Exercise I Scalas4

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$n = 3$ elements

$q = 3$ categories

$$P\left(\underline{n}^{j}_{i} \,\middle|\, \underline{n}\right) = \overbrace{\frac{1 - \delta_{n_i,0}}{3 - z_0(\underline{n})}}^{\text{select}} \cdot \overbrace{\frac{1}{3}}^{\text{move}}$$

SPACE SET:

$$\underset{1}{(0,0,3)},\ \underset{2}{(0,3,0)},\ \underset{3}{(3,0,0)},\ \underset{4}{(1,1,1)},\ \underset{5}{(0,1,2)},\ \underset{6}{(1,0,2)},\ \underset{7}{(1,2,0)},\ \underset{8}{(0,2,1)},\ \underset{9}{(2,0,1)},\ \underset{10}{(2,1,0)}$$

Transition matrix 10×10

$$b_{\substack{1 \\ 4}|k} = \left(\underset{k=1}{\frac{1}{3}}, \underset{k=2}{0}, 0, 0, \underset{k=5}{\frac{1}{3}}, \frac{1}{3}, 0, 0, 0, 0\right)$$

$(0,0,3) \xrightarrow{i=3} (0,0,2)$

$z_0(\underline{n}) = 2$

$P = \frac{1}{3}$ $(n_i \neq 0)$

$j=1 \to (1,0,2)$ 6

$j=2 \to (0,1,2)$ 5

$j=3 \to (0,0,3)$ 1

$$b_{2|k} = \left(0, \frac{1}{3}, 0, 0, 0, 0, \frac{1}{3}, \frac{1}{3}, 0, 0\right)$$

$(0,3,0) \xrightarrow{i=2} (0,2,0)$

$P = \frac{1}{3}$ $(n_i \neq 0)$

$j=1 \to (1,2,0)$ 7

$j=2 \to (0,3,0)$ 2

$j=3 \to (0,2,1)$ 8

$$b_{3|k} = \left(0, 0, \frac{1}{3}, 0, \frac{1}{3}, \frac{1}{3}, 0, 0, 0, 0\right)$$

$(0,0,3) \xrightarrow{i=3} (0,0,2)$

$P = \frac{1}{3}$

$\to (1,0,2)$ 6

$\to (0,1,2)$ 5

$\to (0,0,3)$ 3

$$b_{4|k} = \left(0, 0, 0, \frac{1}{3}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}\right)$$

$(1,1,1)$

$z_0(\underline{n}) = 0$

$P = 1$

$i=1 \to (0,1,1)$: $j=1 \to (1,1,1)$ 4; $j=2 \to (0,2,1)$ 8; $j=3 \to (0,1,2)$ 5

$i=2 \to (1,0,1)$: $j=1 \to (2,0,1)$ 9; $j=2 \to (1,1,1)$ 4; $j=3 \to (1,0,2)$ 6

$i=3 \to (1,1,0)$: $j=1 \to (2,1,0)$ 10; $j=2 \to (1,2,0)$ 7; $j=3$ [illegible]

$1 = \frac{1}{9}$

$m_i \neq 0$

(1,1,1) 4

$b_{5,K} = \left(\frac{1}{6}, 0, 0, \frac{1}{6}, \frac{1}{3}, \frac{1}{6}, 0, \frac{1}{6}, 0, 0\right)$

(0,1,2)

$t_0(m) = 1$

$i=2$ → (0,0,2)
- $i=1$ → (1,0,2) 6
- 2 → (0,1,2) 5
- 3 → (0,0,3) 1

$i=3$ → (0,1,1)
- $i=1$ → (1,1,1) 4
- 2 → (0,2,1) 8
- 3 → (0,1,2) 5

$P = \frac{1}{6}$ $m_i \neq 0$

$b_{6,K} = \left(\frac{1}{6}, 0, 0, \frac{1}{6}, \frac{1}{6}, \frac{1}{3}, 0, 0, \frac{1}{6}, 0\right)$

(1,0,2)

$i=1$ → (0,0,2)
- (1,0,2) 6
- (0,1,2) 5
- (0,0,3) 1

$i=3$ → (1,0,1)
- (2,0,1) 9
- (1,1,1) 4
- (1,0,2) 6

$P = \frac{1}{6}$

$b_{7,K} = \left(0, \frac{1}{6}, 0, \frac{1}{6}, 0, 0, \frac{1}{3}, \frac{1}{6}, 0, \frac{1}{6}\right)$

(1,2,0)

$i=1$ → (0,2,0)
- (1,2,0) 7
- (0,3,0) 2
- (0,2,1) 8

$i=2$ → (1,1,0)
- (2,1,0) 10
- (1,2,0) 7
- (1,1,1) 4

$b_{8,K} = \left(0, \frac{1}{6}, 0, \frac{1}{6}, \frac{1}{6}, 0, \frac{1}{6}, \frac{1}{3}, 0, 0\right)$

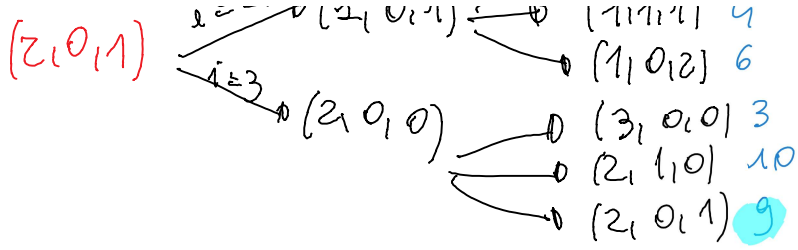
(0,2,1)

$i=2$ → (0,1,1)
- (1,1,1) 4
- (0,2,1) 8
- (0,1,2) 5

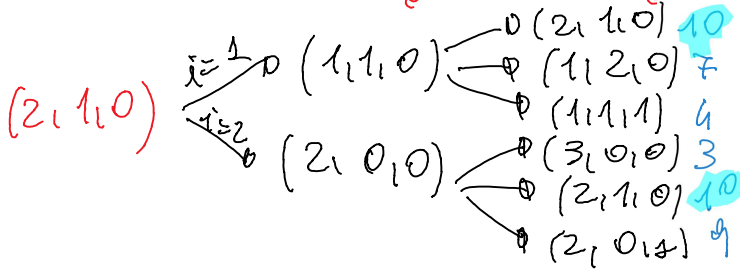
$i=3$ → (0,2,0)
- (1,2,0) 7
- (0,3,0) 2
- (0,2,1) 8

$b_{9,K} = \left(0, 0, \frac{1}{6}, \frac{1}{6}, 0, \frac{1}{6}, 0, 0, \frac{1}{3}, 0\right)$

- (2,0,1) 9



$b_{1,0,K} = \left(0, 0, \frac{1}{6}, \frac{1}{6}, 0, 0, \frac{1}{6}, 0, \frac{1}{6}, \frac{1}{3}\right)$



CHAIN FINITE, IRREDUCIBLE, APERIODIC $P(x,x) \neq 0 \ \forall x$

$\Rightarrow \exists!$ invariant distribution

EQUIL. DISTR

equilibrium vector $\underline{\Pi}$:

$$\underline{1} \cdot \underline{\Pi} = \underline{\Pi} \cdot W \iff \underline{\Pi} - \underline{\Pi} \cdot W = 0$$

left eigenvector of W, related to eigenvalue 1

exact

$$\Pi(\underline{M}) = \frac{1}{18}\left(3 - \varepsilon_0(\underline{M})\right)$$

$$\Pi(\underline{M}) = \left(\underbrace{\frac{1}{18}, \frac{1}{18}, \frac{1}{18}}_{\varepsilon_0(\underline{M})=2}, \frac{1}{6}, \underbrace{\frac{1}{9}, \dots, \frac{1}{9}}_{\varepsilon_0(\underline{M})=1}\right)$$

$\varepsilon_0(\underline{M})=0$

```
#Program BDYdiag.R
#transition matrix
row1<-c(1/3,0,0,0,1/3,1/3,0,0,0,0)
row2<-c(0,1/3,0,0,0,0,1/3,1/3,0,0)
row3<-c(0,0,1/3,0,0,0,0,0,1/3,1/3)
row4<-c(0,0,0,1/3,1/9,1/9,1/9,1/9,1/9,1/9)
row5<-c(1/6,0,0,1/6,1/3,1/6,0,1/6,0,0)
row6<-c(1/6,0,0,1/6,1/6,1/3,0,0,1/6,0)
row7<-c(0,1/6,0,1/6,0,0,1/3,1/6,0,1/6)
row8<-c(0,1/6,0,1/6,1/6,0,1/6,1/3,0,0)
row9<-c(0,0,1/6,1/6,0,1/6,0,0,1/3,1/6)
row10<-c(0,0,1/6,1/6,0,0,1/6,0,1/6,1/3)
W<-matrix(c(row1,row2,row3,row4,row5,
              row6,row7,row8,row9,row10),nrow=10,ncol=10,byrow=TRUE)
#diagonalization
ev<-eigen(t(W))
#eigenvalues (the largest eigenvalues is equal to 1)
ev$val
#eigenvectors (the first column is the eigenvector
#corresponding to 1)
ev$vec
#probability of states (normalization)
p<-abs(ev$vec[,1])/sum(abs(ev$vec[,1]))
p
#exact values of the equilibrium distribution
pt<-c(1/18,1/18,1/18,1/6,1/9,1/9,1/9,1/9,1/9,1/9)
#differences
p-pt
p-p%*%W
```

```
##############################
# Plot of the graph
##############################

library(igraph)

################################
# Rows of the adjacency matrix  #
################################

row1<-c(1,0,0,0,1,1,0,0,0,0)
row2<-c(0,1,0,0,0,0,1,1,0,0)
row3<-c(0,0,1,0,0,0,0,0,1,1)
row4<-c(0,0,0,1,1,1,1,1,1,1)
row5<-c(1,0,0,1,1,1,1,0,1,0)
row6<-c(1,0,0,1,1,1,0,0,0,0)
row7<-c(0,1,0,1,0,0,1,0,0,0)
row8<-c(0,1,0,1,1,0,0,1,0,0)
row9<-c(0,0,1,1,0,0,0,0,1,0)
row10<-c(0,0,1,1,0,0,0,0,0,1)

####################
# Adjacency matrix  #
####################

Arows<-c(row1,row2,row3,row4,row5,row6,row7, row8, row9,row10)
A<-matrix(Arows, ncol=10, nrow=10, byrow=TRUE)
Graph<-graph_from_adjacency_matrix(A, mode=c("undirected"),weighted=NULL,
diag=TRUE, add.colnames=NULL, add.rownames=NA)

####################
# plot of the graph  #
####################

plot.igraph(Graph)
```

```
#Program BDYMV.R
#Number of objects
n<-3
#Number of categories
g<-3
#Number of Montecarlo Carlo steps
T<-100000
#Initial occupation vector
y<-c(1,1,1) #state4
#Frequencies and transitions
state<-4
freq<-c(0,0,0,0,0,0,0,0,0,0)
transmat<-matrix(0,nrow=10,ncol=10,byrow=TRUE)
#Loop of Monte Carlo steps
for(i in 1:T){
  #Random selection of the winner
  #Generate a uniformly distributed integer between 1 and 3
  indexw<-ceiling(3*runif(1)) #j
  #Random selection of the loser
  #Generate a uniformly distributed integer between 1 and 3
  indexl<-ceiling(3*runif(1))
  #Verify if the loser has objects
  while(y[indexl]==0)  indexl<-ceiling(3*runif(1))
  #Dynamic step
  y[indexl]<-y[indexl]-1
  y[indexw]<-y[indexw]+1
  #frequencies
  if(max(y)==1) #(1,1,1)
    newstate<-4
  if(max(y)==3){ #(0,0,3),(0,3,0),(3,0,0)
    if(y[3]==3) newstate<-1
    if(y[2]==3) newstate<-2
    if(y[1]==3) newstate<-3
    #end if
  }
  if(max(y)==2){
    if(y[1]==2){  if(y[2]==1) newstate<-10
    else newstate<-9
    #end if
    }
    if(y[1]==1){
      if(y[2]==2) newstate<-7
      else newstate<-6
      #end if
    }
    if(y[1]==0){
      if(y[2]==1) newstate<-5
      else newstate<-8
      #end if
    }#end if
  }
  #Updates
  freq[newstate]<-freq[newstate]+1
  transmat[state,newstate]<-transmat[state,newstate]+1
  state<-newstate
  #end for
```

```
}
#Normalization
freq<-freq/T
freq
for(j in 1:10) transmat[j,]<-transmat[j,]/sum(transmat[j,])
transmat
#Differences
pt<-c(1/18,1/18,1/18,1/6,1/9,1/9,1/9,1/9,1/9,1/9)
pt-freq
row1<-c(1/3,0,0,0,1/3,1/3,0,0,0,0)
row2<-c(0,1/3,0,0,0,0,1/3,1/3,0,0)
row3<-c(0,0,1/3,0,0,0,0,0,1/3,1/3)
row4<-c(0,0,0,1/3,1/9,1/9,1/9,1/9,1/9,1/9)
row5<-c(1/6,0,0,1/6,1/3,1/6,0,1/6,0,0)
row6<-c(1/6,0,0,1/6,1/6,1/3,0,0,1/6,0)
row7<-c(0,1/6,0,1/6,0,0,1/3,1/6,0,1/6)
row8<-c(0,1/6,0,1/6,1/6,0,1/6,1/3,0,0)
row9<-c(0,0,1/6,1/6,0,1/6,0,0,1/3,1/6)
row10<-c(0,0,1/6,1/6,0,0,1/6,0,1/6,1/3)
w<-matrix(c(row1,row2,row3,row4,row5,
              row6,row7,row8,row9,row10),nrow=10,ncol=10,byrow=TRUE)
w-transmat
```