for Itô stochastic differential equations. These approximations solve random differential equations where the diffusion coefficient is Wick-multiplied by the smoothed white noise. Using a criteria based on the Malliavin calculus we establish absolute continuity and a Fokker-Planck-type equation solved in the distributional sense by the density. The parabolic smoothing effect typical of the solutions of Itô equations is lacking in this approximated framework; therefore, in order to prove absolute continuity, the initial condition of the random differential equation needs to possess a density itself.

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The Vlasov-Fokker-Planck-Navier-Stokes system as a scaling limit of particles in a fluid

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The PDEs system Vlasov-Fokker-Planck-Navier-Stokes (VFPNS) is a model describing particles in a fluid, where the interaction particles-fluid is described by a drag force called Stokes drag force. In the talk I will present a particle system interacting with a fluid that converges in a suitable probabilistic sense to the (VFPNS) system. The interaction betweenparticles and fluid is described by Stokes drag force. I will show how The empirical measure S^N of particles converges to the Vlasov-Fokker-Planck component of the system and the velocity of the fluid u^N coupled with the particles converges in the uniform topology to the the Navier-Stokes component. Moreover some hints on the new uniqueness result for the PDE system will be given.

Fractional Pearson diffusions and continuous time random walks

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We define fractional Pearson diffusions [5,7,8] by non-Markovian time change in the corresponding Pearson diffusions [1,2,3,4]. They are governed by the time-fractional diffusion equations with polynomial coefficients depending on the parameters of the corresponding Pearson distribution. We present the spectral representation of transition densities of fractional Pearson diffusions, which depend heavily on the structure of the spectrum of the infinitesimal generator of the corresponding non-fractional Pearson diffusion. Also, we present the strong solutions of the Cauchy problems associated with heavy-tailed fractional Pearson diffusions and the correlation structure of these diffusions [6].
Continuous time random walks have random waiting times between particle jumps. We define the correlated continuous time random walks (CTRWs) that converge to fractional Pearson diffusions (fPDs) [9,10,11]. The jumps in these CTRWs are obtained from Markov chains through the Bernoulli urn-scheme model, Wright-Fisher model and Ehrenfest-Brillouin-type models. The jumps are correlated so that the limiting processes are not Lévy but diffusion processes with non-independent increments.

This is a joint work with M. Meerschaert (Michigan State University, USA), I. Papic (University of Osijek, Croatia), N.Suvak (University of Osijek, Croatia) and A. Sikorskii (Michigan State University and Arizona University, USA).

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Synchronization in interacting stochastic systems with individual and collective reinforcement

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Reinforced stochastic processes are well known [14] and have applications in many theoretical and applied domains: urns [13], in particular with their use in clinical trials adaptive design [11]; stochastic algorithms and optimisation applications [4,9,10]. We claim they are also of interest in opinion dynamics modeling [5]. In this talk, several models of interacting reinforced processes will be presented, with different kinds of dishomogeneities. They are generalizations of the model introduced in [8] where a (finite) system of Pólya urns is updated in a synchronous way either looking at the individual urn or looking at the global average in the whole system (so called mean field interaction). The time-asymptotics behavior of these systems will be discussed: we proved there is convergence towards a shared a.s. limit, whose nature may be random or deterministic. We may interpret this as a synchronization phenomenon. The different speeds of convergence will be presented with the study of fluctuations, proved through central limit theorems. This talk is based on joint works [6,8,12] with I. Crimaldi, P. Dai Pra, I. Minelli and M. Mirebrahimi. Other recent related works are [1,2,3,7,15].

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Insurance capacity

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Insurance capacity is an important concept for insurance practitioners but it has surprisingly received little attention by academic researchers. It is usually defined as the largest amount of insurance or reinsurance available from a company or the market in general. We build a simple Markovian model of an insurance market with financial frictions in which capacity is endogenously determined as a function of the total capitalisation of the insurance industry and of the concentration of the insurance market. We study the dynamics of this capacity, the corresponding market price of risk and their stationary distribution. The model is able to explain empirical behavior of insurance supply during the Great Recession, once we introduce capital-based regulation.

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Asymptotic results for first-passage times of some exponential processes

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We consider the process $\{V(t) : t \geq 0\}$ defined by $V(t) = v_0 e^{X(t)}$ (for all $t \geq 0$), where $v_0 > 0$ and $\{X(t) : t \geq 0\}$ is a compound Poisson process with exponentially distributed jumps and a negative drift. This process can be seen as the neuronal membrane potential in the stochastic model for the firing activity of a neuronal unit presented in [2]. We also consider the process $\{\tilde{V}(t) : t \geq 0\}$, where $\tilde{V}(t) = v_0 e^{\tilde{X}(t)}$ (for all $t \geq 0$) and $\{\tilde{X}(t) : t \geq 0\}$ is the Normal approximation (as $t \to \infty$) of the process $\{X(t) : t \geq 0\}$. In this paper we are interested in the first-passage times through a constant firing threshold β (where $\beta > v_0$) for both processes $\{V(t) : t \geq 0\}$ and $\{\tilde{V}(t) : t \geq 0\}$; our aim is to study their asymptotic behavior as $\beta \to \infty$ in the fashion of large deviations (see e.g. [1] as a reference on this topic).

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Intermediate and small scale limiting theorems for random fields

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We work with an ensemble of random Gaussian Laplace eigenfunctions, and study their nodal lines. On the 2d flat torus, we consider the process given by the intersection of the nodal line and a straight line segment, and count the nodal intersections. We compute precise asymptotics for the second factorial moment on small scales. The asymptotic remarkably depends on both the line's direction and the limiting spectral measure. Moreover we consider the persistence probability for this process, and establish results that in particular allow us to detect point masses for the corresponding spectral measure.
We next analyse in detail the Cilleruelo random field, with spectral measure supported at four points, and its restriction to a straight line. The persistence probability of the resulting process at certain scales depends on the line's direction. We then establish similar results for a 'Cilleruelo type' field, with spectral measure close to Cilleruelo, via a coupling. This is joint work with Dmitry Beliaev.

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Subordinated fractional Poisson processes

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In this talk, we will present the fractional Poisson process (FPP) time-changed by an independent Lévy subordinator and the inverse of the Lévy subordinator, which we call TCFPP-I and TCFPP-II, respectively. Various distributional properties of these processes will be presented. We will see that, under certain conditions, the TCFPP-I has the long-range dependence property. It can be seen that the TCFPP-II is a renewal process and its waiting time distribution can be derived. The bivariate distributions of the TCFPP-II will be discussed. Some specific examples for both the processes and simulations of the sample paths of these processes will be presented.

Lipschitz continuity of probability kernels and applications to Bayesian inference

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We focus on Lipschitz continuity of conditional probability distributions with respect to the observable value. Such conditional laws are often obtained by disintegrating a joint probability distribution γ on a product space $(\mathbb{X} \times \Theta, \mathscr{B}(\mathbb{X} \times \Theta))$. Here, the *sample space* $\mathbb{X}$ is a convex set of a Banach space with norm $\| \cdot \|_{\mathbb{X}}$, while the *parameter space* Θ is an open set of $\mathbb{R}^d$ (corresponding to parametric statistical models). Our main results provide sufficient conditions in order to guarantee that the probability kernel $\pi(\cdot \mid \cdot) : \mathscr{B}(\Theta) \times \mathbb{X} \to [0, 1]$, defined by disintegrating γ with respect to its first marginal γ_1 as

$$\int_A \pi(B \mid x)\, \gamma_1(\mathrm{d}x) = \gamma(A \times B) \quad \text{for any } A \in \mathscr{B}(\mathbb{X}) \text{ and } B \in \mathscr{B}(\Theta),$$

possesses a version $\pi^*(\cdot \mid \cdot)$ satisfying

$$\mathcal{W}_2\big(\pi^*(\cdot \mid x); \pi^*(\cdot \mid y)\big) \leq C \|x - y\|_{\mathbb{X}} \qquad \forall\, x, y \in \mathbb{X}$$

where $\mathcal{W}_2$ denotes the square-Wasserstein distance from optimal transport theory, defined on the space of probability measures over Θ.

Our approach is fully based on the dynamical formulation of the optimal transport distance, and leads to solve a degenerate elliptic PDE with mixed boundary conditions. Therefore, owing to the theory of weighted Sobolev spaces, we obtain the above estimate by requiring the finiteness of Poincaré or Muckenhoupt constants.

Finally, moving to the nonparametric case (Θ being the space of probability measures over $\mathbb{X}$), we exploit the de Finetti's representation theorem for exchangeable sequences of random variables, allowing to reduce the problem to the parametric case.

As possible applications, we show some stability results in Bayesian statistical inference, with specific focus on exponential models and Pareto models.

Derivative-free optimization by Wasserstein natural gradient

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The Stochastic Relaxation of a cost function, i.e., the optimization of its expected value over a statistical model, allows to look for minima using derivative-free optimization methods. For continuous cost functions, a common approach consists in computing the expected value of the function with respect to a Gaussian distribution, and look for local minima by gradient descent. However, the Euclidean geometry may not be the most convenient for this type of optimization, and effective algorithms have been designed to compute Riemannian gradients based on Fisher-Rao geometry for Gaussian distributions, known in the literature as Riemannian natural gradient. In this talk we present an optimization algorithm for continuous functions which exploits an alternative geometry for Gaussian distributions based on the Wasserstein distance. Indeed, it is known that the Wasserstein distance defines a Riemannian geometry over the manifold of normal distributions, and thus it allows the computation of the Wasserstein natural gradient of the Stochastic Relaxation.

Asymptotic results for the Fourier estimator of the integrated quarticity

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In this paper we prove a central limit theorem for the Fourier quarticity estimator. We obtain a new consistency result and we show that the estimator reaches the parametric rate $\rho(n)^{1/2}$, where $\rho(n)$ is the discretization mesh and n the number of points of such a discretization. The optimal variance is obtained, with a suitable choice of the number of frequencies employed to compute the Fourier coefficients of the volatility, while the limiting distribution has a bias. As a by-product, thanks to the Fourier methodology, we obtain consistent estimators of any even power of the volatility function as well as an estimator of the spot quarticity. We assess the finite sample performance of the Fourier quarticity estimator in a numerical simulation with different market microstructure frictions.

Weak well-posedness for some degenerate SDEs driven by stable processes

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In our work, we investigate the effects of the propagation of a stable noise through a chain of n d-dimensional oscillators. More in details, we establish the weak well-posedness for a stochastic differential equation (SDE in short) of the following form:

$$\begin{cases} dX_t^1 &= F_1(t, X_t^1, \dots, X_t^n)dt + dZ_t, \\ dX_t^2 &= F_2(t, X_t^1, \dots, X_t^n)dt, \\ dX_t^3 &= F_3(t, X_t^2, \dots, X_t^n)dt, \\ \vdots \\ dX_t^n &= F_n(t, X_t^{n-1}, X_t^n)dt, \end{cases} \tag{15}$$

where Z is a d-dimensional, symmetric α-stable process with symbol comparable to $-|\lambda|^\alpha$, for α in $(1, 2)$. We suppose that the coefficients $(F_i)_{i\in[\![2,n]\!]}$ satisfy a weak Hörmander-like non-degeneracy condition, i.e. we assume that the matrices $D_{x_{i-1}}F_i$ have full rank. The major issue especially comes from the specific degenerate framework considered here: the noise in the i-th component only comes from the $(i-1)$-th component through the non-degeneracy of the gradients $(D_{x_{i-1}}F_i(t, x))_{i\in[\![2,n]\!]}$ (components which transmit the noise).
We nevertheless show that the system is well-posed in a weak sense, when the coefficients $(F_i)_{i\in[\![2,n]\!]}$ lie in some suitable anisotropic Hölder spaces with multi-indices of regularity. Denoting by $(\beta_{i,j})_{i,j\in[\![2,n]\!]}$ the Hölder index of the i-th component F_i of the drift with respect to the j-th variable, we assume:

$$\beta_{i,j} \text{ is in } \Big(\frac{1+\alpha(i-2)}{1+\alpha(i-1)}; 1\Big].$$

Furthermore, we show through suitable counter-examples that the regularity exponents that ensure weak well-posedness for the SDE are almost sharp. Such counter-examples can be seen as natural modifications of those considered in [3] and [1] to our degenerate, stable framework.
Our approach relies on a perturbative method based on forward parametrix expansions to show global Schauder-type estimates for the associated integro-partial differential equation (IPDE in short). Such a method was firstly used in [2] to derive analogous estimates in a degenerate Gaussian framework.
Roughly speaking, the main steps of the perturbative approach are the following: we start choosing a suitable proxy for the IPDE of interest, (i.e. an operator whose associated semigroup and density are known and that is close enough to the original one) and we then exhibit the suitable regularization properties associated with the proxy (especially, we show that it satisfies Schauder estimates). Thus, we expand a solution of the original IPDE around the proxy through a Duhamel-like formula and eventually show that the expansion error only brings a negligible contribution so that the Schauder estimates still hold for the original equation. To control the expansion error, we need to exploit some duality results between appropriate Besov Spaces. This is due to the low regularizing properties given by our degenerate setting and to some integrability constraints linked to the stability index. We finally point out that when dealing with an unbounded first order perturbations, it seems rather natural to include in the proxy a deterministic flow associated with the drift in (15) in order to establish Schauder estimates as firstly used by Krylov and Priola [4] in the diffusive non-degenerate setting.
Eventually, we manage to obtain results for some super-critical indices α in $(0, 1]$ in the specific case of perturbed Ornstein-Uhlenbeck dynamics.

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On the elastic telegraph process

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Finite velocity random motions provide a good alternative to diffusion processes for modeling the continuous time evolution of natural phenomena in life sciences. Indeed, diffusion processes exhibit some features, such as the unboundedness of the first variation, that often make them unsuitable to describe the true dynamics. For this reason, in the last decades, attention of scientists has been drawn to random evolutions with finite velocity and their applications in many contexts, such as biology, physics, engineering and mathematical finance.
The integrated telegraph process is one of the basic models for the description of a random motion on the real line. It describes a motion characterized by finite (constant) velocity, the direction being reversed at the random epochs of a homogeneous Poisson process. Since the seminal papers of Goldstein [2] and Kac [3], many generalizations of the telegraph process have been provided in a quite large literature (see, for instance, [4] and [5]).
In the present contribution, we study the one-dimensional telegraph process $\{X(t); t \geq 0\}$, with initial state $X(0) = x \geq 0$, in the presence of an elastic boundary at the origin. When the particle hits the origin, it is either absorbed, with probability α, or reflected upwards, with probability $1-\alpha$, for $0 < \alpha < 1$. In the case of exponentially distributed random times between consecutive changes of direction, we obtain the distribution of the renewal cycles and of the absorption time at the origin. This investigation is performed both in the case of motion starting from the origin and non-zero initial state. Finally, we study the conditional distribution of $X(t)$ within a renewal cycle.

(This contribution is based on a joint work with A. Di Crescenzo and S. Zacks.)

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Path dependent HJB equations via BSDEs

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In this talk we present semilinear path-dependent Kolmogorov equations in the space of continuous functions, extending the infinite dimensional approach developed in [1] for the linear case. Under suitable assumptions on the coefficients and on the terminal condition we provide existence of smooth (*classical*) solutions: the starting point is a Feynman-Kac formula obtained introducing a well-suited forward-backward stochastic system, extending the methods introduced in [2]. Towards [2] the novelty is a deep study of differentiability up to the second order of the backward stochastic differential equation.
Semilinear path dependent Kolmogorov equations arise naturally in connection with stochastic differential equations with delay and with stochastic optimal control problems related: thanks to the regularity of the solution we are able to prove existence of optimal controls in strong sense.

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Existence of nonnegative vortex sheets for 2D stochastic Euler equations

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In his 1991 paper, J.-M. Delort proved existence of solutions to the 2D Euler equations with H^{-1}-valued nonnegative vorticity; this includes the case of initial nonnegative vorticity concentrated on a line (vortex sheet).
Here we prove the analogue result for the stochastic case, with transport noise on the vorticity. Namely, we consider 2D stochastic Euler equations in vorticity form

$$\partial_t \xi + u \cdot \nabla \xi + \sum_k \sigma_k \cdot \nabla \xi \circ \dot{W}^k = 0,$$
$$\xi = \text{const} + \text{curl} u,$$

where σ_k are given (divergence-free) regular vector fields and W^k are independent Brownian motions. Our main result is existence of a weak (in the probabilistic sense) H^{-1}-valued nonnegative solution ξ.

On the Itô-Alekseev-Gröbner formula for stochastic differential equations

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In this talk we introduce a nonlinear integration-by-parts formula for stochastic differential equations which generalizes both Itô formula and the classical Alekseev-Gröbner lemma for deterministic differential equations. We well focus on the fact that our formula yields a new perturbation theory, i.e. allows to estimate the global error between the exact solution of an SDE and a general Itô process in terms of the local characteristics (and their Malliavin derivatives). The proof is based on expressing anticipating stochastic integrals as Skorokhod integrals. If time permits, we then discuss applications for deriving strong convergence rates for perturbations or approximations of stochastic (partial) differential equations.

Examining whether brain types typical of males are typical of women, and vice versa

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Analysis of (1): distributions of brain measurements differentiated by gender (pure types or mixtures?), and (2): gender disparity in clustering that ignores gender. Emphasis will be placed on exposure of the statistical methodology.
Joint work with Daphna Joel (Psychology, Tel Aviv University) and colleagues.

Approccio soggettivista alla probabilità per la formazione degli insegnanti

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Negli ultimi anni, la probabilità ha acquisito un ruolo importante nei curricula di matematica di tutto il mondo con un crescente interesse di ricerca nell'ambito dell'educazione matematica. Infatti, la probabilità gioca un ruolo cruciale nell'alfabetizzazione matematica degli individui (ad es., OCSE, 2016 e MIUR, 2012) e i ragionamenti probabilistici sono alla base sia di molti problemi decisionali quotidiani che di interessanti questioni scientifiche. Anche se lo sviluppo della consapevolezza della valutazione di una probabilità è qualcosa di culturalmente e socialmente rilevante, la ricerca rivela difficoltà sia da parte degli studenti, ma, dato ancora più allarmante, anche da parte degli insegnanti (ad es. Batanero, Godino & Roa, 2004).
In particolare, le diverse impostazioni per la definizione della probabilità (classica, frequentista e soggettivista) forniscono diversi sistemi di concetti e procedure che servono ad analizzare situazioni in ambito di incertezza. Gli insegnanti dovrebbero essere consapevoli della diversità di questi approcci perché essi influenzano il ragionamento degli studenti nel confrontarsi con problematiche inerenti il pensiero probabilistico. In questo senso, è fondamentale pensare a nuovi modi/approcci per sviluppare conoscenza e consapevolezza negli insegnanti riguardo alla probabilità sia durante la loro formazione iniziale che in servizio. Nel perseguire tale obiettivo, abbiamo preso come punto di partenza l'approccio soggettivistico della probabilità, guardando a esso come un giudizio soggettivo condizionato dalle informazioni e dalla conoscenze in possesso del valutatore. Questo approccio, dal nostro punto di vista, "cattura" la base psicologica sottostante alla valutazione di una probabilità e può essere collegato in maniera significativa e utile con gli altri approcci.
In questo scenario, abbiamo progettato un particolare *task* per indagare e, allo stesso tempo, sviluppare conoscenze e consapevolezza negli insegnanti sia in formazione che in servizio. Il task è costruito all'interno di particolari contesti di scommesse e prevede la richiesta agli insegnanti di costruire delle "quote" in funzione della puntata di un potenziale giocatore: ad esempio, nel contesto del lancio di due dadi, quanto si è disposti a pagare per il realizzarsi dell'evento che esca un determinato valore della somma dei punteggi. La richiesta è abbastanza aperta in modo da lasciare gli insegnanti liberi di attribuire le quote e di esprimerle in modo additivo o moltiplicativo; le loro ipotesi sono poi rielaborate attraverso discussioni collettive guidate dal formatore.

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Integration by parts formulae on open convex sets in Wiener spaces

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In an abstract Wiener space (with Malliavin approach), a set is of finite perimeter if an integration by parts formula can be introduced, involving a vector measure said perimeter measure which variation is the Feyel-de la Pradelle measure. An open convex set is of finite perimeter, and we provide an explicit representation of the perimeter measure in terms of the Minkowski functional.

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Finite velocity random motions with jumps governed by an alternating fractional Poisson process

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We consider the random motion on the real line of a particle that moves with constant finite velocities $c > 0$ and $-v < 0$. At the random time epochs of a fractional alternating Poisson process, the particle performs an upward (downward) jump of deterministic amplitude if it follows a period of forward (backward) motion. It then reverses its direction and changes its speed. We give the exact joint distribution of the position and the velocity of the particle in terms of uniformly convergent series of generalized Mittag-Leffler functions. We then analyze the special case of constant jump size and random initial velocity, discussing the effects of "fractionality". Finally, we present the formal distribution of the first-passage time through a fixed positive level and then focus on some suitable lower bounds.

(This contribution is based on a joint work with A. Di Crescenzo.)

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The generalized moment method for parameters estimate in stochastic fibre processes

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Statistical methods related with stochastic geometries, and in particular with fibre processes, can be applied to address a big set of biomedical problems, ranging from quantifying dose/effect ratios in medical treatments, to automatic diagnosis of pathologies related with the *shape* of the fibre process under study, whose description can be highly complex. In this context, statistical methods based on suitable descriptors of the geometry of the fibre process are needed to compare quantitatively patterns arising in different experimental or pathological conditions, also taking into account spatial heterogeneities of the patterns, which are very frequent in real applications [5].

Fibre processes are also used to model dynamic phenomena like angiogenesis, vasculogenesis, formation of neuronal networks, etc. In such cases some of the stochastic dynamical models available in literature [1, 2] describe the evolution of tips of vessels, coupled with the evolution of some underlying fields of nutrients. The parameters of these models are strictly connected with the geometry of the generated vessels. Statistical techniques for parameter estimation of such models, based on suitable descriptors of the geometry of the fibres, are needed to validate the models themselves.

In this talk we will address some of these problems and provide a parameter estimation method based on a suitable revisitation of the Generalized Moment Method [3, 4]. The properties of the proposed parameter estimators will be studied both theoretically and on simplified simulated test cases.

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Optimal stopping of the exponential of a Brownian bridge

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We study an optimal stopping problem where the gain function is the exponential of a Brownian bridge X pinned at zero at time $t = 1$. This problem was posed in a paper by Ernst and Shepp [4], who highlighted its difficult analytical tractability. Optimal stopping of a Brownian bridge has been a canonical problem in the optimal stopping literature for half a century, with early work by Shepp [5], for the case of a linear gain function.
From the point of view of applications to bond trading [2] (as well as stock trading [1]), letting the underlying asset price be denoted by $P_t := \exp(X_t)$ at time $t \in [0, 1]$, avoids the inconvenience of negative prices. However, from a mathematical point of view it prevents explicit solvability of the problem (contrarily to the setting of, e.g., [3], [4], [5]).
In this work we rely on a probabilistic approach to free boundary problems and we prove that the optimal stopping rule prescribes to stop when the Brownian bridge exceeds a time-dependent, continuous, decreasing, optimal boundary. It is worth noticing that, to the best of our knowledge, this is one of very few examples of fully solved stopping problems for *time-inhomogeneous* diffusions. We also obtain the maximal regularity of the value function. Namely, we show that the value function of our problem is globally C^1 with second order spatial derivative continuous in both the continuation region and the stopping region.
Finally, we solve numerically the Volterra nonlinear integral equation that uniquely characterises the optimal boundary.

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Opinion dynamics on random networks evolving via preferential attachment

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We discuss convergence results for for a system of N interacting reinforced random walks. Each walk has a reinforcement mechanism which is influenced by interaction with the other walks and (eventually) by a forcing input. Such a system can be interpreted as a model for opinion dynamics on a random dynamic environment, consisting of N networks evolving via a preferential attachment rule [6]. For any N, the walks converge almost surely to the same, possibly random, limit. By means of central limit theorems in functional form, we find different rates of convergence of the walks to their common limit, depending on the size of reinforcement and on the strength of interaction.
When specialized to the non-interacting case and for specific values of the parameters, our results correspond to functional central limit theorems well known in the field of urn models and stochastic approximation (see [4], [5], [7]).
In particular, our interest is to compare the rate of convergence of the walks with the rate of *synchronization*, i.e., the rate at which their mutual distances converge to zero. We show that, under certain conditions, synchronization is *faster* than convergence.
In the context of opinion dynamics, this phenomenon can be interpreted as a strong form of consensus: indeed, interacting communities share the same opinion even when such opinion is far form its limiting value, so that we observe "synchronized fluctuations".
The talk is based on joint works with I.Crimaldi, P. Dai Pra and P.-Y. Louis.

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A new approach to forecast market interest rates

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Modelling interest rates is the object of many studies and also attracts the attention of economists and practitioners. Through the time, different extensions of the well known Cox, Ingersoll and Ross (CIR) model [1] have been introduced and analysed. However, to the usual challenges (e.g. regime switching, clustered volatility, skewed tails, etc.) it has been added by the current market environment the need to model negative interest rates. The talk will focus on some new results obtained by the authors in joint works ([2], [3], [4]) to forecast future expected interest rates based on rolling windows from observed financial market data. The main goal is to propose a novel methodology that preserves the structure of the original CIR model, even with negative interest rates. The novelty consists in: (1) an appropriate partitioning of the data sample; (2) calibrating the CIR parameters by replacing the Brownian motion in the random term of the model with normally distributed standardized residuals of the "optimal" ARIMA model suitably chosen for each sub-group partitioning the historical market data. This allows capturing all the statistically significant time changes in volatility of interest rates, thus giving an account of jumps in market dynamics. The efficiency of the new approach with respect to the original CIR model in forecasting future interest rates is tested for different term structures in EUR and USD currency.

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Equivalent characterizations of BV on domains of Wiener spaces

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We provide three different characterizations of the space $BV(O,\gamma)$ of the functions of bounded variation with respect to a centered Gaussian measure γ on the open domain O in Wiener spaces. Throughout these different characterizations we deduce a sufficient condition for belonging to $BV(O,\gamma)$ by means of the Ornstein–Uhlenbeck semigroup and we provide an explicit formula for one–dimensional sections of functions of bounded variation. Finally, we apply our technique to Fomin differentiable probability measures ν on a Hilbert space X, inferring a characterization of the space $BV(O,\gamma)$ of the functions of bounded variation with respect to ν on open domains $O \subset X$.

A Bayes nonparametric prior for semi-Markov processes

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We introduce the semi-Markov beta-Stacy process (SMBS), a novel Bayesian nonparametric prior for discrete-time semi-Markov processes. Since semi-Markov processes are widely used in applications to predict the evolution of a discrete system, we fully characterize the predictive distributions associated to the SMBS prior. These provide a rule to perform probabilistic predictions for the state of a semi-Markov process at the next time point conditionally on its observed history up to the present moment. We show that the predictive distributions of the SMBS process, which are available in closed form, correspond to the transition kernels of a system of reinforced urns.

Joint life insurance pricing using extended Marshall-Olkin models

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Bivariate copula functions have been widely used to model the dependence structure between the residual lifetimes of the two individuals in a couple. However, considered copulas are absolutely continuous and do not allow for the case of a simultaneous death due to some catastrophic event. In this talk we will analyze the Extended Marshall-Olkin model (introduced in Pinto and Kolev, 2015) which is based on the combination of two approaches: the absolutely continuous copula approach, where the copula is used to capture dependencies due to environmental factors shared by the two lives, and the classical Marshall-Olkin model, where the association is given by accounting for a fatal event causing the simultaneous death of the two lives. More precisely we assume that the residual lifetimes T^m and T^f of the individuals in a couple are modeled through the Marshall-Olkin type stochastic representation

$$(T^m, T^f) = (\min(X^m, Z), \min(X^f, Z))$$

where the three underlying random variables (X^m, X^f, Z) are distributed according to the joint survival function

$$\bar{F}_{X^m,X^f,Z}(x, y, z) = C\left(\bar{F}_{X^m}(x), \bar{F}_{X^f}(y)\right)\bar{F}_Z(z)$$

whith C a bivariate absolutely continuous copula function.
Important properties of the Extended Marshall-Olkin model will be analyzed and the behavior of the induced mortality intensities studied. The model is finally applied to a sample of censored residual lifetimes of couples of insureds extracted from a dataset of annuities contracts of a large Canadian life insurance company. The above analysis is the content of the paper Gobbi et al. (2019).

Finally, possible extensions of the model in order to take into account for a delayed effect of the common shock will be discussed.

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Hitting time asymptotics for hard-core interactions on bipartite graphs

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We study the metastable behaviour of a stochastic system of particles with hard-core interactions in a high-density regime. Particles sit on the vertices of a bipartite graph. New particles appear subject to a neighbourhood exclusion constraint, while existing particles disappear, all according to independent Poisson clocks. We consider the regime in which the appearance rates are much larger than the disappearance rates, and there is a slight imbalance between the appearance rates on the two parts of the graph. Starting from the configuration in which the weak part is covered with particles, the system takes a long time before it reaches the configuration in which the strong part is covered with particles. We will describe results in collaboration with den Hollander and Taati concerning a sharp asymptotic estimate for the expected transition time and show that the transition time is asymptotically exponentially distributed, and identify the size and shape of the critical droplet representing the bottleneck for the crossover. For various types of bipartite graphs the computations are made explicit. Proofs rely on potential theory for reversible Markov chains, and on isoperimetric results. We compare our results with the ones in [2] where the authors considered the same hard-core model, evolving according to Metropolis dynamics on finite grid graphs and investigated the asymptotic behavior of the first hitting time between its two maximum-occupancy configurations (called tunneling time). In particular they show how the order-of-magnitude of this first hitting time depends on the grid sizes and on the boundary conditions by means of a novel combinatorial method. The analysis also proved the asymptotic exponentiality of the scaled hitting time and yields the mixing time of the process in the low-temperature limit as side-result. In order to derive these results, the authors extended the model-independent framework in [1] for first hitting times to allow for a more general initial state and target subset.

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Hard shocks in (T)ASEP

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We consider the asymmetric simple exclusion process (ASEP) on $\mathbf{Z}$ with an initial data such that in the large time particle density $\rho(\cdot)$ a discontinuity at the origin is created, where the value of ρ jumps from zero to one, but $\rho(-\varepsilon), 1-\rho(\varepsilon) > 0$ for any $\varepsilon > 0$. We consider the position of a particle x_M macroscopically located at the discontinuity, and show that its limit law has a cutoff under $t^{1/2}$ scaling. Inside the discontinuity region, we show that a discrete product limit law arises, which bounds from above the limiting fluctuations of x_M in the general ASEP, and equals them in the totally ASEP. Sending $M \to \infty$, we recover the $F_{\mathrm{GUE}} \times F_{\mathrm{GUE}}$ distribution previously observed at shocks in TASEP.

Stationary half-space last passage percolation

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joint work with D. Betea[a] and P.L. Ferrari[a]

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Last passage percolation (LPP) in half-space (quadrant), a variant of Johannson's full-quadrant corner growth model, is a known model in the 1-dimensional Kardar–Parisi–Zhang (KPZ) universality class. It was introduced by Baik–Rains [1] in 2000 and considered further by Sasamoto–Imamura [6], Forrester–Nagao–Rains [5], Baik–Barraquand–Corwin–Souidan [2] and Betea–Bouttier–Nejjar–Vuletic [4]. Given a sequence of i.i.d. random variables on the half-space quadrant of integers, the last passage percolation time is defined as the maximum over sums of the random variables along up-right paths in the half-quadrant. The model is equivalent to the last passage percolation on the full quadrant where the weights are symmetric with respect to the diagonal and it is integrable when the weights are geometric/exponential random variables or come from a Poisson process of constant intensity. It was proved that the last passage time has KPZ-like $n^{1/3}$ fluctuations, where n is the size of the lattice. In the critical scaling, the limiting distribution $F(s; w)$ — first obtained in [1], is known to interpolate between the Tracy–Widom GSE ($w = -\infty$) and GOE ($w = 0$) distributions (both observed in random matrix theory) as well as the Gaussian one ($w = \infty$). Here w represents the limiting strength of the diagonal bounding the half-quadrant.

Jointly with Dan Betea and Patrik Ferrari, we study stationary half-space last passage percolation with exponential weights; stationarity means that the LPP increments are given by sums or i.i.d. random variables. Using integrable probability (pfaffian) techniques based on the theory of Schur processes with free boundaries, analytic continuation, and asymptotic analysis —with a strategy similar to Baik–Rains [1] and Baik–Ferrari–Péché [3], we obtain that the $n^{1/3}$ KPZ fluctuation limit for the last passage time in the critical scaling regime obeys a law analogous to the stationary Baik–Rains distribution law F_0 from the case of stationary full-space LPP. We further point out that unlike the Baik–Rains distribution F_0, ours depends on a parameter, the (limiting) strength of the diagonal bounding the half-quadrant. We believe this distribution to have similar universality behavior to the one first discovered by Baik and Rains.

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Some optimal control problems for non-local random systems

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The aim of the talk is to study the relations between various formulations of an optimal control problem for interacting particles systems. Starting with a random evolution of N particles we firstly show the equivalence of the Lagrangian and Eulerian formulations at the level of the value functions. Then we introduce a general setting and we discuss the Γ-convergence as the number of particles diverges $N \uparrow +\infty$. To deal with optimal control problems in space of measure we take advantage of the optimal transportation theory and of the superposition principle.

Nonparametric Bayesian estimation of the extremal dependence

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Predicting the extremes of multiple variables is important in many applied fields for risk management. The extreme-value theory proposes several approaches for modelling multivariate extremes. According to the componentwise block-maxima approach, one possibility is to focus on the so-called max-stable models, i.e. a class of asymptotic distributions for suitable normalised componentwise maxima of random vectors (e.g. Beranger and Padoan 2015).

Max-stable distributions are characherized by an extreme-value copula and margins that are members of the so-called univariate generalized extreme-value family of distributions. In particular, the extreme-value copula depends on an infinite-dimensional parameter, which is a function called the angular measure (i.e. a probability measure subjected to some constraints) that permits an interpretation of the amount of dependence (see Falk et al. 2010 for details). Thus, the estimation of the dependence structure (extremal dependence) is not straightforward. However, several nonparametric estimators for the extremal dependence have been proposed in the last few decades, see e.g. Gudendorf and Segers (2011), Bücher et al. (2011), Marcon et al. (2017) to name a few. These estimators are designed for a suitable reparametrization of the angular measure, known as the Pickands dependence function, which is easier to interpret (see Falk et al. 2010 for details).

Statistical prediction can be performed in practice most naturally via the Bayesian approach. Recently, Marcon et al. (2016) proposed a fully nonparametric Bayesian estimation method for bivariate max-stable distributions, where both dependence parametrizations are represented by means of polynomials in Bernstein form. Furthermore, assuming that the extreme-value copula is a specific parametric model, Dombry et al. (2017) proposed a Bayesian inferential method for fitting max-stable distributions in arbitrary dimensions (greater than two). These two outcomes are further extended in Padoan and Rizzelli (2019). Firstly, we describe a similar estimation framework to that in Marcon et al. (2016), but where the dependence parameterizations are described through splines. Then, we discuss the asymptotic properties of the inferential procedure for both frameworks and dependence parametrizations. Next, we describe an extension of the framework introduced in Marcon et al. (2016) for max-stable distributions in arbitrary dimensions and we discuss the asymptotic properties for the resulting inferential procedure.

The asymptotic results are derived assuming that a data sample comes from a max-stable distribution with known margins. However, in practice max-stable distributions are asymptotic models, for sufficiently large sample sizes and the margins are known apart from some unknown parameters. Finally, we discuss how the asymptotic results extends to the case where the data come from a distribution that is in a neighbourhood of a max-stable distribution and to the case where the margins of the max-stable distribution are heavy-tailed with unknown tail indices.

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From the linear superposition of Langevin-driven Brownian particles to the fractional Brownian motion

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A heterogeneous ensemble of Brownian particles whose trajectories are governed by the Langevin equation has been proposed for studying anomalous diffusion in biological systems [1]. The heterogeneity of the ensemble is characterised by a population of friction coefficients, such that from different populations different stochastic processes can be constructed [2]. When the frequency histogram of the friction coefficients follows a proper power-law, the linear superposition of these Brownian trajectories converges in probability to the fractional Brownian motion (fBm). This construction provides a dynamical framework that allows for a discussion of the fBm under physical perspectives. This result is first related with the randomly scaled Gaussian processes in the sense of the so-called generalized gray Brownian motion (ggBm), and later the application of the ggBm for modelling anomalous diffusion in biological systems is discussed [3].

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Residual varentropy of random lifetimes

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We aim to illustrate various results on the entropy and the varentropy of absolutely continuous random variables. The analysis of the entropy deserves interest in several applications related to information theory. Generally, the entropy gives a measure of the information content while the varentropy gives a measure of the concentration of information. The study of dynamic versions of such information measures is relevant in reliability theory. For instance, with reference to the residual lifetimes of systems or components, large attention is given in the literature to the so-called “residual entropy”. Along this line, we carry out the analysis of the varentropy for residual lifetime distributions (named “residual varentropy”). Specifically, we investigate:

(i) conditions for which the residual varentropy is constant;

(ii) the behaviour of the residual varentropy under linear transformations;

(iii) a kernel estimation of the residual varentropy.

We also deal with a first application oriented to a modified Ehrenfest model, in order to analyse the residual varentropy of a first-passage-time density for the Ornstein-Uhlenbeck jump-diffusion process. Furthermore, we deal with an application of the residual varentropy to the proportional hazard rates model, which in turn can be employed to the reliability analysis of series system.

(This contribution is based on a joint work with A. Di Crescenzo.)

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Birth-death and diffusion processes to model the logistic growth: analysis and comparisons

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Stochastic models obtained by including randomness in deterministic growth models turn out to be very useful in the context of population dynamics. Specifically, we deal with the logistic model, which is one of the most known and studied. For the logistic growth, population expansion decreases as resources become scarce and it finally levels off when the carrying capacity (that is the maximum population size the environment can sustain) is reached.

The aim of this contribution is to study this growth model in detail both from a deterministic and stochastic point of view. First, we describe some relevant properties of the deterministic equation and its solution, such as the inflection point, the monotony, the concavity, the maximum specific growth rate, the lag time and the threshold crossing time problem, aiming to perform comparisons with other growth models (Gompertz, Korf and modified Korf).

We also investigate various stochastic counterparts of the logistic model. We first deal with a time-inhomogeneous linear birth-death process. A sufficient and necessary condition is shown in order to attain a logistic conditional mean. Because of the existence of an absorbing endpoint (the state zero) which is unrealistic in certain applications, we further investigate a time-inhomogeneous linear birth process, which is more suitable to describe a growth behavior. Adapting the previous results to this case, the conditional mean, the conditional variance and other indices of dispersion, such as the Fano factor and the coefficient of variation are also studied. The first-passage-time problem concludes the analysis of the simple birth process.

Then we focus on diffusion processes obtained by means of the following suitable strategies:

(i) considering the limit of sequences of discrete processes described by stochastic difference equations that approximate the growth deterministic equation,

(ii) considering the limit of sequences of discrete processes that approximate the solution of the logistic equation,

(iii) considering different parametrization leading to a process having a conditional logistic mean.

Many comparisons between the diffusion processes at hand can be made through the analysis of the stochastic differential equations, the probability density function and the probability distribution. The conditional mean, the conditional variance and some indices of dispersion are determined and compared: the time-inhomogeneous linear simple-birth process has a logistic conditional mean, as well as the diffusion process obtained in case (iii), but the last one is characterized by a greater variability.

(This contribution is based on a joint work with A. Di Crescenzo.)

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Quasi-infinitely divisible processes and measures

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In this talk I will introduce quasi-infinitely divisible (QID) random measures and processes. The class of QID processes is strictly larger than the class of infinitely divisible (ID) processes. I will present explicit spectral representations and Lévy-Khintchine formulations for QID processes and random measures. Then, I will show that QID random measures are dense in the space of completely random measures under convergence in distribution. Throughout the talk, I will present many examples.

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The convergence problem for finite state mean field games

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Mean field games represent limit models for symmetric non-zero sum dynamic games when the number N of players tends to infinity. We address the problem of convergence of the feedback Nash equilibria to their mean field game limits through the so-called master equation. If there is uniqueness of mean field game solutions, i.e. under monotonicity assumptions, then the master equation possesses a smooth solution which can be used to prove the convergence of the value functions of the N players, a propagation of chaos property for the associated optimal trajectories and refined asymptotics for the empirical measures. In the non-uniqueness scenario, we consider an example with binary state space and anti-monotonous costs, in which the mean field game has exactly three solutions. We prove that the N-player game always admits a limit, which depends on the initial distribution of the players. Moreover, the value functions are shown to converge to the entropy solution to the master equation.

On stochastic Langevin and Fokker-Planck equations

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We consider a stochastic version of the Langevin equation in the form

$$d_{\mathbf{Y}} u_t(x,v) = \frac{a_t(x,v)}{2}\partial_{vv} u_t(x,v)dt + \sigma_t(x,v)\partial_v u_t(x,v)dW_t, \qquad \mathbf{Y} = \partial_t + v\partial_x. \tag{16}$$

We discuss the filtering problem associated to (16) and show existence, regularity and Gaussian-type estimates of a stochastic fundamental solution of the SPDE.

Our method is based on a Wentzell's reduction of the SPDE to a PDE with random coefficients to which we apply a revised parametrix technique to construct a fundamental solution. This approach avoids the use of the Duhamel's principle for the SPDE and the related measurability issues that appear in the stochastic framework as discussed, for instance, in [4].

This procedure also motivated the development of a technique which gives global, pointwise estimates of Itô processes, which extend some results in [2] under stronger assumptions. These are crucial to control the effects of the transform on the drift $\mathbf{Y}$ and the regularity of the coefficients in the new variables.

In the deterministic case, the parametrix method has been applied to degenerate Fokker-Planck equations, including (16) with $\sigma \equiv 0$, by several authors, [3], [1] using the so called *intrinsic* Hölder spaces. We address the lack of a suitable notion of intrinsic regularity in the stochastic framework by employing a *time-dependent* parametrix.

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Decisions under different scenarios in a finitely additive framework

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In many decision problems under uncertainty we need to choose between uncertain consequences in a set X that are contingent on the states of the world in the set S. In such situations, a finitely additive framework has shown to be the most "natural" environment, as it allows to overcome any measurability restriction, since finitely additive probabilities can always be extended, generally not in a unique way, to any larger domain. The finitely additive framework has also been extensively used for modelling "ambiguity", by considering classes of finitely additive probabilities (together with their envelopes): as a by-product, this has paved the way to the study of non-additive uncertainty measures [5, 9].
One of the most popular classes of non-additive uncertainty measures is the class of totally monotone measures [2, 10]. As shown in [8], every multivalued mapping defined on a finitely additive probability space induces a totally monotone measure. We also have that every totally monotone measure φ defined on $\wp(X)$ determines a closed convex set $\mathbf{core}(\varphi)$ of finitely additive probabilities on $\wp(X)$. Moreover, every finitely additive probability on $\wp(\mathcal{U})$, where $\mathcal{U} = \wp(X) \setminus \{\emptyset\}$, represents a totally monotone measure φ defined on $\wp(X)$ through a suitable integral expression, as shown in [4, 6].

In this work we deal with a two-stage decision process where decisions are modelled as generalized Anscombe-Aumann acts [1] mapping S to the set $\mathbf{M}(\mathcal{U})$ of finitely additive probabilities on $\wp(\mathcal{U})$, and let $\mathcal{F}$ be a convex subset of $\mathbf{M}(\mathcal{U})^S$ containing the constant acts.
Here we consider a family of preference relations $\{\precsim_H\}_{H\in\wp(S)^0}$ on $\mathcal{F}$ indexed by the set $\wp(S)^0 = \wp(S) \setminus \{\emptyset\}$ of non-impossible events. Every preference relation $\precsim_H$ can be interpreted as comparing acts under the hypothesis H.
We search for a representation in terms of a conditional functional Φ defined, for every $f \in \mathcal{F}$ and every $H \in \wp(S)^0$, as

$$\Phi(f|H) = \int_S \left[\int_{\mathcal{U}} \left(\inf_{x\in B} u(x)\right) f(s)(\mathrm{d}B)\right] P(\mathrm{d}s|H),$$

where $P(\cdot|\cdot)$ is a finitely additive full conditional probability [3] on $\wp(S) \times \wp(S)^0$ and $u : X \to \mathbb{R}$ is a bounded utility function. As shown in [4], for every $s \in S$, the finitely additive probability measure $f(s)$ on $\wp(\mathcal{U})$ determines a totally monotone measure $\varphi_{f(s)}$ on $\wp(X)$, so, the above conditional functional can be expressed as

$$\Phi(f|H) = \int_S \left[\oint_X u(x)\varphi_{f(s)}(\mathrm{d}x)\right] P(\mathrm{d}s|H),$$

where the inner integral is of Choquet type. Thus, the above conditional functional consists in a mixture with respect to a finitely additive full conditional probability of Choquet expected utilities contingent on the states of the world. In particular, due to the properties of the Choquet integral, every state-contingent Choquet expected utility is actually a lower expected utility with respect to the finitely additive probabilities in $\mathbf{core}(\varphi_{f(s)})$.
The present model generalizes the conditional version of the Anscombe-Aumann model given in [7] by considering arbitrary S and X and by introducing a form of ambiguity for the uncertainty evaluations given on consequences.
We provide a set of axioms for the family $\{\precsim_H\}_{H\in\wp(S)^0}$ assuring the existence of a unique finitely additive full conditional probability $P(\cdot|\cdot)$ and a cardinal bounded utility function u such that the corresponding functional Φ *represents* the preferences, i.e., for every $f, g \in \mathcal{F}$ and every $H \in \wp(S)^0$, $f \precsim_H g \iff \Phi(f|H) \le \Phi(g|H)$. It turns out that a rational agent in this model behaves as a Φ maximizer, so, as a maximizer of a conditional expected value of state-contingent lower expected utilities.
We finally discuss a generalization of the present model obtained by replacing the finitely additive full con-

ditional probability $P(\cdot|\cdot)$ with a family $\{\mathcal{C}_H\}_{H\in\wp(S)^0}$ of closed convex sets of finitely additive probabilities on $\wp(S)$, whose lower envelopes satisfy a form of chain rule. This gives rise to the conditional functional $\underline{\Phi}$, defined as

$$\underline{\Phi}(f|H) = \min_{P\in\mathcal{C}_H} \int_S \left[\int_{\mathcal{U}} \left(\inf_{x\in B} u(x)\right) f(s)(\mathrm{d}B)\right] P(\mathrm{d}s).$$

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Variance reduction for fast ABC using resampling

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Approximate Bayesian computation (ABC) is the state-of-art methodology for likelihood-free Bayesian inference. Its main feature is the ability to bypass the explicit calculation of the likelihood function, by only requiring access to a model simulator to generate many artificial datasets. In the context of pseudo-marginal ABC-MCMC [1], generating $M > 1$ datasets for each MCMC iteration allows to construct a kernel-smoothed ABC likelihood which has lower variance, this resulting beneficial for the mixing of the ABC-MCMC chain, compared to the typical ABC setup which sets $M = 1$. However, setting $M > 1$ implies a computational bottleneck, and in [1] it was found that the benefits of using $M > 1$ are not worth the increasing computational effort. In [2] it was shown that, when the intractable likelihood is replaced by a *synthetic likelihood* (SL, [3]), it is possible to use $M = 1$ and resample many times from this single simulated dataset, to construct computationally fast SL inference that artificially emulates the case $M > 1$. Unfortunately, this approach was found to be ineffective within ABC, as the resampling generates inflated ABC posteriors.
In this talk we show how to couple *stratified sampling* with the resampling idea of [2]. We construct an ABC-MCMC algorithm that uses a small number of model simulations ($M = 1$ or 2) for each MCMC iteration, while substantially reducing the additional variance in the approximate posterior distribution induced by resampling. We therefore enjoy the computational speedup from resampling approaches, and show that our stratified sampling procedure allows us to use a larger than usual ABC threshold, while still obtaining accurate inference.

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On a fractional Ornstein-Uhlenbeck process with a stochastic forcing term and applications

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We refer to a linear stochastic differential equation (SDE) driven by a fractional Brownian motion whose solution is a fractional Ornstein-Uhlenbeck process; in particular, here, we consider the SDE with a stochastic forcing term in the drift. Mean and covariance functions are determined and their asymptotic behavior are studied. A sort of short- or long-range dependence, under specified hypotheses on the covariance of the forcing process, is revealed. We also discuss applications of this process in neuronal modeling, providing an example of a stochastic forcing term as a linear combination of Heaviside functions with random center. Simulations of sample paths of this process are finally performed.

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Information geometry of the Gaussian Space

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This talk is based on the conference papers [1,2,3]. It presents an overview of the topic and some of the current developments.

The exponential manifold [4,5] on the finite-dimensional Gaussian space [1] has special features namely, the existence of a finite entropy and finite moments of all orders for all densities in the manifold. Moreover, it is possible to discuss the continuity of translations, Poincareé inequalities, and the generalized differentiability for densities. As a consequence, it is possible to define an exponential manifld for densities belonging to a given Orlich-Sobolev space with Gaussian weight.

A field of application is the study of the dimensionality reduction for of evolution equations in the sense of D. Brigo [2] i.e., the projection of the solutions onto a finite-dimensional exponential family.

The basic exponential representation of densities in the exponential manifold can be modified by the use of the so-called deformed exponentials for example, the Nigel Newton exponential [6]. The linear growth of the deformed exponential allows for a simplified treatment of the manifold of densities in a Sobolev space with Gaussian weight.

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On SDEs with additive noise driven by stable processes

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We deal with multidimensional stochastic differential equations (SDEs) of the form

$$X_t = x + \int_0^t b\,(X_s)\,ds + L_t, \quad x \in R^d, \;\; t \geq 0,$$

where $L = (L_t)$ is a non-degenerate d-dimensional Lévy process of stable type. In particular we concentrate on the case when $L = (L_t^1, \ldots, L_t^d)$ and $L^1, \ldots, L^d$ are independent one-dimensional symmetric stable processes of index α, $\alpha \in (0, 2)$. We consider some regularity properties of the solution X_t with respect to the initial condition x when $b : R^d \to R^d$ is smooth enough. Moreover we show well-posedness of the SDE when b is only Hölder continuous under a suitable condition on the Hölder exponent. We also present some open problems.

Measuring linear correlation between random vectors

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We introduce a new coefficient to measure linear correlation between random vectors which preserves all the relevant properties of Pearson's correlation in arbitrary dimensions. We build an empirical estimator of the newly defined correlation, give its limiting distribution and illustrate its relevance in some simulation studies. We also give some auxiliary results of independent interest in matrix analysis and mass transportation theory.

The buck-passing game on networks

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We consider a game where each player lives on a vertex of a directed graph. The strategy of each player is a probability vector on the out-edges of her vertex. Hence, a strategy profile induces a Markov chain on the directed graph. The cost of each player is the stationary measure of this Markov chain.
First we study a first class of games that admit an ordinal potential structure, therefore, ensuring the existence of pure Nash Equilibria. The proof, as a byproduct, suggests some randomized algorithms to find pure Nash Equilibria for such games.
Then we extend the results about ordinal potential to a larger class of games and characterize the potential function by means of three different representations—combinatorial, probabilistic, and algebraic—emphasizing some deep connection with the modern theory of Markov chains.

Analytical approximation of counterparty value adjustment

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Vulnerable options are derivatives that are subject to some default event, which concerns the solvability of the option's issuer (counterparty credit risk). In this case, the option price needs an adjustment to include in its quotation also the possibility of default. This is what is called the **Credit Value Adjustment (CVA)**. Typically the default event is characterized by means of a random time, representing the time of default. At the time of default there might be a total loss for the investor or a partial recovery of the investment current value might be possible.
The difficulty in the evaluation of the CVA is twofold. First of all the default time might be not completely measurable with respect to the information generated by the market prices, since it reflects also other exogenous factors. Secondly, even under full knowledge of this random time, the derivative's evaluation will call for the joint distributions of the random time and the price processes, usually very difficult to know, related to the so-called Wrong/Right-Way-Risk: that is, a decrease/increase in the credit quality of the counterparty produces a higher exposure in the portfolio of the derivative's holder.
In this framework, we shall consider the pricing of a vulnerable European option. If a so called intensity approach is used in order to characterize the distribution of the default time, conditionally to the information generated by the market prices, under appropriate conditions we may describe the joint dynamics of the asset prices, of the default time and of the other stochastic factors as a Markovian system, whose components may exhibit correlation. This correlation is going to be modeled by means of a set of parameters linking the processes driving the dynamics and the usual theory of stochastic calculus allows to set up a PDE system, whose solution, though not easily computable, may be analytically approximated.
We present different approximations as valuable alternatives to standard Monte Carlo estimates. Firstly, we reconsider a method, introduced in the papers [1] and [2], which expands theoretically the solution of the PDE system in a Taylor's series with respect to the correlation parameters (see [3]). Indeed, under quite general hypotheses, it is straightforward to verify that the solution to the PDE is regular with respect to the correlation parameters and therefore it can be expanded in series around the zero value for all of them. The coefficients of the series are characterized, by using Duhamel's principle, as solutions to a chain of PDE problems and they are therefore identified by means of Feynman-Kac formulas and expressed as expectations, that turn to be easier to compute or to approximate. Alternatively, by exploiting a keen integration by parts formula we may obtain a different representation of CVA to enucleate the contribution due to the correlation. Finally, in the recent paper [4] a method is proposed for addressing the CVA computational problem under WWR based on a change of measures, e.g. Girsanov's theorem, in the stochastic-intensity default setup. We present numerical results of these approaches in the case of GBM asset price dynamic and CIR default intensity.

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Piecewise linear processes with Poisson-modulated switching times and market models

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We introduce a class of double stochastic piecewise linear processes with a Poisson modulated exponential distributions of persistent epochs.

The first layer of stochasticity is a usual telegraph process based on an alternating Poisson process $N(t)$, and the second one (driving the change of patterns) is characterised by exponentially distributed holding times with a $N(t)$-modulated parameter.

To describe the first, consider the sequence of independent Poisson processes $N_m = N_m(t)$, which are driven by two alternating sequences of parameters, $\{\lambda_n^{(0)}\}_{n\geq 0}$ and $\{\lambda_n^{(1)}\}_{n\geq 0}$. The second is the sequence of independent holding times $\{T_m\}_{m\geq 1}$, having the Poisson-modulated exponential distribution, $\text{Prob}\{T_m > t\} = \mathbb{E}\exp\left(-\int_0^t \mu_{N_{m-1}(s)}^{(i)}\mathrm{d}s\right)$.

The model is based on a piecewise linear process $L = L(t)$, which follows two patterns alternating after the holding times T_m, that is

$$L(t) = \sum_{m=1}^{M(t)} l_{m-1}(T_m) + l_{M(t)}\left(t - T^{(+,M(t))}\right), \qquad t \geq 0,$$

where $M(t)$ is counting the number of pattern switchings, $l_m(t) = \int_0^t c^{(\varepsilon_m)}(N_m(s))\mathrm{d}s$, $m \geq 0$; $T^{(+,m)} := T_1 + \ldots + T_m$. Here $\varepsilon_m \in \{0, 1\}$ is the current state; $c^{(0)}(n)$ and $c^{(1)}(n)$ are real sequences describing the trends.

The jump component added to this process also consists of two parts. The first one calculates the jumps occurring at the arrival times of the embedded Poisson processes,

$$j(t) = \sum_{m=1}^{M(t)} j_{m-1}(T_m) + j_{M(t)}\left(t - T^{(+,M(t))}\right),$$

where $j_m(t) = \sum_{n=1}^{N_m(t)} r_m(n-1)$, $m \geq 0$. Here $\{r_m(n)\}$, $n \geq 0, m \geq 0$, are independent random jumps, occurring at tendency switching times, which are independent of counting process N_m. Assume that the distributions of jumps are alternating, such that the processes j_m with even (odd) m are identically distributed.

The second jumping part is determined by the jumps that occur when the pattern changes, at times $T^{(+,m)}$, $m \geq 1$. We assume that the jump amplitudes depend on the number of interventions $N_{m-1}(T_m)$, during the elapsed time T_m,

$$J(t) = \sum_{m=1}^{M(t)} R_{m-1}\left(N_{m-1}(T_m)\right).$$

Here, independent random variables $\{R_m(n)\}$ are the jump magnitudes which are independent of T_m, N_m and $\{r_m(n)\}$, $n \geq 0$, $m \geq 0$. Assume that $R_m(\cdot)$ are of the alternating distributions.

Theorem. *Under some regularity conditions the process $L(t) + j(t) + J(t)$, $t \geq 0$, is a martingale if and only if for the both states the parameters of the model satisfy*

$$\Delta(n) := c(n) + \lambda_n\overline{r(n)} + \mu_n\overline{R(n)} = 0, \qquad n \geq 0.$$

This class of processes is exploited for the purposes of financial modelling. In particular, we study an incomplete financial market model based on the process $L + j + J$.
The dynamics of the considered stochastic process is characterised by two alternating types of tendencies, whose holding times have Poisson-modulated exponential distribution. Moreover, the model includes two different kinds of jumps, one occurring at the tendency switchings and the other at the changes of patterns. A relevant aspect of this model is the presence of external shocks, which affect the rates of the trend switchings. This feature ensures that the model is largely flexible and thus it is suitable to describe a wide family of financial market scenarios. Specifically, it can be used to describe a financial market model whose log-returns are influenced both by global market forces and by efforts of speculators.
The market model might be interpreted as follows. Assume that the interest rates are zeros. Let

$$S_0\mathcal{E}_t(L+J) = S_0 e^{L(t)} \prod_{m=1}^{M(t)} \left(1 + R_{m-1}\left(N_{m-1}(T_m)\right)\right), \qquad t \geq 0,$$

be the essential component of asset price, which is determined by inherent market forces. This component takes into account only the jumps accompanying the pattern's switchings.
We assume that in the both states the tendency $c(n)$ and the accompanying jump amplitude $\overline{R(n)}$ are of the opposite signs:

$$c(n)/\overline{R(n)} < 0, \qquad \forall n. \qquad (*)$$

This assumption seems natural, since the market model with the stock price defined above is arbitrage-free if and only if this condition holds, see [1-2].
The jumps defined by

$$z_m(t) := \prod_{n=1}^{N_m(t)} \left(1 + r_m(n)\right), \qquad m \geq 0,$$

accompanying each tendency fluctuation inside the current pattern, can be considered as the result of external interventions of small markets players. Finally, let the stock price follow

$$S(t) = S_0\mathcal{E}_t(L+J) \times \left[\prod_{m=1}^{M(t)} z_{m-1}(T_m) \cdot z_{M(t)}\left(t - T^{(+,M(t))}\right)\right], \qquad t \geq 0.$$

If condition (*) fulfilled, then the model is still arbitrage-free, in spite of the efforts of small players.

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Hybrid non parametric priors for clustering

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Bayesian statistics has been proven effective in borrowing information between different groups or studies. Subjects in different studies may share the same unknown distribution. A well-known BNP prior to flexibly cluster probability distributions for the partially exchangeable case is the nested Dirichlet process (NDP) which is known to degenerate to the fully exchangeable case when there are ties across samples. Generalizations of the NDP have been proposed to address this issue. We propose a novel hybrid nonparametric prior which solves the problem by combining two different discrete nonparametric random structures. We derive a closed form expression of the induced random partition distribution which allows to gain a deeper insight on the theoretical properties of the model and, further, yields a MCMC algorithm for evaluating Bayesian inference of interest. Finally a BNP test of homogeneity between different groups will be displayed and it complemented by illustrative examples.

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On discrete-time semi-Markov processes

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In the last years many authors studied a class of continuous-time semi-Markov processes obtained by time-changing Markov processes by hitting times of independent subordinators. Such processes are governed by integro-differential convolution equations of generalized fractional type, having the form

$$\int_0^\infty (p(x,t-\tau)-p(x,t))\nu(d\tau) = G_x p(x,t) \tag{17}$$

where G_x is the generator of the original Markov process, while the operator on the left side is usually called generalized fractional derivative. The reason of this name is that in the case where the random time is an inverse β-stable subordinator, equation (17) reduces to

$$\frac{d^\beta}{dt^\beta} p(x,t) = G_x p(x,t) \tag{18}$$

where $\frac{d^\beta}{dt^\beta}$ is the fractional derivative of order $\beta \in (0,1)$, and the sojourn time in any state follows a Mittag-Leffler distribution.

We develop a discrete-time version of such a theory. Indeed, since their introduction, semi-Markov processes have been mostly studied in continuous time case, while discrete time processes are almost absent from the literature. Yet, in many applications, the time scale is intrinsically discrete, and for this reason the discrete time case merits a further investigation.

Our processes can be constructed as discrete-time Markov chains subordinated via discrete-time renewal processes. They converge weakly to those in continuous time under suitable scaling limits. We obtain governing equations of the form

$$\sum_{\tau=0}^{\infty} \big(p(x,t-\tau)-p(x,t)\big)\mu(\tau) = G_x p(x,t) \qquad t \in N \tag{19}$$

which is the discrete version of (17). In the case where the sojourn times follow a so-called dicrete Mittag-Leffler distribution, equation (19) reduces to

$$(I-B)^\beta p(x,t) = G_x p(x,t) \qquad t \in N \tag{20}$$

where B is the backward shift operator in the time-variable, $I-B$ is the discrete time derivative, and $(I-B)^\beta$ is its fractional power.

It is well known that (17) arises from a suitable continuous time random walk limit. In this spirit, our work proves that a connection to fractional calculus also exists before taking such a continuous-time limit.

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On the existence of continuous processes with given one-dimensional distributions

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In what follows, a *process* is always meant as a real stochastic process indexed by $[0, 1]$. A process is continuous (cadlag) if almost all its paths are continuous (cadlag).

Let $\mathcal{P}$ be the collection of Borel probability measures on $\mathcal{R}$, equipped with the weak* topology, and let $\mu : [0, 1] \to \mathcal{P}$ be a continuous map. We focus on the problem:

(*) Is there a continuous process X such that $X_t \sim \mu_t$ for each $t \in [0, 1]$?

Question (*) arises naturally in some frameworks, such as gradient flows and certain partial differential equations; see e.g. Chapter 8 of [1]. In addition, (*) is intriguing from a foundational point of view. A positive answer to (*), for instance, could be regarded as a strong version of Skorohod representation theorem.

By a result of Blackwell and Dubins [2], there always exists a process X such that, for each fixed t, $X_t \sim \mu_t$ and almost all the X-paths are continuous at t. Despite this fact, however, the answer to (*) is generally no. A simple example is $\mu_t = (1 - t)\delta_0 + t\delta_1$.

Say that μ is *presentable* if question (*) has a positive answer, namely, $X_t \sim \mu_t$ for some continuous process X and all $t \in [0, 1]$. To investigate presentability of μ, there is an obvious process to work with. Let $\mathcal{B}$ be the Borel σ-field on $(0, 1)$, λ the Lebesgue measure on $\mathcal{B}$, and

$$F_t(x) = \mu_t(-\infty, x] \text{ for all } t \in [0, 1] \text{ and } x \in \mathcal{R}.$$

Define a process Q on the probability space $((0, 1), \mathcal{B}, \lambda)$ as

$$Q_t(\alpha) = \inf\{x \in \mathcal{R} : F_t(x) \geq \alpha\} \qquad \text{for all } t \in [0, 1] \text{ and } \alpha \in (0, 1).$$

Such a Q may be called the "quantile process" and its finite dimensional distributions are

$$\lambda\Big(Q_{t_1} \leq x_1, \ldots, Q_{t_k} \leq x_k\Big) = \min_{1 \leq i \leq k} F_{t_i}(x_i)$$

where $k \geq 1$, $t_1, \ldots, t_k \in [0, 1]$ and $x_1, \ldots, x_k \in \mathcal{R}$. In particular, $Q_t \sim \mu_t$ for all t so that μ is presentable if Q is continuous.

This note is devoted to problem (*). Two main results are proved:

- Q is continuous if and only if $\lambda^*(J) = 0$, where λ^* is the λ-outer measure and

  $$J = \{\alpha \in (0, 1) : F_t(x) = F_t(y) = \alpha \text{ for some } t \in [0, 1] \text{ and } x < y\}.$$

  Among other things, this fact provides an useful sufficient condition for presentability. Indeed, μ is presentable whenever μ_t is supported by an interval (possibly, by a singleton) for all but countably many t.

- μ is presentable if and only if Q is continuous. Practically, this means that, to decide whether μ is presentable, there is no loss of generality in investigating continuity of Q.

We finally mention two open problems.

One is to replace ”continuous” with ”cadlag” in problem (*). Namely, to assume $\mu : [0, 1] \to \mathcal{P}$ cadlag and investigate the problem

(**) Is there a cadlag process X such that $X_t \sim \mu_t$ for each $t \in [0, 1]$?

Incidentally, we are not aware of any μ which provides a negative answer to (**). In particular, if $\mu_t = (1-t)\delta_0 + t\delta_1$, a cadlag process X satisfying $X_t \sim \mu_t$ for all t is

$$X_t = 1_{[U,1]}(t),$$

where the random variable U is uniformly distributed on $(0, 1)$.

A second open problem arises if $\mathcal{R}$ is replaced with an arbitrary metric space S. Let $\mathcal{P}(S)$ be the set of Borel probability measure on S, equipped with the weak* topology, and let $\mu : [0, 1] \to \mathcal{P}(S)$ be a continuous map. Then, problem (*) turns into

(***) Is there a continuous, S-valued process X such that $X_t \sim \mu_t$ for each $t \in [0, 1]$?

If $S = \mathcal{R}$ and each μ_t is supported by an interval, then μ is presentable. Hence, a question is whether (***) has a positive answer under some (reasonable) assumption on the supports of the μ_t.

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How to interpret probability using a purely mathematical approach

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1. Introduction

Authors discuss and compare the classical and Bayesian statistics. For example, Jacob Steinhardt holds that when Bayesian updating can be performed computationally efficiently, then Bayes is the optimal approach. Even when some of these assumptions fail, Bayes can still be a fruitful approach. However, by working under weaker or adversarial assumptions, classical statistics can perform well in very complicated domains [1].
Theorists have not reached a shared conclusion and at the practical level statisticians usually decide to apply the classical or the Bayesian methods on the basis of pragmatic criteria [2]. In fact, several experts are inclined to minimize the differences between the two statistical schools. Although this stance turns out to be very questionable, the frequentist and subjective theories of probability underpinning the two statistical approaches prove to be irreconcilable in point of logic. The frequentist theory defines probability as the limit of the relative frequency in a large number of trials; the subjective theory sees probability as an individual person's measure of belief that an event will occur. Even when the two statistical methods provide the same numerical value of probability, the two numbers denote incompatible concepts:
How to solve this contradiction?
A recent textual analysis [3] demonstrates that the principal constructs of probability are strongly influenced by philosophical arguments and personal convictions, that is why debates go ahead without reaching sherable conclusions. I made an attempt, using a purely mathematical approach, to establish the meaning of probability in applications and a rule for the adoption of the most suitable statistical method.

2. The Mathematical Approach

Let the Kolmogorov postulates be true, and the random event A is the argument of probability $P(A)$ in abstract. In applications let us suppose that the random events make a sequel of Bernoulli trials; that is, the experiment outcome can be either of two possible results: 'success' and 'failure'. By convention, the symbol A denotes the success; $F(A_n)$ is the relative frequency of success in n trials; and $F(A_1)$ is the relative frequency of success in a single trial. Take the following alternative assumptions:

a) $n \to \infty$,
b) $n = 1$.

Two theorems prove [4][5] that:

If **a** = true, then $F(A_n) \to P(A_n)$;
If **b** = true, then $F(A_1) \neq P(A_1)$.

The first is the strong form of the *theorem of large number* (TLN) demonstrated by Borel that shows how the probability can be confirmed or falsified by means of practical observations; $P(A_n)$ can be controlled, at least in principle, in the physical reality and thus $P(A_n)$ is a real quantity. The second, called *theorem of a single number* (TSN), demonstrates that the probability of a single experiment cannot be validated in the physical world. It is not a question of inaccuracy or testing methods, TSN demonstrates that nobody can corroborate $P(A_1)$. This fault entails that $P(A_1)$ should be discarded from the scientific domain,as long as the scientific method does not accept a parameter which cannot be controlled by means of tests. Although many people are concerned with the probability of a single experiment.
How to circumvent the untestability of $P(A_1)$?
Semiotics teaches us that words and numbers are pieces of information. By definition a piece of information stands for something. For example, the number 0.32[$= P(A_n)$] represents a physical quantity. The number 0.32[$= P(A_1)$] does not represent a physical quantity, yet it has a semantic value. As a consequence

of the informational position held by $P(A_1)$, subjectivist authors recycle $P(A_1)$ that should be refused. They ascribe a subjective significance to $P(A_1)$, that is employed to qualify a personal credence about the occurrence of A_1.
The present method does not establish the 'true' significance of probability; TLN and TSN prove that probability has double meaning depending on the specific hypothesis, that is to say specific practical circumstances. Assumptions **a** and **b** do not overlap, and thus the theorems legitimate both the frequentist probability and the subjective probability with the support of the semantic concepts.

3. Implication and Conclusion

The present approach does not fix the characteristics of probability on the basis of personal choices; it does not present a dogmatic view which inevitably fires endless philosophical debates. TLN and TSN lead to the following conclusions:

1) Probability $P(A)$ is unique in abstract, whereas it has two different meanings in applications depending on the arguments A_1 and A_n.

2) The frequentist and subjective definitions are compatible because they refer to the ensuing distinct assumptions **a** and **b**.

3) Experts are allowed to use either the classical or the Bayesian statistics in the working environment, however the theorems of large numbers and a single number furnish rigid criteria for selecting the most appropriate statistics. The following rules derive from the theorems:

- Classical statistics must be used only under assumption **a**;
- Bayesian statistics is available only under assumption **b**.

Since decades philosophers spend energies in sustaining various interpretations of probability without reaching a sharable conclusion. The present mathematical approach assigns distinct meanings to probability using TLN and TSN and proves how those meanings are not irreconcilable as usually credited.

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Nodal lengths of random spherical harmonics

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In this talk we study the asymptotic behaviour of the nodal length of random spherical harmonics f_ℓ of high degree $\ell \to \infty$, i.e. the length of their zero set $f_\ell^{-1}(0)$. We will prove that the nodal lengths are asymptotically equivalent, in the L^2-sense, to the "sample trispectrum", i.e., the integral of $H_4(f_\ell(x))$, the fourth-order Hermite polynomial of the values of f_ℓ. A particular by-product of this is a Quantitative CLT (in Wasserstein distance) for the nodal length, in the high energy limit. If time permits we will investigate how much the nodal length behaviour characterizes the full geometry of eigenfunctions, i.e., the behaviour of excursion sets for arbitrary levels. This talk is mainly based on [1] and [2].

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Non-geometric rough paths on manifolds

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Rough paths were introduced by Lyons in [1] as enhanced irregular paths, suited to drive differential equations. In the two decades since its conception, this theory has been shown to give a rigorous pathwise meaning to equations driven a wide variety of stochastic processes beyond classical semimartingales. A rough path is geometric when it satisfies a first order chain rule: the prototypical example in stochastic analysis is a semimartingale X together with its Stratonovich iterated integrals $\int_s^t (X_u - X_s) \otimes \circ dX_u$. Recent works explore the theory of geometric rough paths on manifolds [2,3]. Adopting the framework of [4] and using old ideas of Schwartz and Meyer [5], we explain how to extend the theory to non-geometric rough paths of bounded $3 > p$-variation, such as the Itô lift of a semimartingale. A connection on the manifold M defines an integration theory for (controlled) rough paths, and rough integrals are independent of the connection when the rough path is geometric. Rough differential equations, for which both the driver and the solution take values in a manifold, are addressed in this setting. The theory is revisited in the extrinsic setting, i.e. when M is isometrically embedded in Euclidean space. Finally, we illustrate how parallel transport and Cartan development can be defined for non-geometric rough paths: these constructions involve lifting the connection on M to the manifold TM. This is work in progress, supervised by Damiano Brigo, Thomas Cass and John Armstrong.

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Sub-ballistic random walks among biased random conductances in one dimension: a story of wells and walls

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We consider a random walk among i.i.d. random conductances in one dimension and add an external bias. It is known that, when both the expectation of the conductances and of their inverse is finite, X_n/n converges a.s. to a strictly positive constant. We study instead the sub-ballistic regime of the walk, where the conductances decay polynomially at $+\infty$ with exponent α_∞ and at 0 with exponent α_0, satisfying $\alpha := \min\{\alpha_\infty, \alpha_0\} < 1$. We show that X_n/n^α converges to the inverse of an α-stable subordinator. The fine properties of the tails of the conductances at $+\infty$ and at 0 determine what parts of the environment are responsible for the critical slow-down of the walk: they can be either very large conductances (*wells* of the potential), either very small conductances (*walls* of the potential) or, in particular cases, a combination of the two (*wells-and-walls*). In this sense, the trapping mechanism is richer than the one observed in higher dimension in [4], where only large conductances can block the walk, and the one of i.i.d. random environments of [3], where the depth of potential valleys has a more regular behavior.

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Logical operations among conditional events: theoretical aspects and applications

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We generalize the notions of conjunction and disjunction of two conditional events to the case of n conditional events. These notions are defined, in the setting of coherence, by means of suitable conditional random quantities with values in the interval [0, 1]. We also define the notion of negation, by verifying De Morgan's Laws. Then, we give some results on coherence of prevision assessments for some families of compounded conditionals and we show that some well known properties which are satisfied by conjunctions and disjunctions of unconditional events are also satisfied by conjunctions and disjunction of conditional events. We also examine in detail the coherence of the prevision assessments related with the conjunction of three conditional events. We consider the relation between the conjunction and other different definitions of conjunction among conditional events. In particular, we consider the notion of quasi-conjunction which is largely studied in non monotonic reasoning. Based on conjunction, we also give a characterization of p-consistency and of p-entailment, with applications to the inference rules *And*, *Cut*, *Cautious Monotonicity*, and *Or* of System P. Then, we examine some non p-valid inference rules (*transitivity* and an example from Boole) by also illustrating two methods which allow to suitably modify non p-valid inference rules in order to get inferences which are p-valid. We introduce a notion of iterated conditional and we characterize p-entailment by showing that a (p-consistent) family $\mathcal{F} = \{A|H, B|K\}$ p-entails $E|H$ if and only if $(E|H)|\mathcal{C}(\mathcal{F}) = 1$, where $\mathcal{C}(\mathcal{F})$ is the conjunction of the conditional events in $\mathcal{F}$.

Keywords
Conditional events, conditional random quantities, conjunction, disjunction, negation, coherent prevision assessments, coherent extensions, quasi conjunction, probabilistic reasoning, p-entailment, inference rules, iterated conditionals, System P.

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Hydrodynamics and duality in dynamic random environment

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For the simple exclusion process evolving in a symmetric dynamic random environment, we derive the hydrodynamic limit from the quenched invariance principle of the corresponding random walk. For instance, if the limiting behavior of a test particle resembles that of Brownian motion on a diffusive scale, the empirical density, in the limit and suitably rescaled, evolves according to the heat equation.
Our goal is to make this connection explicit for the simple exclusion process and show how self-duality of the process enters the problem. This allows us to extend the result to other conservative particle systems (e.g. IRW, SIP) which share a similar property.

Limit theorems for the non-homogeneous fractional Poisson process

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The fractional non-homogeneous Poisson process was introduced by a time-change of the non-homogeneous Poisson process with the inverse alpha-stable subordinator. We propose a similar definition for the (non-homogeneous) fractional compound Poisson process. We give both finite-dimensional and functional limit theorems for the fractional non-homogeneous Poisson process and the fractional compound Poisson process. The results are derived by using martingale methods, regular variation properties and Anscombe's theorem. Eventually, some of the limit results are verified in a Monte Carlo simulation.

Optimal control of a stochastic phase-field model for tumor growth

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We study well-posedness and optimal control of a diffuse interface model for tumor growth with stochastic perturbations. The system couples a Cahn-Hilliard-type equation for the order parameter (difference in volume fractions between the healthy and necrotic cells) with a reaction-diffusion equation for the nutrient (glucose). Important biological phenomena as tumor-proliferation, tumor-apoptosis, nutrient-consumption, and effects of cytotoxic drugs are taken into account. The stochastic perturbations account for the random microscopic uncertainties of the system, and act both on the evolution of the order parameter and on the one of the nutrient. After introducing an objective cost-functional arising naturally from applications, we prove existence of an optimal control and first-order necessary conditions for optimality through the analysis of the linearized state system and the adjoint system. This study is based on a joint work with Elisabetta Rocca (Università di Pavia, Italy) and Carlo Orrieri (Università di Trento, Italy).

Representation of multivariate Bernoulli distributions with a given set of specified moments

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Dependent binary variables play a key role in many important scientific fields such as clinical trials and health studies. The problem of the simulation of correlated binary data is extensively addressed in the statistical literature. The simulation problem consists of constructing multivariate distributions for given Bernoulli marginal distributions and a given correlation matrix ρ. We propose a new but simple method to represent multivariate Bernoulli variables belonging with some specified moments. This method represents the mass functions of the given class of multivariate Bernoulli distributions as points of the convex hull whose generators are mass functions which belong to the same class. Our main contribution is to provide a method and develop an algorithm to find the extreme rays of this convex hull. We also provide the bounds that all the moments must satisfy to be compatible and the possibility to choose the best distribution according to a certain criterion. For the special case of the Fréchet class of the multivariate Bernoulli distributions with given margins we find a polynomial characterization of the class. This representation is fully characterized, since our approach allows us to find necessary and sufficient conditions on the polynomial parameters to have a mass function in the Fréchet class. Our characterization allows us to have bounds for the higher order moments. It is worth noting that this method puts no restriction either on the number of variables or on the specified moments. The range of applications is limited only by the amount of computational effort required, because the number of extremal rays increases very quickly as the dimension of the multivariate Bernoulli variables increases. This method provides a new computational procedure to simulate multivariate distributions of binary variables with some given moments. We also show how this method offers the opportunity to choose the *best* distribution according to a certain criterion. For example, as the moments of multivariate Bernoulli are always positive, it could be of interest to find one of the distributions with the smallest sum of all the moments with order greater than 2. This problem can be efficiently solved using linear programming techniques.
This work is based on Fontana, Semeraro (2008).

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Can one define conditional expectations for probability charges?

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The concept of Conditional Expectation (=CE) is central in σ–additive probability theory as was already highlighted in Kolmogorov's monograph. However, CEs do not seem to have achieved the same status in the case of *finitely additive* probabilities (also called probability charges). The reason for this difference lies in the fact that the Radon–Nikodym theorem, indispensable for σ–additive measures in proving the existence of CEs, has no *exact* general equivalent in the finitely additive case.
In this contribution we rely heavily on the paper by de Amo et al. (1999) for an approximate Radon–Nikodym theorem and on the book by the Bhaskara Rao's (1983) on charges, namely finitely additive measures, and propose a definition of finitely additive CEs — thus trying to answer in the positive the question of the title — and study their properties according to this definition. We show that a Conditional Expectation in this setting turns out to be a sequence of simple functions rather than a single random variable; however, we also show that most of the properties of countably additive CEs carry over to the new context.

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Queue-based activation protocols for random-access wireless networks with bipartite interference graphs

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We study queue-based activation protocols in random-access networks. The network is modeled as an interference graph. Each node of the graph represents a server with a queue. Packets arrive at the nodes as independent Poisson processes and have independent exponentially distributed sizes. Each node can be either active or inactive. When a node is active, it deactivates at unit rate. When a node is inactive, it activates at a rate that depends on its current queue length, provided none of its neighboring nodes is active. Thus, two nodes that are connected by a bond cannot be active simultaneously. This situation arises in random-access wireless networks where, due to interference, servers that are close to each other cannot use the same frequency band. In the limit as the queue lengths at the nodes become very large, we compute the transition time between the two states where one half of the network is active and the other half is inactive. We consider networks with a general bipartite graph as interference graph. We decompose the transition into a succession of nucleations on complete bipartite subgraphs. The total transition time depends in a delicate way on the architecture of the graph and is achieved in a greedy way. We formulate a greedy algorithm that takes the graph as input and gives as output the set of transition paths the system is most likely to follow. Each of these paths is described by a sequence of subsets of activating nodes forming complete bipartite subgraphs. Along these paths we compute both the mean transition time of the graph and the law of the transition time on the scale of its mean. In a previous work, the authors have studied the transition in the situation of complete bipartite interference graphs. We notice three regimes of behaviour depending on the aggressiveness of the activation rates.

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Minimization of the Kullback-Leibler divergence over a log-normal exponential arc

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The geometry of nonparametric exponential models and its analytical properties in the topology of Orlicz spaces started with the paper of Pistone and Sempi [4]. In their framework, the starting point is the notion of *maximal exponential model* centered at a given positive density p, which is defined using the Orlicz space associated an exponentially growing Young function. One of the main result in the subsequent work by Cena and Pistone [1] states that any density belonging to the maximal exponential model centered at p is connected by an *open* exponential arc to p and viceversa (by *open*, we essentially mean that the two densities are not the extremal points of the arc). Further upgrades have been proved in in Santacroce, Siri and Trivellato [5], [6], [7] and [8]. In [5], the equivalence between the equality of the maximal exponential models centered at two (connected) densities p and q and the equality of the Orlicz spaces referred to the same densities is proved.
Applications of statistical exponential models built on Orlicz spaces can be found in several fields, such as differential geometry, algebraic statistics, information theory. In mathematical finance, applications to convex duality have been recently given by [7] and [8], while in physics by, e.g., [3] and [2].
Here, we briefly recall some results on exponential connections by arc and their relation to Orlicz spaces. We then deal with the minimization of the Kullback-Leibler divergence of a given log-normal density from a log-normal open exponential arc. The optimal density is explicitly computed as well as the corresponding minimal divergence. It turns out that the optimal divergence is robust with respect to homogeneous transformations which leave the correlation of the random variables involved unchanged.
Our results are then specified in the Merton's financial market model. The minimization of the entropy over the exponential arc is the dual problem of a classical exponential utility maximization problem in which an uncertainty exists on the choice of the reference probability measure. Modeling this uncertainty by varying the reference measure over an open exponential arc ensures a robustness result in the terms specified before.

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Estimating functions for discretely observed diffusion processes conditioned to nonabsorption

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In the framework of this talk, we study a diffusion process evolving in a domain with forbidden states. The process is said to be killed when it hits the trap. Such diffusion models arise in any setting where an absorbing state is considered, such as extinction events in chemical reaction networks and population dynamics, or firing in stochastic neuronal models. Inspired by [3], we introduce a class of estimating functions which are proved consistent under the theory of quasi-stationary distributions for diffusion processes, see [1,2,4]. The practical behavior of the estimators is studied through simulated examples as well.

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Hilbert modules in probability

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Hilbert modules are an invaluable tool in the study of dynamical systems both classical and quantum. Starting from the description of the *universal Markov process* of a semigroup of transition kernels in terms of Hilbert modules, motivating in this way carefully also how the notion emerges naturally, we illustrate some of the consequences. Several of these consequences relate a Markov process (that is, a classical irreversible dynamical system) to questions about irreversible and reversible quantum dynamical systems.

Random denials in rumor spreading models

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Recently, models for the spread of rumor have attracted large interest because of their role in various contexts of the real life: modern technology, business marketing and sociology (cf. [2], [4], [5], [8], [10]). There are a lot of applications of such processes, for example the virus propagation in social and computer networks, the diffusion of innovations, the occurrence of information cascades in social and economic systems, information diffusion in a society through the word of mouth mechanism ([11] and references therein, [12], [13]). Moreover, a rumor spreading can shape the public opinion in a country, greatly impacts financial markets and causes panic in a society. To analyze the spreading and cessation of the rumor, models are often formulated as social contagion processes (cf., for example, [1], [6]). The DK-m was the first classical rumor spreading model. It was proposed by Daley and Kendal in 1960s (cf. [3]). Then, Maki and Thomson developed an other classical model (MT-m) (cf. [9]). In these models people are divided into three groups: the Spreaders know and transmit the rumor, the Ignorants do not know the rumor and the Stiflers know the rumor but do not transmit it; the rumor spreads through pair-wise contacts between Spreaders and the other people. In the DK-m, Spreader-Ignorant contact will convert the Ignorant to Spreader; Spreader-Spreader contact will convert both Spreaders to Stiflers and Spreader-Stifler contact will stifle the spreader. In the MT-m, when a Spreader contacts another Spreader, only the initiating one becomes a Stifler. A large amount of works have studied the dynamics and limit behaviors of these systems and their variants (for example cf. [7]).
In this scenario we insert the concept of denial. The denials occur at random instants of time, they reset the system to the initial condition (i.e. only one person is able to spread the rumor and all others are ignorant) and then the process starts following the previous rules. Generally, during the spreading of a rumor one can consider the effect of an external entity that denies the rumor so the process is reseted to the initial state, i.e. there is only one spreader, the initial one, that renews the spreading process. For example, if we consider the rumor as a worm, the denial represents the effect of an anti-virus that restores the initial condition in which the hacker reinforces the virus (or he designs a new virus). In business marketing, the rumor is the advertisement of a product, the denial can be an information that discredits the product (in this case the society improves the product or defends oneself from the accuses), or the launch of a new concurrent product. In both the cases after the denial the rumor restarts with a new advertisement. In a political campaign, we can explain the rumor as the promoting of a candidate, the denial can be the consequence of a scandal, the restarting is the refusal of the scandal.
We introduce the denials in two models: the classic DK-m, and in a variant in which each Spreader can transmit the rumor at most k times before becoming a Stifler. Obviously, if k tends to infinity the model B can be connected to the DK-m. We study the stationary density of the DK-m with denials and we show that at most the half of the population can be informed about the rumor. Then, we consider the model B subject to denials and we discuss the stationary density. In this case the percentage of ignorants depends on k, in addition the rate at which the denials occur. Moreover, when k grows the model behaves like the DK-model with denials and a good match is found already for $k = 6$.

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Role of multivariate conditional hazard rates in the analysis of non-transitivity and aggregation/marginalization paradoxes for vectors of non-negative random variables

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The talk will start with a brief review about well-known examples concerning phenomena of non-transitivity and aggregation/marginalization paradoxes, in the field of Probability. As it is well-known, this theme is rather classic not only in Probability, but also in Statistics and in Decision Theory, where it has even more relevance. In recent times, a renewed interest has arisen, for different respects.
After demonstrating some relations among such phenomena and such (apparent) paradoxes, we will specifically concentrate attention on n non-negative random variables $X_1, ..., X_n$, under the condition of absolute continuity for their joint probability distribution.
Starting from $X_1, ..., X_n$ we will consider random variables of the form $M_A := \min_{j\in A} X_j$ and events of type $(M_A = X_i)$, for $A \subseteq [n] \equiv \{1, 2, ..., n\}$, $i \in A$. We will point out some controversial aspects that may emerge in this context and, more in particular, in establishing stochastic orderings among the variables M_A's, for different subsets A.
Incidentally, we notice that the simplest case is the one of independent and exponential (non-identically distributed) variables $X_1, ..., X_n$, where no controversial aspect can be met. Actually, from an heuristic view-point, some aspects of the other cases may appear controversial just because intuition is often based on the independent-exponential case.
Under absolute continuity for the joint probability distribution of $X_1, ..., X_n$, stochastic dependence can be described in terms of the set of multivariate conditional hazard rate (m.c.h.r.) functions.
We will show that, under stochastic dependence, such a tool allows the analysis of the variables M_A's to be simplified by considering a series of cases only involving independent variables. We will also point out that the same tool is specially convenient to understand how apparently paradoxical situations can arise and to single out conditions under which such situations can be excluded.
This work is in particular based on a collaboration with Emilio De Santis and Yaakov Malinovsky.

Analitycal valuation of surrender options in life insurance contracts with minimum guaranteed

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We consider a life insurance contract based on partecipating policies, whose investment profits are credited to the policy holder through a continuously compounded interest rate crediting scheme with a minimum guaranteed, taking into account the risk of default for the insurance company. The policy holder may exit the contract at any time before maturity by exercising an American-type option, the so called surrender option. The value function of the corresponding finite-time horizon optimal stopping problem is known as the "surrender value". As expected, the optimal exercise boundary makes the state space split into two regions: the stopping region where it is optimal to immediately surrender the option, and the continuation region where instead it is optimal to keep holding the contract. An interesting feature of our problem is that the stopping region is disconnected, a novelty in the literature of partecipating policies with surrender option. We characterize the optimal exercise boundary in terms of a continuous, piece-wise monotonic curve on $\mathbb{R}_+$. Moreover, thanks to the geometry of the optimal boundary we establish global C^1 regularity of the value function.

Spatio-temporal spike pattern detection in experimental parallel spike trains using SPADE

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The cerebral cortex consists of a network of highly interconnected neurons, which communicate through the transmission of action potentials, or spikes. It is hypothesized that neurons organize and co-activate in groups, called cell assemblies [1], which act as information processing units. One possible signature of the expression of such active assemblies are spatio-temporal patterns (STPs), i.e. precise temporal sequences of spikes of multiple neurons. Thus, one occurrence of an STP of two spikes is identified by the two neurons $\{n_1, n_2\}$ firing, and by the time delay $\delta_{1,2}$ between the two spikes. Moreover, it is hypothesized that the information of spiking activity lies in the time of spike occurrence [2], therefore spiking activity can be mathematically expressed as point processes. Thus, the STP detection problem can be reduced to looking for significant repetitions of patterns in multi-dimensional point processes, where each dimension represents the activity of one recorded neuron.

We developed a method to find and statistically evaluate such structures, called SPADE (Spike PAttern Detection and Evaluation) [3,4]. SPADE involves three steps: first it identifies repeating STPs using Frequent Itemset Mining [5]. Then, it evaluates the detected patterns for significance through a Monte Carlo technique using surrogate data under the null hypothesis of mutual independence of the parallel point processes representing the firing of each neuron. Finally, SPADE identifies and removes patterns being a byproduct of true patterns and background activity by evaluating the conditional significance of each pattern against all patterns partially overlapping with it.

We hypothesize that different assemblies are active at different points in time in relation to behavior. Therefore, we analyze neuronal activity recorded from pre-/motor cortex of a macaque monkey performing a reach-to-grasp task [6]. After an instructed preparatory period, monkeys had to pull and hold an object by using either a side or a precision grip, and using either high or low force (four behavioral conditions). The data in form of parallel spike trains are segmented according to different behavioral phases during the experiment, and are analyzed separately with the SPADE method. Each significant STP detected by SPADE is identified by its neuron composition, the times of the spikes involved in the pattern, and the total number of occurrences of that particular STP.

Our results show that STPs occur throughout all trial phases, though they occur more often during the movement period. Moreover, we find that the pattern compositions vary with respect to the behavioral conditions (side or precision grip), suggesting that different neuronal assemblies are active for the performance of the different behaviors. We also find that same neurons participate in different STPs, however with different combinations of lags between the spikes in the pattern. This means that individual neurons are involved in different patterns at different points in time. We call these neurons hub neurons, suggesting that they have a primary role in the coordination of pattern activity. Finally, we show that individual spikes of some neurons may take part in different patterns.

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‘Essential enhancements’ for activated random walks

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This talk considers an interacting particle system with conserved number of particles known as *activated random walk* (ARW), where *active particles* perform a continuous time simple random walk and become inactive with rate $\lambda \in (0, \infty)$, while *inactive particles* do not move become active whenever they share the vertex with an active particle. The motivation of studying this model is two-fold. Firstly, ARW was introduced in the physics literature as a more mathematically tractable approximation of the stochastic sandpile model, and is one of the paradigm examples of the widely studied phenomenon of *self-organized criticality*. Moreover, it can be regarded as a special case of driven *diffusive epidemic process* introduced by Spitzer in 1970s and studied in a number of papers after that.
This model undergoes an *absorbing-state phase transition* at a critical particle density $\mu_c = \mu_c(\lambda)$, which separates a regime where the activity dies out (low density) from a regime where the activity is sustained (high density). In the last years significant effort has been made for proving basic properties of the critical curve $\mu_c(\lambda)$, which is known to be *non-trivial* and *universal* on vertex-transitive graphs. We prove a new general fact of $\mu_c(\lambda)$, namely that it is a *continuous* function of the parameter λ, and provide a general bound on its curvature. Our results hold in great generality, namely on any graph with bounded degree, and are achieved introducing the method of ‘essential enhancements’ in the context of Abelian networks, which we believe is of independent interest.

Spectral density-based and measure-preserving ABC for partially observed diffusion processes

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Over the last decades, stochastic differential equations (SDEs) have become an established and powerful tool for modelling time dependent, real world phenomena that underlie random effects. Besides the modelling, it is of primary interest to estimate the underlying model parameters. This is particularly difficult when the n-dimensional stochastic process is only partially observed through a function of its coordinates. Moreover, due to the increasing model complexity, needed to understand and reproduce the data, the underlying likelihoods are often unknown or intractable. Among likelihood-free inference methods, here we focus on the Approximate Bayesian Computation (ABC) approach [1,2].

When applying ABC to stochastic processes, two major difficulties arise. First, different realisations from the output process with the same choice of parameters may show a large variability due to the stochasticity of the model. Second, exact simulation schemes are rarely available, requiring the derivation of suitable numerical methods for the synthetic data generation. To reduce the randomness in the data coming from the SDE, we propose to build up the statistical method (e.g., the choice of the summary statistics) on the underlying structural properties of the model. Here, we focus on the existence of an invariant measure, and we map the data to their estimated invariant density and invariant spectral density. Then, to ensure that the model properties are kept in the synthetic data generation, we adopt a structure-preserving numerical scheme that, differently from the commonly used Euler-Maruyama method, preserves the properties of the underlying SDE.

The derived Spectral Density-Based and Measure-Preserving ABC method is illustrated on the broad class of partially observed Hamiltonian SDEs [3], both with simulated and with real electroencephalography (EEG) data, and on the stochastic FitzHugh-Nagumo model (FHN) [4,5]. Both models are ergodic, resulting in output processes admitting an invariant measure that is preserved by a properly derived numerical splitting scheme.

Note that, the proposed ABC method can be directly applied to all SDEs characterised by an invariant distribution, for which a measure-preserving numerical scheme can be derived.

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The non local diffusion equation and the aggregation of Brownian motion

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A class of integro-differential Volterra equations whose convolution kernel depends on the vector variable are considered and a connection of these equations with a class of semi-Markov processes is presented. The variable order $\alpha(x)$-fractional diffusion equation is a particular case of our analysis and we show that it is associated with a suitable (non-independent) time-change of the Brownian motion. The resulting process is semi-Markovian and its paths have intervals of constancy, as it happens for the delayed Brownian motion, suitable to model trapping effects induced by the medium. However in our scenario the interval of constancy may be position dependent and this means traps of space-varying depth as it happens in a disordered medium. The strength of the trapping is investigated by means of the asymptotic behaviour of the process: it is proved that, under some technical assumptions on $\alpha(x)$, traps make the process non-diffusive in the sense that it spends a negligible amount of time out of a neighborhood of the region $\text{argmin}(\alpha(x))$ to which it converges in probability under some more restrictive hypotheses on $\alpha(x)$.

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Stein-Malliavin approximation for local geometric functionals of random spherical harmonics

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Recently, considerable interest has been drawn by the analysis of geometric functionals for the excursion sets of random eigenfunctions on the unit sphere (spherical harmonics). Hence, to describe these regions, the so-called Lipschitz-Killing Curvatures have been investigated. In dimension 2, they correspond to the area, half of the boundary length and the Euler-Poincaré characteristic. Since they are random features, the interest is to compute their mean, their variance and to establish a Limit Theorem. In this talk we extend the results known for the 2-dimensional sphere to spherical caps. In particular, we focus on the asymptotic behavior of the excursion area, in the case of level sets different from zero, and of the nodal lines (level equal to zero) for random spherical harmonics restricted to shrinking domains. Then, a Central Limit Theorem is established in both cases exploiting Stein-Malliavin techniques.

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A diffusion process related to a Gompertz curve with multiple inflection points

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Growth curves with sigmoidal behavior constitute one of the most widely used tools to analyze data in multiple fields of application. The great diversity of existing curves of this type, and the various possible approaches that have been established in their study, make countless mathematical models that have been proposed for study and, therefore, the literature on this subject is large.

Independently of the model being considered, all of them are built to study the behavior of variables that present, in their evolution, some common characteristics: at the beginning a slow growth is observed, followed by a rapid growth (of exponential type) that is slowing down gradually until reaching an equilibrium value (usually called *carrying capacity* or level of saturation of the system). However, there are multiple real situations in which the maximum level of growth is reached after successive stages, in each of which there is a deceleration followed by an explosion of exponential type. This means that the usual sigmoidal growth curve is replaced by another one in which more than one inflection point is observed indicating the growth changes mentioned. A typical example of this behavior is observed in the growth of various fruit species (Álvarez, A. and Boche, S., 1999). But not only in this context the double or multiple sigmoid models have a place. For example, Cairns et al. (2008) have used double-sigmoidal models to study fatigue profiles in mouse muscles, while Amorim et al. (1993) detected this type of behavior in the different phases in which the fungus Ustilago Scitaminea Sydow infects the sugarcane producing the typical coal of said plant.

Usually, most of the multi-sigmoidal growth models cited above are deterministic and do not incorporate information apart from that provided by the variable under study. In this work we address this problem through the introduction of a diffusion process whose mean function obeys a pattern of pluri-sigmoidal behavior. In particular, we will deal with the case of a type-Gompertz growth with multiple inflection points, following the idea embodied by Amorim et al. (1993) in the case of the monomolecular curve and generalized Gompertz curve.

Let $Q_{\boldsymbol{\beta}}(t) = \sum_{l=1}^{p} \beta_l \, t^l$ a polynomial of degree $p > 1$, where $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)^T$ denotes a real-valued parametric vector with $\beta_p > 0$. We define the plurisigmoidal Gompertz curve as

$$f_{\boldsymbol{\theta}}(t) = f_{\boldsymbol{\theta}}(t_0) \exp\left(-\alpha\left(e^{-Q_{\boldsymbol{\beta}}(t)} - e^{-Q_{\boldsymbol{\beta}}(t_0)}\right)\right), \; t \geq t_0 \geq 0; \; \alpha, \; f_{\boldsymbol{\theta}}(t_0) > 0; \; \boldsymbol{\theta} = (\alpha, \boldsymbol{\beta}^T)^T. \qquad (21)$$

Assuming that (21) presents at least one inflection point, this function is found under the conditions listed in Román and Torres (2015), which ensures that the growth phenomenon represented by the Gompertz curve can be modeled by an inhomogeneous lognormal diffusion process whose average function is $f_{\boldsymbol{\theta}}(t)$.

Following the notation used in the aforementioned article, let $\{X(t); t \in I\}$ a diffusion process that takes values in $\mathbb{R}^+$ and with infinitesimal moments

$$\begin{array}{l} A_1(x,t) = h_{\boldsymbol{\theta}}(t)x \\ A_2(x) = \sigma^2 x^2, \qquad \sigma > 0 \end{array} \qquad (22)$$

where $I = [t_0, +\infty)$ is a real interval ($t_0 \geq 0$) and $\Theta \subseteq \mathbb{R}^{p+1}$ an open set, being $\boldsymbol{\theta} = (\alpha, \boldsymbol{\beta}^T)^T \in \Theta$ and

$$h_{\boldsymbol{\theta}}(t) = \alpha \, P_{\boldsymbol{\beta}}(t) \, e^{-Q_{\boldsymbol{\beta}}(t)}. \qquad (23)$$

With regard to the inference in the process, the estimation of the parameters by maximum likelihood is adapted from the development carried out in Román et al. (2018). There are two problems for its application: on one hand, the determination of the degree of the polynomial (which in general will be unknown) and, on the other hand, the obtaining of initial solutions for the system of likelihood equations. For both problems, in this work useful strategies are proposed. Finally, some examples illustrate the development carried out.

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Progressive enlargement of filtrations by the reference filtration of a general semi-martingale: results and applications

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Let $\mathbb{F}$ and $\mathbb{H}$ be two filtrations on the same probability space $(\Omega, \mathcal{F}, P)$, where $\mathbb{F}$ is the reference filtration of a semi-martingale $\mathbf{X}$ with values in $\mathbb{R}^m$ and $\mathbb{H}$ is the reference filtration of a semi-martingale $\mathbf{Y}$ with values in $\mathbb{R}^n$. Denote by $\mathbf{M}$ and $\mathbf{N}$ the martingale parts of $\mathbf{X}$ and $\mathbf{Y}$ respectively, and assume that for all $i = 1, \ldots, m$, the i-component of $\mathbf{M}$, M^i, is $\mathbb{F} \vee \mathbb{H}$-strongly orthogonal to the j-component of $\mathbf{N}$, N^j, for all $j = 1, \ldots, n$. Under these conditions independence of $\mathbb{F}$ and $\mathbb{H}$ follows and, according to different choices of probability on $(\Omega, \mathcal{F})$, two different martingale representations for the enlarged filtration $\mathbb{F} \vee \mathbb{H}$ hold. The first representation is expressed in terms of $\mathbf{M}$, $\mathbf{N}$ and the covariation process $[\mathbf{M}, \mathbf{N}]^V$, which is the vector process defined as any fixed sort order of the family $([M^i, N^j],\ i = 1, \ldots, m, j = 1, \ldots, n)$. The second martingale representation involves $\mathbf{X}$, $\mathbf{Y}$ and $[\mathbf{X}, \mathbf{Y}]^V$.
Some application of these results are discussed, in particular an extension of the classical Kusuoka's theorem of credit risk theory is given. In this result $\mathbb{H}$ is the natural filtration of the occurrence process of a general random time τ.

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Brownian motion governed by the telegraph process in stochastic modeling of the inflation and deflation episodes of Campi Flegrei

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The Campi Flegrei active caldera is located near the City of Naples in Italy. This region, famous worldwide for its slow vertical motion recorded since Roman times, is characterized by the longest ground deformation time series near a volcanic region. To describe the alternating random trend exhibited by such series, we propose herein a suitable stochastic model, consisting of a Brownian motion process driven by a generalized telegraph process. We carried out estimates of some basic parameters regulating the inflation/deflation processes, such as velocities and time constants, via linear regression with constraints. The knowledge of the probability law of the process allows to predict the measure of the ground displacements and changes in the motion tendency at future time instants. The goodness of the model has been confirmed by a statistical test on the Brownian component.

(This contribution is based on a joint work with A. Di Crescenzo, B. Martinucci and R. Scarpa.)

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An infinite dimensional Gaussian random matching problem

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We investigate upper and lower bounds for the Wasserstein-Kantorovich distance between random empirical measures and the common law for a sequence of i.i.d. infinite dimensional Gaussian random variables taking values in a Hilbert space. The technique uses random PDE's in Gaussian Hilbert spaces. As an application, we obtain quantitative rates of convergence for samples of Brownian paths to the Wiener measure. Joint work with E. Stepanov.

Gaussian mean field lattice gas

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In this talk I present a 0,1 version of the Sherrington-Kirckpatick spin glass model defined by the Hamiltonian $H(\eta) = \frac{1}{\sqrt{N}} \sum_{i,j} J_{ij} \eta_i \eta_j$ where the couplings J_{ij} are independent standard Gaussian Random variables and $\eta \in 0, 1^N$. We are mainly interested in the ground state this system. In the discrete optimization literature this model is known as Unconstrained Binary Quadratic Programming and belongs to the class of NP-hard problems. Further, I introduce a heuristic algorithm, based on a Probabilistic Cellular Automaton, to find the minima of H. This algorithm can be effectively implemented on a GPU and appears to have very promising performances.

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The isotropic constant of random polytopes

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For an n-dimensional isotropic convex body K we consider the isotropic constant L_{K_N} of the symmetric random polytope K_N generated by N independent random points which are distributed according to the cone probability measure on the boundary of K. We show that with overwhelming probability $L_{K_N} \leq C\sqrt{\log(2N/n)}$, where $C \in (0,\infty)$ is an absolute constant. If K is unconditional we argue that even $L_{K_N} \leq C$ with overwhelming probability and thereby verify the hyperplane conjecture for this model. The proofs are based on concentration inequalities for sums of sub-exponential or sub-Gaussian random variables, respectively, and, in the unconditional case, on a new ψ_2-estimate for linear functionals with respect to the cone measure in the spirit of Bobkov and Nazarov, which might be of independent interest.

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Strong Kac's chaos in the mean-field Bose-Einstein condensation

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A stochastic approach to the (generic) mean-field limit in Bose-Einstein Condensation is briefly described as well as the convergence of the ground state energy in the termodynamic limit. A strong form of Kac's chaos on path-space for the k-particles probability measures are derived from the previous energy convergence by purely probabilistic techniques notably using a simple chain-rule of the relative entropy. The Fisher's information chaos of the fixed-time marginal probability density under the generic mean-field scaling limit and the related entropy chaos result are also deduced.

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The role of the atomic decoherence-free subalgebra in the study of Quantum Markov Semigroups

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Given a uniformly continuous Quantum Markov Semigroup (QMS) $\mathcal{T} = (\mathcal{T}_t)_{t\geq 0}$ on the algebra of all linear and bounded operators acting on a separable Hilbert space, we study the decoherence-free subalgebra $\mathcal{N}(\mathcal{T})$, where maps $\mathcal{T}_t$ act as automorphisms. We showed in [2] that, if $\mathcal{N}(\mathcal{T})$ is atomic, it induces a decomposition of the system into its noiseless and purely dissipative parts, determining the structure of invariant states, as well as decoherence-free subsystems and subspaces [4].
We prove now that, for a QMS with atomic decoherence-free subalgebra and a faithful invariant state the following results holds.

1. Environment induced decoherence ([1, 3]) holds if and only if $\mathcal{N}(\mathcal{T})$ is atomic. In this case the decoherence-free subalgebra is generated by the set of eigenvectors corresponding to modulus one eigenvalues of the completely positive maps $\mathcal{T}_t$.

2. The decoherence-free subalgebra and the set of so-called *reversible states*, i.e. the linear space generated by eigevectors corresponding to modulus 1 eigenvalues of predual maps $\mathcal{T}_{*t}$ are in the natural duality of a von Neumann algebra with its predual.

3. We find a spectral characterization of the decomposition of the fixed point algebra.

Loosely speaking one can say that, for QMSs with a faithful invariant state, the same conclusions can be drawn replacing finite dimensionality of the system Hilbert space by atomicity of the decoherence-free subalgebra.

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Finitely additive, set-valued measures and applications in Economic Theory

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Set-valued measures with convex range are of great interest in a variety of fields including statistical decision theory, optimal control and game and general equilibrium theory. Among the others, they play an important role in classical results such as Dvoretsky, Wald and Wolfowitz' purification principle, the Bang Bang principle and the Core-Walras equivalence in large, competitive economies.
In this work we present some convexity results for the range of set-valued measures that (1) are only finitely additive, (2) have an infinite dimensional, locally convex range-space. We will base our approach on a a reformulation of the concept of *saturated measure space*, introduced in [1] and recently employed in several extensions of Lyapunov's Theorem on the range of vector measures and its applications (see [2], [3] for references).
We will then show how our result can be used in the study of classical issues in economic theory such as the veto-power of small coalitions in perfectly competitive economies and the purification of mixed strategies in games with incomplete information.

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Wasserstein geometry on Gaussian densities with trace one covariance matrix

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Starting from the Wasserstein distance for Gaussian densities, one can derive a Riemannian metric W on the space of symmetric positive definite matrices, $\mathrm{Sym}^{++}(n)$, see [1], [2], and [3]. At $\Sigma \in \mathrm{Sym}^{++}(n)$, this metric is given by:

$$W_\Sigma(U,V) = Tr(\mathcal{L}_\Sigma[U]\Sigma\mathcal{L}_\Sigma[V]) = \frac{1}{2}Tr(\mathcal{L}_\Sigma[U]V)$$

where $\mathcal{L}_\Sigma[W] = X$ denotes the solution to the Lyapunov equation $\Sigma X + X\Sigma = W$. If we let $\bar{g}$ denote the trace metric on $M(n)$, it turns out that the map:

$$\begin{aligned} \sigma : (GL(n), \bar{g}|_{GL(n)}) &\to (\mathrm{Sym}^{++}(n), W) \\ A &\mapsto AA^* \end{aligned}$$

is a Riemannian submersion. This map can be used to carry over the geometrical structure of $GL(n)$ to our manifold $(\mathrm{Sym}^{++}(n), W)$, [3]. The geodesics and Levi-Civita connection can be found in explicit matrix form, [2].

We are interested in the geometrical structure of the submanifold $\mathcal{D}(n) := \{\rho \in \mathrm{Sym}^{++}(n) : Tr(\rho) = 1\}$. More generally, quantum information theory is occupied with the study of the complex analogue of this set. Note that $\sigma^{-1}(\mathcal{D}(n)) = \{A \in GL(n) : Tr(AA^*) = 1\}$. Since $\bar{g}$ induces a distance measure d on $M(n)$ such that $d(0, A) = Tr(AA^*)$, we can interpret $\sigma^{-1}(\mathcal{D}(n))$ as the intersection of the unit sphere in $M(n)$ with $GL(n)$. The goal of this paper is to adapt the argument in [1] in order to obtain the induced Riemannian structure on $\mathcal{D}(n)$.

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Optimal installation of solar panels: a two-dimensional singular control problem

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We consider a price-maker company which produces energy and sells it in the spot market. The company can increase its level of installed power by irreversible installations of solar panels. In absence of any actions of the company, the energy's spot price evolves as an *Ornstein-Uhlenbeck process*, and therefore it has a mean-reverting behavior. The current level of the company's installed power has a permanent impact on the energy's price and affects its mean-reversion level. The company aims at maximizing the total expected profits from selling energy in the market, net of the total expected proportional costs of installation. This problem is modeled as a two-dimensional singular stochastic control problem in which the installation strategy is identified as the company's control variable. We follow a *guess-and-verify approach* to solve the problem. We find that the optimal installation strategy is triggered by a curve (which is called the *free boundary* and is characteristic of this problem) which separates the *waiting region*, where it is not optimal to install additional panels, and the *installation region,* where it is. Such a curve is a strictly increasing function which depends on the current level of the company's installed power. We prove that the free boundary is the unique global solution of a suitable ordinary differential equation, and we obtain some properties, like monotonicity and boundedness. Finally, our study is complemented by a numerical analysis of the dependency of the optimal installation strategy on the model's parameters.

Key words: singular stochastic control; irreversible investment; variational inequality; Ornstein-Uhlenbeck process; market impact.

OR/MS subject classification: Dynamic programming/optimal control: applications, Markov; Probability: stochastic models applications, diffusion.

JEL subject classification: C61; Q42.

MSC2010 subject classification: 93E20; 49L20; 91B70; 91B76; 60G99.

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Fourth moment theorems on the Poisson space in any dimension

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In a recent paper by C. Döbler and G. Peccati (2018), an exact quantitative fourth moment theorem (FMT) for multiple Wiener-Itô integrals on the Poisson space is established. In particular, the authors extended the spectral framework initiated by Ledoux (2012) from the situation of a diffusive Markov generator to the non-diffusive Ornstein-Uhlenbeck generator on the Poisson space. On the other hand, very recently, I. Nourdin and G. Zheng (2017) reproved FMTs in the Gaussian setting via an innovative exchangeable pairs couplings construction. In this talk, we will show how we obtained, by adapting the exchangeable pairs couplings construction to the Poisson framework, an optimal improvement of the quantitative FMT in the univariate case, proving it under the weakest possible assumptions of finite fourth moments, as well as an extension of it to any dimension, namely a Peccati-Tudor type theorem.

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Tail optimality and preferences consistency for stochastic optimal control problems

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Given a stochastic optimal control problem over a time interval $[t_0, T]$ and a control plan associated to it, we introduce the four notions of local and global tail optimality of the control plan, and local and global preferences consistency of the optimizer. While the notion of tail optimality of a control plan is not new in control problems, the notion of preferences consistency of an optimizer seems novel.
We prove that, in the case of a *linear* time-consistent problem where dynamic programming can be applied, the optimal control plan is globally tail-optimal and the optimizer is globally preferences-consistent. Opposite, in the case of a *non-linear* problem that gives rise to time-inconsistency, we find that global tail optimality and global preferences consistency do not coexist. We analyze three common ways to attack a time-inconsistent problem: (i) precommitment approach, (ii) dynamically optimal approach, (iii) consistent planning approach. We find that for the precommitment approach there is local tail optimality and local preferences consistency at initial time t_0; for the dynamically optimal approach there is global preferences consistency, but there is no local tail optimality at any time; for the consistent planning approach there is neither local tail optimality nor local preferences consistency at any time with respect to the original non-linear problem, but there is global tail optimality and global preferences consistency with respect to a different linear problem. To illustrate the theoretical results, we analyze a notable case example, the mean-variance portfolio selection problem.
This analysis should shed light on the price to be paid in terms of tail optimality and preferences consistency with each of the three approaches currently available for time inconsistency.

On the estimation of the mean density of lower dimensional germ-grain models in R^d

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The mean density of a random closed set is a crucial notion in stochastic geometry, in fact it is a fundamental tool in a large variety of applied problems, such as image analysis, medicine, computer vision, etc. Hence the estimation of the mean density is a problem of interest both from a theoretical and computational standpoint. The mean density, say λ_{Θ_n}, of a random closed set Θ_n in R^d with integer Hasudorff dimension $n < d$ is defined as the density of the measure $E[\mathcal{H}^n(\Theta_n \cap \cdot)]$ with respect to $\mathcal{H}^d$, whenever it exists.
In this talk we consider inhomogeneous random sets of the type

$$\Theta_n(\omega) = \bigcup_{(\xi_i, Z_i) \in \Phi(\omega)} \xi_i + Z_i, \qquad \omega \in \Omega,$$

where Φ is a marked point processes in R^d with marks in the space of compact subsets of R^d, and we give an overview on some recent results on the existence of the mean density λ_{Θ_n}, and on its estimation when an i.i.d. random sample of Θ_n is available. The proposed estimation technique may be consider as a generalization to the n dimensional case of the well known probability density estimation of random variables. Finally, strong consistency and asymptotic Normality properties will be derived by means of large and moderate deviation principles.

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A mean-field model with discontinuous coefficients and spatial interaction

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We consider a system of particles; each particle is identified by its position X^j (that we allow to be random) and a second quantity V^j that evolves in time, subject to the effect of the interaction with the other particles and to noise. The interaction term for the j-th particle depends on the positions X^i of the other particles in a continuous way, but also on their values V^i in a discontinuous way.
We study well-posedness of the system of particles and its mean-field limit as the number of particles goes to infinity, described by the solution of a Fokker-Planck-type PDE, for which we prove an existence and uniqueness result. As a consequence we obtain well-posedness also for the associated McKean-Vlasov stochastic equation. The main novelty of our approach lies in the limit procedure that allows to deal with discontinuities.
Our work is motivated by a new approach to the study of the electric behavior of networks of neurons that interact with each other following individually an integrate-and-fire dynamics. In this model the quantity V^j represents the voltage of the j-th neuron. We are able to introduce in the dynamics the refractary period of each neuron, but this forces the interaction term to be discontinuous in the variable V. The limit Fokker-Planck-type PDE describes then the voltage density in space-time along the network.
The talk is mainly based on the paper "A mean-field model with discontinuous coefficients for neurons with spatial interaction".

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Surface measures and integration by parts formula on levels sets induced by functionals of the Brownian motion in $\mathbb{R}^n$

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On the infinite dimensional space of continuous paths from $[0, 1]$ to $\mathbb{R}^n$, $n \geq 3$, endowed with the Wiener measure μ, we construct a surface measure defined on level sets of the L^2-norm of n-dimensional processes that are solutions to a general class of stochastic differential equations, and provide an integration by parts formula involving this surface measure. We follow the approach to surface measures in Gaussian spaces proposed via techniques of Malliavin calculus in [1] and in particular we rely on the procedure recently introduced in [2] and [3].

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Existence of Gibbsian fields via entropy methods

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In 1988, Hans-Otto Georgii introduced a new method of proving the existence of an infinite-volume Gibbs measure, that uses the specific entropy as a "Lyapunov function".
Drawing inspiration from examples in stochastic geometry, we use this approach in the framework of *marked* Gibbs point processes, and present an existence result for a wide class of unbounded interactions.

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Inverse first passage time for some two-dimensional diffusion processes

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In several applications the dynamics of the variables of interest is described via suitable stochastic processes constrained by boundaries. Often the focus is on the first passage time (FPT) of Markov processes through constant or time varying boundaries. However explicit solutions to the first-passage problem are known only in a limited number of special cases and efficient algorithms for their study were proposed in literature [2].

Generally the dynamics of the involved variables are described via a suitable stochastic process $\{X_t, t \geq 0\}$ constrained by an assigned boundary $b : (0, \infty) \to \mathbb{R}$ and the distribution features of the FPT

$$\tau_b = \inf\{\, t > 0 \mid W_t \geq b(t)\,\} \tag{24}$$

of X_t over b are investigated. This is the direct FPT problem. However there are also instances when the underlying stochastic process is assigned, one knows or estimates the FPT distribution F_b and wishes to determine the corresponding boundary shape. This is the inverse first passage time (IFPT) problem. In [4] we studied this problem in the case of a Wiener process constrained by a single boundary. We proposed an algorithm to determine the unknown time dependent boundary when the distribution of the FPT is assigned and we studied its convergence properties. In [3] we generalized the approach in order to apply it to an Ornstein-Uhlenbeck process and we proposed its use for a classification problem in neuroscience.

Here we extend the IFPT method to multivariate Gauss-Markov processes and we investigate the boundary shape corresponding to given FPT distributions for suitable choices of the parameters. Special attention is given to the Integrated Brownian motion and the two-dimensional Ornstein-Uhlenbeck process. Some examples of applications of the proposed methods will be illustrated [1].

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