Commutators in residually finite groups

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Thus, our latest result is an improvement since it shows that t can be taken 68 always.

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I will now explain why 68 commutators are easier to deal with.

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The fact that the class of all groups G such that G' is locally finite and every product of 68 commutators has order dividing n is a variety is another result in the spirit of the RBP.

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