FIXED POINTS OF COPRIME AUTOMORPHISMS OF FINITE GROUPS

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Let A be a finite group acting coprimely on a finite group G. It is well known that the structure of the centralizer $C_G(A)$ (the fixed-point subgroup) of A has strong influence over the structure of G. Following the solution of the restricted Burnside problem it was discovered that the exponent of $C_G(A)$ may have strong impact over the exponent of G. In this talk we will discuss the following result.

Theorem. *Let q be a prime, m a positive integer and A an elementary abelian group of order q^r with $r \geq 2$ acting on a finite q'-group G. If, for some integer d such that $2^d \leq r-1$, the dth derived group of $C_G(a)$ has exponent dividing m for any $a \in A \setminus \{1\}$, then the dth derived group $G^{(d)}$ has exponent bounded only in terms of m, q, and r.*

The result was already known for $r = 2$ (Khukhro - Shumyatsky, 1999) and for $r = 3$ (Guralnick - Shumyatsky, 2001). The novelty of our approach consists in introducing the concept of A-special subgroups. We discuss it in the talk.

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* Joint work with Pavel Shumyatsky

A NOTE ON FINITE GROUPS IN WHICH C-NORMALITY IS A TRANSITIVE RELATION

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A subgroup H of G is c-normal in G if there exists a normal subgroup N of G such that $HN = G$ and $H \cap N \leq H_G$. A group G is called CT-group if c-normality is transitive relation in G. A number of new characterizations of finite solvable CT-groups are given.

PRODUCTS OF LOCALLY SUPERSOLUBLE GROUPS

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Some results on the structure of groups

$$G = AB = AC = BC$$

with a triple factorization by locally supersoluble subgroups A, B, C are proved.

* Joint work with Francesco de Giovanni

MODULES WITH MAXIMAL GROWTH OVER FREE GROUP ALGEBRAS *

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A number of publications of Professor Narain Gupta have been devoted to Free Group Rings (for instance see [1]). These important objects appear in a different context in two recent joint papers [2] and [3].

Let $\mathcal{F}_r$ be the group algebra of a free group F_r of rank $r > 1$, over a field, Δ the augmentation ideal of $\mathcal{F}_r$. A right module M over $\mathcal{F}_r$ is called *nil* (resp., *periodic*) if for any $(x, u) \in M \times \Delta$ (resp., $(x, g) \in M \times F_r$) there is $n > 0$ such that $xu^n = 0$ (resp., $xg^n = x$).

A finitely generated module M over $\mathcal{F}_r$ whose growth function $g(n)$ with respect to a finite generating set satisfies $g(n) > C(2r-1)^n$, for some $C > 0$, is said to be of *maximal growth.* In general, we require that M has a cyclic submodule of maximal growth. All infinite-dimensional finitely presented $\mathcal{F}_r$-modules have maximal growth. Each of these latter modules has a free submodule of finite codimension. But in general the following are true.

Theorem. *Any $\mathcal{F}_r$-module M presented by p generators and q relators, such that $p - q > 0$, can be mapped onto an infinite-dimensional finitely generated nil-$\mathcal{F}_r$-module. This latter can be mapped onto a module, which is still infinite-dimensional and nil but also residually finite-dimensional.*

The proof uses direct limits of *large* modules, specific modules of maximal growth, each having a submodule of finite codimension, which can be mapped onto a nonzero free module. The next result is a by-product of our study of F_r-*actions* of maximal growth.

Theorem. *There exists a simple and periodic $\mathcal{F}_r$-module of maximal growth.*

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* This talk is dedicated to the memory of Narain Gupta

PARABOLIC SUBGROUPS OF CHEVALLEY GROUPS OVER A DEDEKIND RINGS OF ARITHMETIC TYPE

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Description of parabolic subgroups of algebraic groups over fields is due to Tits. It was later generalised to semi-local rings by Borewicz, Suzuki, and the second author. In the present talk we make a next step, and describe overgroups of Borel subgroups in Chevalley groups over a Dedekind ring of arithmetic type with infinite multiplicative group, under some mild additional restrictions. As the main step we prove that an overgroup of the standard Borel subgroup either contains non-trivial root elements in all positions, or is contained in a proper parabolic subgroup in the usual sense. Thus, we can either reduce rank, or invoke results for the semi-local case. The proofs rely on representation theory and algebraic K-theory.

* Joint work with Nikolai Vavilov

COCHARACTERS OF $UT_2(E)$

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Let A be a F-algebra over a field F of characteristic 0 and $T(A)$ the ideal of all polynomial identities of A. If the characteristic of F is 0, $T(A)$ is generated, as a T-ideal, by the subspaces $V_n \cap T(A)$ of multilinear identities. It is more efficient to study the factor space $V_n(A) = V_n/(V_n \cap T(A))$ in fact, although the intersection $V_n \cap T(A)$ is huge as n goes to infinity, if A is a PI-algebra, $V_n(A)$ grows exponentially. An effective tool for the study of $V_n(A)$ is provided by the representation theory of the symmetric group. Indeed, one can notice that $V_n(A)$ is an S_n-module, then we denote by $\chi_n(A)$ its character, called the n-th cocharacter of A. The explicit form of the multiplicities of $\chi_n(A)$ is known for few algebras only, among them the Grassmann algebra E (Olsson and Regev [4]), the 2×2 matrix algebra $M_2(F)$ (Formanek [3], Drensky [2]), the algebra $U_2(F)$ of the 2×2 upper triangular matrices (Mishchenko, Regev and Zaicev [5]), the tensor square $E \otimes E$ of the Grassmann algebra (Popov [6], Carini and Di Vincenzo [1]). In this paper we compute firstly the Hilbert series of $UT_2(E)$, using the tool of proper Hilbert series and, as a consequence, its cocharacter sequence.

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GROUPS WITH MANY INERT SUBGROUPS *

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This communication talk is about results from a work in progress. Two subgroups H and K of a group G are told *commensurable* iff the index of $H \cap K$ in both H and K is finite. A subgroup is told *inert* if it is commensurable with all conjugates of its.

We consider the property *cn* for a subgroup H of a group G of being *commensurable with a normal* subgroup, i.e. $\exists N \triangleleft G \ : \ |HN/(H \cap N)| < \infty$. A *cn* subgroup is inert, clearly.

We talk about the class of groups whose all subgroups are *cn*, which is a subclass of the class of *totally inert groups*, that is groups whose all subgroups are inert (see [1], [3] and [5]), and contains the class of *CF-groups*, that is groups G in which all subgroups H are cf (core-finite), i.e. $|H/H_G| < \infty$, considered by J.Wiegold et alii ([2], [6]).

The class of groups in which *cn* is a *transitive relation* coincides with the class of groups in which *subnormal* subgroups are *cn*. We regard this class as a subclass of the class of groups in which *subnormal subgroups are inert*. Recall that groups in which cf is a transitive relation were studied in [4].

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* This talk is dedicated to the memory of J.Weigold.

** Joint work with Silvana Rinauro, Dipartimento di Matematica e Informatica Università della Basilicata, silvana.rinauro@unibas.it

GROUPS WITH ALL SUBGROUPS PERMUTABLE OR OF FINITE RANK

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The structure of groups with all subgroups permutable is quite well-known. The structure of groups with all proper subgroups of finite rank is not quite so transparent, although for certain classes of groups, again the structure is known. In this talk I will give a short survey of known results and then discuss recent work where we meld the permutability and finite rank conditions. This requires restrictions on the class of groups under consideration; even locally graded groups of finite rank are not well-understood, so we have to consider a smaller class of groups which we denote by $\mathfrak{X}$. In this recent work the following theorem is proved.

Theorem. *Let $G \in \mathfrak{X}$ be a group of infinite rank in which every subgroup of infinite rank is permutable. Then every subgroup of G is permutable.*

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* Joint work with Yalcin Karatas

FINITE GROUPS WITH A SPLITTING AUTOMORPHISM

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Thompson proved that a finite group with a fixed point free automorphism of prime order is nilpotent. An automorphism ϕ of G is called a **splitting automorphism** if for every $x \in G$

$$xx^{\phi}x^{\phi^2}...x^{\phi^{n-1}} = 1$$

where n is the order of ϕ. Clearly, a fixed point free automorphism of a finite group is a splitting automorphism. Kegel generalized Thompson's result to splitting automorphisms, namely he proved that a finite group with a splitting automorphism of prime order is nilpotent. Rowley proved that a finite group with a fixed point free automorphism is solvable. However, there are non-solvable finite groups admitting a splitting automorphism.

Example. Observe that the cyclic group $\mathbb{Z}_{31}$ has a fixed-point-free automorphism α of order 30. Now, define $G = \mathbb{Z}_{31} \times A_5$ and consider

$$\begin{aligned} \phi :\ \mathbb{Z}_{31} \times A_5 &\longrightarrow \mathbb{Z}_{31} \times A_5 \\ (x, y) &\longrightarrow (x^{\alpha}, y). \end{aligned}$$

Now, one can observe that ϕ is a splitting automorphism of G of order 30, but G is not solvable.

By Kegel's result, a finite group admitting a splitting automorphism of prime order is nilpotent. Moreover, Jabara proved that, a finite group with a splitting automorphism of order 4 is solvable. Hence, one might ask the following question:

Question. *Let m be a natural number which is not divisible by the exponent of any non-abelian finite simple group. Let G be a finite group admitting a splitting automorphism of order m. Is G necessarily solvable?*

In this talk we will give a partial answer to this question. In particular, we will prove the following result:

Theorem. *[1] A finite group with a splitting automorphism of odd order is solvable.*

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COVERING MONOLITHIC GROUPS WITH PROPER SUBGROUPS

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Let G be a finite group. A "cover" of G is a family $\mathcal{H}$ of proper subgroups of G such that $\bigcup_{H\in\mathcal{H}} H = G$. Define $\sigma(G)$ to be the smallest cardinality of a cover of G, and $\sigma(G) := \infty$ if G is cyclic. This concept has been introduced by Cohn in [2]. The behaviour of the function σ which assigns to a finite group G the number $\sigma(G)$ has been studied by many authors. If N is a normal subgroup of G then $\sigma(G) \leq \sigma(G/N)$. This implies that the information about σ is encoded in the groups G with the property that $\sigma(G) < \sigma(G/N)$ for every non-trivial normal subgroup N of G. We call such groups "σ-elementary". Recall that a group is said to be "monolithic" if it has exactly one minimal normal subgroup. Lucchini and Detomi [1] proved that every σ-elementary finite group is a subdirect product of monolithic groups and conjectured that every non-abelian σ-elementary finite group is monolithic. The solvable case is true by a result of Tomkinson [3]. We show how the direct study of monolithic groups is related to a possible solution of this conjecture. If X is a monolithic group with non-abelian socle N define O to be the set of generating sets of X/N and for $\Omega \in O$ define $\sigma_\Omega(X)$ to be the smallest number of supplements of N in X needed to cover the union of the elements of Ω. Define

$$\sigma^*(X) := \min\{\sigma_\Omega(X) \mid \Omega \in O\}.$$

Bounding the quantity $\sigma(X) - \sigma^*(X)$ from above turns out to be very helpful. This is done for some particular case in [4] and [5].

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FINITE DIMENSIONAL LIE SUPERALGEBRAS AND CODIMENSION GROWTH

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Let A be a non-necessarily associative algebra over a field of characteristic zero and let $c_n(A), n = 1, 2, \ldots$, be its sequence of codimensions, measuring the growth of the polynomial identities satisfied by A ([1]). If A is a finite dimensional algebra, such sequence is exponentially bounded. We have recently proved in [2] that if A is a finite dimensional simple algebra then $\exp(A) = \lim_{n\to\infty} \sqrt[n]{c_n(A)}$, the PI-exponent of A, exists and is bounded from above by $\dim A$.

It is well known that for associative or Lie or Jordan algebras, the equality $\exp(A) = \dim A$ holds provided that the base field is algebraically closed. Since simple Lie superalgebras are simple in a non-graded sense, their PI-exponent exists and here we prove that for the infinite family of Lie superalgebras of type $b(t)$, $t \geq 3$, the PI-exponent is strictly less than the dimension. Finally we exhibit a 7-dimensional Lie superalgebra whose PI-exponent is strictly between 6 and 7.

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ON SOME DIFFERENT TOPICS: AUTOMORPHISMS, TEST SETS, POLYNILPOTENT SERIES, EQUATIONALLY NOETHERIAN, PARTIALLY COMMUTATIVE METABELIAN GROUPS

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A brief survey of few different concepts of combinatorial group theory will be given bringing up to some recent results.

SOLVABLE GROUPS WITH SUBNORMAL NORMALIZERS OF SUBNORMAL SUBGROUPS

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In many classes of finite solvable groups (for instance T-groups, PST-groups, PT-groups) we have the property that all normalizers of subnormal subgroups are subnormal. That leads to the question what can be said of groups satisfying this more general condition, briefly called NSS-groups. In contrast to the classes mentioned before, NSS-groups need not be metanilpotent, but they are not too "distant" from them. In fact we were able to show the following.

Theorem. *For the NSS-group G the following statements are true:*
(a) G is of p-length 1 for all primes p;
(b) if $F_1 = F(G)$ and $F_2/F_1 = F(G/F_1)$, then G/F_2 is nilpotent of squarefree exponent;
(c) denoting $G^{\mathcal{N}}F(G)/F(G)$ by Q, then Q' and $Q/Z(Q)$ are of exponent two;
(d) $G^{\mathcal{N}\mathcal{N}}$ is nilpotent.

The talk will exhibit some of the arguments used and some more information on $G/F(G)$ and Q. More details are contained in the reference.

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* Joint work with J. C. Beidleman

SMALL DOUBLING IN ORDERED GROUPS

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We prove that if a finite subset S of an ordered group generates a non-abelian subgroup, then $|S^2| = |\{xy \mid x, y \in S\}| > 3|S| - 3$. This generalizes a classical result from the theory of set addition.

* This is a joint paper with Gregory Freiman, Patrizia Longobardi and Mercede Maj.

FINITE SIMPLE QUOTIENTS OF GROUPS SATISFYING PROPERTY (T)

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Let X be a graph and let $\epsilon > 0$ be a real number. We say that X is an ϵ-*expander* if, for all sets A consisting of at most half the vertices of X, we have $|\partial A| \geq \epsilon |A|$, where ∂A denotes the boundary of A, i.e. the vertices of distance 1 from A. An infinite family $\{X_i\}_{i\in\mathbb{N}}$ of k-regular finite graphs is called a *family of expanders* if there exists $\epsilon > 0$ such that all the graphs X_i are ϵ-expanders and the number of vertices of X_i tends to infinity. The first proof of the existence of families of expanders by Pinsker is based on counting arguments. For applications one wants explicit constructions. Margulis realized that using infinite residually finite groups satisfying Kazhdan's property (T) one can construct explicit examples of families of expanders.

The construction is done in the following way. Let Γ be infinite residually finite groups satisfying Kazhdan's property (T) (for example, $\Gamma = \mathrm{SL}_3(\mathbb{Z})$). Let S be a symmetric finite generating set of Γ and $\{\Gamma_i\}_{i\in\mathbb{N}}$ a family of normal subgroups of Γ of finite index such that $|\Gamma/\Gamma_i|$ tends to infinity. Then the Cayley graphs $X_i = \mathrm{Cay}(\Gamma/\Gamma_i; S)$ form a family of expanders.

We say that a family $\{G_i\}$ of finite groups is a *family of expanders* if there are $k \in \mathbb{N}$ and $\epsilon > 0$ such that every group G_i has a symmetric subset S_i of k generators for which $\mathrm{Cay}(G_i; S_i)$ is an ϵ-expander. It was conjectured by Babai, Kantor and Lubotzky that the family of all the finite (nonabelian) simple groups is a family of expanders. The conjecture first was proved by Kassabov, Nikolov and Lubotzky for all the simple groups with the exception of the Suzuki groups and recently for the Suzuki groups by Breuillard, Green and Tao.

In my talk I want to consider the following question.

Question. Let $\mathcal{F}$ be a family of finite simple groups. Is there a group Γ satisfying Kazhdan's property such that the groups from $\mathcal{F}$ are quotients of Γ?

Our main result is the following:

Theorem. *There exists a group* Γ *satisfying property* (T) *such that every finite simple group of Lie type of rank at least 2 is a quotient of* Γ.

* Joint work with Mikhail Ershov and Martin Kassabov

FINITE COVERINGS: A JOURNEY THROUGH GROUPS, LOOPS, RINGS AND SEMIGROUPS *

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A group is said to be covered by a collection of subsets if each element of the group belongs to at least one subset in the collection: the collection of subsets is called a covering of the group.

On the bottom of page 105 of Derek Robinson's "Finiteness Conditions and Generalized Soluble Groups I", there are two theorems which served as my roadmap for exploring finite coverings of groups, loops, rings and semigroups. The first one, an unpublished result by Reinhold Baer, is stated as follows.

Baer's Theorem. *A group is central-by-finite if and only if it has a finite covering by abelian subgroups.*

The second one, due to Bernhard Neumann, is stated as follows.

Neumann's Lemma. *Let G be a group having a covering by finitely many cosets by not necessarily distinct subgroups. If we omit any cosets of subgroups of infinite index, the remaining cosets will still cover the group.*

In my talk I will report on my journeys through groups, loops, rings and semigroups, on what I discovered there about finite coverings together with several fellow travelers and on some discoveries which might still lie ahead.

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* This talk is dedicated to James Wiegold

AUTOMORPHISMS OF FINITE p-GROUPS WITH A PARTITION

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For a finite p-group P the following three conditions are equivalent:

(a) to have a (proper) partition, that is, to be the union of some proper subgroups with trivial pairwise intersections;

(b) to have a proper subgroup outside of which all elements have order p; this obviously means that P has proper Hughes subgroup: $H_p(P) = \langle g \in P \mid g^p \neq 1\rangle \neq P$;

(c) to be a semidirect product $P = P_1 \rtimes \langle\varphi\rangle$ with P_1 being a subgroup of index p and φ a splitting automorphism of order p of P_1, which means that $\varphi^p = 1$ and $xx^{\varphi}x^{\varphi^2}\cdots x^{\varphi^{p-1}} = 1$ for all $x \in P_1$.

Theorem 1. *If a finite p-group P with a partition admits a soluble group of automorphisms A of coprime order such that the fixed-point subgroup $C_P(A)$ is soluble of derived length d, then P has a maximal subgroup that is nilpotent of class bounded in terms of p, d, and $|A|$.*

The proof is based on a similar result of the author and Shumyatsky for the case where P has exponent p and on the trick of "elimination of automorphisms by nilpotency", which was used earlier by the author, in particular, for studying finite p-groups with a splitting automorphism of order p.

Theorem 2. *If a finite p-group P with a partition admits a group of automorphisms A that acts faithfully on $P/H_p(P)$, then the exponent of P is bounded in terms of the exponent of $C_P(A)$.*

The proof of this result is based on the author's positive solution of the analogue of Restricted Burnside Problem for finite p-groups with a splitting automorphism of order p.

Both theorems yield corollaries for finite groups admitting a Frobenius group of automorphisms whose Frobenius kernel has prime order and acts by splitting automorphisms.

CENTRALIZERS IN SIMPLE LOCALLY FINITE GROUPS

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The structure of the centralizers of elements played an important rôle in the classification of the finite simple groups and we have considerable information about the centralizers of elements in finite simple groups. A group is called a **locally finite group** if every finitely generated subgroup is a finite group. For the centralizers of elements in infinite simple locally finite groups, we may use the information in the centralizers of elements in finite simple groups with the help of **Kegel Sequences**. Kegel proved that every countable infinite simple locally finite group has a sequence of finite subgroups $G_1 \leq G_2 \leq G_3 \dots$ where $G = \bigcup G_i$, $|G_i| < \infty$ and $N_i \lhd G_i$, G_i/N_i is simple and $G_i \cap N_{i+1} = 1$. The sequence (G_i, N_i), $i \in \mathbb{N}$ is called a **Kegel sequence** of G.

A characterization of simple locally finite subgroups in which every proper subgroup has a locally soluble subgroup of finite index is given by Kleidman and Wilson in [2]. The examples of simple non-linear locally finite groups with an element whose centralizer is a p-group is constructed by Meierfrankenfeld in [3]. We will discuss the affect of the restriction on centralizers of elements to the simple locally finite group and the following theorem.

Theorem. *(Ersoy K., Kuzucuoğlu M.) Let G be a non-linear simple locally finite group which has a Kegel sequence $\mathcal{K} = \{(G_i, 1) : \quad i \in \mathbb{N} \}$ consisting of finite simple subgroups. Then for any finite $\mathcal{K}$-semisimple subgroup F, the centralizer $C_G(F)$ is an infinite group.*

Moreover $C_G(F)$ has an infinite abelian subgroup A isomorphic to the restricted direct product of Z_{p_i} for infinitely many distinct prime p_i.

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SOME QUESTIONS ARISING FROM THE STUDY OF THE GENERATING GRAPH

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For a finite group G let $\Gamma(G)$ denote the graph defined on the non-identity elements of G in such a way that two distinct vertices are connected by an edge if and only if they generate G. In the talk we will present a series of questions related to the study of this graph.

- Many deep results on the generation of the finite simple groups G can be equivalently stated as theorems that ensure that $\Gamma(G)$ is a rich graph, with several good properties. We consider $\Gamma(G^\delta)$ where G is a finite non-abelian simple group and G^δ is the largest 2-generated power of G, with the aim to investigate whether the good generation properties of G still affect the behaviour of $\Gamma(G^\delta)$. In particular we prove that the graph obtained from $\Gamma(G^\delta)$ by removing the isolated vertices is 1-arc transitive and connected and we investigate the diameter of this graph.
- A Hamiltonian cycle is a cycle in an undirected simple graph which visits each vertex exactly once. If G is a sufficiently large finite simple group or a sufficiently large symmetric group, then the graph $\Gamma(G)$ contains a Hamiltonian cycle. The following conjecture was proposed: let G be a finite group with at least 4 elements, then the graph $\Gamma(G)$ contains a Hamiltonian cycle if and only if G/N is cyclic for all non-trivial normal subgroups N of G. We will describe some conditions on the positive integer m that ensure that $\Gamma(G)$ contains a Hamiltonian cycle when $G = S \wr C_m$ is the wreath product of a finite simple group S and a cyclic group of order m.
- For $2 \leq d \in \mathbb{N}$, denote by $\mu_d(G)$ the largest m for which there exists an m-tuple of elements of G such that any of its d entries generate G. We obtain a lower bound for $\mu_d(G)$ in the case when G is a d-generated finite solvable group. Our result implies in particular that if G is d-generated then the difference $\mu_d(G) - d$ tends to infinity when the smallest prime divisor of the order of G tends to infinity.

ON GROUPS WITH GIVEN PROPERTIES OF FINITE SUBGROUPS *

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We consider the following matters.

Theorem. *Suppose that in every finite even order subgroup F of a periodic group G for every involution u of F and every element x of F the equality $[u,x]^2 = 1$ holds. Then the subgoup I, which is generated by all the involutions of G, is a locally finite 2-group. Besides, the normal closure in G of every order 2 subgroup is abelian.*

The authors do not know weather or not the nilpotence class or derived length of the subgroup I from the conclusion of the Theorem is bounded.

Corollary. *Suppose that in a group G the order of a product of every two involutions is finite and every finite even order subgroup of G is nilpotent or has exponent 4. Then the Sylow 2-subgroup T of G is normal in G and either nilpotent of class 2 or has exponent 4. If moreover G is periodic, then $G = TC_G(T)$.*

The proof of the Theorem rests on the following result, which states that a group G, generated by three involutions, such that the order of the product of every two involutions of G devides 4, is finite.

Proposition. *Let G be a group generated by three involutions. If the order of the product of every two involutions of G devides 4, then G is a finite 2-group.*

The authors do not know wheather the same statement is true for the groups which are generated by four or more involutions.

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* The work is dedicated to Narain D. Gupta
** Joint work with Viktor Mazurov

SOME INTEGRAL REPRESENTATIONS OF FINITE GROUPS

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Let K/F be a Galois extension of finite degree of the field of rationals $\mathbf{Q}$, let O_K and O_F be the maximal orders of K and F, and let Γ be the Galois group of K/F. We study the realization fields $K = F(G)$ obtained via adjoining to F all matrix coefficients of all matrices $g \in G$ for some finite subgroup $G \subset GL_n(O_K)$.

The following theorem was proven using the results for finite flat group schemes over $\mathbf{Z}$, the ring of rational integers, annihilated by a prime p, obtained by V. A. Abrashkin and J.-M. Fontaine:

Theorem. *Let $K/\mathbf{Q}$ be a normal extension with Galois group $\Gamma = Gal(K/\mathbf{Q})$, and let $G \subset GL_n(O_K)$ be a finite Γ-stable subgroup. Then $G \subset GL_n(O_{K_{ab}})$ where K_{ab} is the maximal abelian over $\mathbf{Q}$ subfield of K. Moreover, $\mathbf{Q}(G) = \mathbf{Q}(\zeta_t)$ for some root ζ_t of 1.*

This theorem can be specified using a generalization of the concept of permutation lattices and permutation modules implemented by A. Weiss and K.W. Roggenkamp to study Zassenhaus Conjecture for group rings, and the structure of representations of the subgroups $G \subset GL_n(O_{K_{ab}})$ in the theorem above can be given more explicitly.

Similar results for totally real extensions and CM-fields $K/\mathbf{Q}$ are interesting for classification problems of quadratic and Hermitian lattices, and also for Galois cohomology.

For instance, the theorem above implies that for definite arithmetic groups $G \subset GL_n(O_K)$ the kernel of the natural cohomology map

$$H^1(Gal(K/\mathbf{Q}), G) \to \prod_v H^1(Gal(K_v/\mathbf{Q_v}), G_v)$$

is trivial.

CURIOUSER AND CURIOUSER (SAID ALICE TO PHILIP HALL)

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In 1938 Hall published an identity involving the orders of finite abelian p-groups and of their automorphisms groups. Alternative proofs were given by several people (including the speaker). Hall characterized his identity as "rather curious". We will discuss related identities for non-abelian p-groups; some of these apply the Rogers-Ramanujan identities from number theory. It seems that Hall was already aware of such identities, at least implicitly.

ON BUTLER $B(n)$-GROUPS

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A Butler $B(n)$-*group* is a torsionfree Abelian group which is the sum of a finite number of its rank 1 subgroups, tied by n independent linear relations. It is thus determined by a linear choice: the n relations - and by an order-theoretical choice: the isomorphism *types* of the rank one groups.

$B(0)$-groups are thus finite direct sums of rank 1 torsionfree Abelian groups, known as *completely decomposable* (= c.d.) groups, studied by Reinhold Baer in 1937.

The definition of the class of Butler groups dates to a paper by Michael Butler of 1967, showing that it has an alternative definition, as the class of pure subgroups of c.d. groups. The most studied subclass is that of $B(1)$-groups, where the order-theoretical side prevails to yield informations on the structure of the group: besides DVM, Arnold and Vinsonhaler are the foremost contributors among many other algebraists.

As soon as the relations become two or more ($n \geq 2$), the linear side shows its relevance, to the point of becoming almost determinant.

I will outline some of the recent developments of this surprising, unusual and fascinating class of groups.

* Joint work with Clorinda De Vivo

FITTING HEIGHT AND CHARACTER DEGREE GRAPHS *

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Given a finite group G and its set of irreducible characters $Irr(G)$, a prime p is a vertex of the character degree graph $\Gamma(G)$ if $p|\chi(1)$ for some $\chi \in Irr(G)$. Two primes p and q are adjacent if $pq|\chi(1)$ for some $\chi \in Irr(G)$. We define a vertex complete if it is adjacent to all the others. Since its introduction in [3], the role of the Fitting height $h(G)$ has been extensively analysed. For G solvable, $h(G) \leq 4$ if $\Gamma(G)$ is disconnected ([2]), as well as if $\Gamma(G)$ has 4 vertices and none of them complete ([1]). We generalize these results.

Theorem A *If a solvable finite group G has a character degree graph $\Gamma(G)$ with no complete vertices, then $h(G) \leq 4$, and this is the best possible bound.*

In [4] it is proved that if there is at most a complete vertex then $h(G) \leq 31$. We improve this estimation.

Theorem B *If a solvable finite group G has a character degree graph $\Gamma(G)$ with exactly one complete vertex, then $h(G) \leq 6$.*

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EMBEDDING THEOREMS FOR AMENABLE GROUPS *

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Let G be a group with a finite generating set X and $|g|_X$ the length of an element g with respect to X. If H is a subgroup of G and ℓ it the restriction of $|\cdot|_X$ to H, then the function $\ell : H \to \{0, 1, 2, \dots\}$ obviously satisfies the conditions

($\mathbf{L_1}$) $\ell(h) \geq 0$, and $\ell(h) = 0$ iff $h = 1$.
($\mathbf{L_2}$) $\ell(h) = \ell(h^{-1})$ for any $h \in H$.
($\mathbf{L_3}$) $\ell(gh) \leq \ell(g) + \ell(h)$ for any $g, h \in H$.
($\mathbf{L_4}$) There is $a > 1$ such that $\#\{h \in H \mid \ell(h) \leq n\} \leq a^n$ for any $n \geq 1$.

Conversely [1], if H is a group and ℓ is a function satisfying ($\mathbf{L_1}$)–($\mathbf{L_4}$), then H is a subgroup of a group G with a finite set of generators X such that for some positive constants c_1 and c_2 and every $h \in H$, we have
$c_1|h|_X \leq \ell(h) \leq c_2|h|_X$.

Now we prove the same statement under the assumption that both H and G belong to a class of groups $\mathcal{C}$, e.g. $\mathcal{C}$ is the class of all (a) solvable groups or (b) amenable groups, or (c) elementary amenable groups, or (d) groups satisfying non-trivial identities, or (e) groups satisfying the property A.

We cannot anymore exploit the small cancellation techniques as in [1] since such constructions endows G with many free subgroups. Instead of this we develop the method of wreath products going back to Neumanns' work [2]. As applications of the theorem, we get answers to some known questions.

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* This talk is dedicated to the memory of Narain Gupta and James Wiegold
** Joint work with Denis Osin

GROUPS WITH FEW ISOMORPHISM TYPES OF DERIVED SUBGROUPS

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A *derived subgroup* in a group G is the derived, i.e., commutator, subgroup of some subgroup of G. This work is part of a program to assess the importance of the set of derived subgroups of a group within the lattice of all subgroups. Here we investigate groups in which there are at most two isomorphism types of derived subgroup. This class contains groups of many diverse types, including abelian groups, free groups with countable rank, Tarski groups and a wide variety of soluble groups. We are able to construct all groups with just two isomorphism classes of derived subgroups in the case where the commutator subgroup is finite, and also, somewhat less precisely, when the group is soluble with finite rank. The classifications raise some interesting number theoretic problems.

* This is joint work with Patrizia Longobardi, Mercede Maj and Howard Smith.

ABOUT THE GROUP OF NORMALIZED UNITS OF INTEGRAL GROUP RINGS

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Using the Luthar-Passi method and results of M. Hertweck, we study the long-standing conjecture of H. Zassenhaus for normalized units in integral group rings $\mathbb{Z}A_n$ of alternating groups $A_n, n \leq 10$. As a consequence of our results, we confirm the W. Kimmerle's conjecture about prime graphs for those groups.

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SUPERSOLUBLE CONDITIONS AND TRANSFER CONTROL

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Definition. *Let G be a finite group and $P \in Syl_p(G)$. Let P^* be a strongly closed subgroup of P with respect to G. We say that*

$$\Phi(P^*) = V_o \trianglelefteq V_1 \trianglelefteq \ldots \trianglelefteq V_n = P^*$$

is a Φ-chain of P^* if $\Phi(P^*)$ is strongly closed in P with respect to G, V_i is weakly closed in P with respect to G and $|V_i : V_{i-1}| = p$, $i = 1, \ldots n$.

In this talk we present the following characterization of supersoluble groups.

Proposition. *A finite group G is supersoluble if and only if for every $p \in \pi(G)$ if $P \in Syl_p(G)$, then P possesses a strongly closed subgroup P^* which has a Φ-chain and P/P^* is cyclic.*

Moreover some results on transfer control and on the existence of normal complements will be given.

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* Joint work with Anna Luisa Gilotti

OVERGROUPS OF SUBSYSTEM SUBGROUPS IN EXCEPTIONAL GROUPS

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In the present talk we discuss recent results on overgroups of subsystem subgroups in exceptional Chevalley groups over commutative rings. Such results were known over fields from the work of Aschbacher, Liebeck, Seitz, King, Li Shangzhi, and others. However, until very recently over rings similar results were available only for classical groups, no known approaches would work for exceptional ones. However, now the advances of Saint Petersburg school of the last decade allow to successfully address this problem for exceptional cases. We state some general results, such as determination of levels and normalisers of intermediate subgroups. The talk culminates in a complete description of overgroups, in the simplest case of $\mathbf{A_5} + \mathbf{A_1}$ in $\mathbf{E_6}$.

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ON VERBAL SUBGROUPS IN FINITE AND PROFINITE GROUPS

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The circle of problems discussed in this talk can be described as follows.

Given a word w and a group G, assume that certain restrictions are imposed on the set of all w-values in G. How does this influence the properties of the verbal subgroup $w(G)$?

In this talk we deal with profinite groups G such that all w-values in G are contained in a union of finitely many subgroups with certain prescribed properties. We show that $w(G)$ has similar properties.

FINITARY AUTOMORPHISMS OF GROUPS

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We study the structure of groups of *finitary automrphisms* of an arbitrary group G. These groups were introduced in [1] as follows:

Definition. *An automorphism φ of a group G is said to be* finitary *if φ acts trivially on a subgroup of finite index in G, i.e.*

$$|G : \mathrm{C}_G(\varphi)| < \infty.$$

The set of all finitary automorphisms of a group G is denoted $\mathrm{FAut}\, G$ and is referred to as *the group of all finitary automorphisms* of G. It is clear that $\mathrm{FAut}\, G$ is indeed a group, and furthermore, it is a normal subgroup in $\mathrm{Aut}\, G$.

Theorem. *For an arbitrary group G there is a normal series in* $\mathrm{FAut}\, G$*:*

$$1 \leqslant H_1 \leqslant H_2 \leqslant H_3 \leqslant H_4 = \mathrm{FAut}\, G$$

such that

(i) H_1 *is nilpotent of class* $\leqslant 4$*;*

(ii) H_2/H_1 *is abelian;*

(iii) H_3/H_2 *is a* ZA*-group;*

(iv) H_4/H_3 *can be embedded into the restricted direct product of groups K_i, such that each K_i is a subgroup of* $\mathrm{FGL}(V_i)$*, where V_i is a vector space over a prime field.*

Corollary. *Let G be an arbitrary group, and let A/B be an infinite simple section of* $\mathrm{FAut}\, G$*. Then there exist a prime p and a vector space V over* $\mathrm{GF}(p)$ *such that A/B is isomorphic to a section of* $\mathrm{FGL}(V)$*.*

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* Joint work with V.V. Belyaev, Department of Mathematics, MIPT.

ON GROUPS OF ODD ORDER ADMITTING AN ELEMENTARY 2-GROUP OF AUTOMORPHISMS

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Let G be a group and A a group of automorphisms of G. The subgroup of all elements of G fixed by A is usually denoted by $C_G(A)$. It is well-known that very often the structure of $C_G(A)$ has strong influence over the structure of the whole group G. The influence seems especially strong in the case where G is a finite group of odd order and A is an elementary abelian 2-group. In particular in [1] it was proved that if G is a finite group of derived length k on which an elementary abelian group A of order 2^n acts fixed-point-freely, then G has a normal series $G = N_0 \geq \cdots \geq N_{n-1} \geq N_n = 1$ all of whose quotients are nilpotent and the class of N_{i-1}/N_i is bounded by a function of k and i (see also [2] for a short proof of this result).

What we can say if $C_G(A)$ is of finite exponent? In [3] we proved the following result:

Theorem. *Let G be a finite group of odd order and of derived length k. Suppose that G admits an elementary abelian group A of automorphisms of order 2^n such that $C_G(A)$ has exponent e. Then G has a normal series*

$$G = G_0 \geq T_0 \geq G_1 \geq T_1 \geq \cdots \geq G_n \geq T_n = 1$$

such that the quotients G_i/T_i have $\{k, e, n\}$-bounded exponent and the quotients T_i/G_{i+1} are nilpotent of $\{k, e, n\}$-bounded class.

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* Joint work with Karise G. Oliveira and Pavel Shumyatsky

UNITRIANGULAR FACTORISATIONS OF CHEVALLEY GROUPS

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Lately, the following problem has attracted a lot of attention in various contexts: find the shortest factorisation $G = UU^{-}UU^{-} \dots U^{\pm}$ of a Chevalley group $G = G(\Phi, R)$ in terms of the unipotent radical $U = U(\Phi, R)$ of the standard Borel subgroup $B = B(\Phi, R)$ and the unipotent radical $U^{-} = U^{-}(\Phi, R)$ of the opposite Borel subgroup $B^{-} = B^{-}(\Phi, R)$. So far, the record over a finite field was established in a 2010 paper by Babai, Nikolov, and Pyber, where they prove that a group of Lie type admits unitriangular factorisation $G = UU^{-}UU^{-}U$ of length 5. It was recently proved by the authors that over any ring of stable rank 1 one has unitriangular factorisation $G = UU^{-}UU^{-}$ of length 4.

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* Joint work with Nikolai Vavilov and B. Sury

THE COXETER COMPLEX AND THE EULER CHARACTERISTICS OF A HECKE ALGEBRA

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For any Hecke algebra $\mathcal{H} = \mathcal{H}_q(W, S)$ associated to a Coxeter group (W, S), of parameter q, we introduce a finite-dimensional chain complex $C_\bullet$ of $\mathcal{H}$-modules which reflects many properties of the Coxeter complex of (W, S), e.g., it is acyclic if, and only if, W is infinite (i.e., non spherical).

Proposition. *For infinite W, the complex $C_\bullet$ is a finite dimensional resolution of the trivial $\mathcal{H}$-module. Under suitable conditions on q, projectivity is ensured.*
By an inductive argument, the augmented algebra $\mathcal{H}$ is of type FP.

There is a canonical trace (generalizing the Markov trace) which, evaluated on the Hattori–Stallings rank of the trivial $\mathcal{H}$-module, can be considered as an Euler characteristics $\chi_{\mathcal{H}}$ of $\mathcal{H}$.

Theorem. *For q generic, $\chi_{\mathcal{H}}$ is the inverse of the Poincaré series of (W, S).*

In particular, a combinatorial object such as the Poincaré series of a Coxeter groups (W, S) has a cohomological interpretation as Euler characteristic of another suitable object, the Hecke algebra of (W, S).

This also explains the alternating sum formula often used to compute Poincaré series of Coxeter groups.

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* Joint work with Th. Weigel

ON GROUPS ADMITTING A WORD WHOSE VALUES ARE ENGEL

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It is a long-standing problem whether any n-Engel group is locally nilpotent. Following Zelmanov's solution of the restricted Burnside problem, Wilson proved that this is true if G is residually finite [3]. Later Shumyatsky showed that, if all γ_k-values of a residually finite group G are n-Engel, then $\gamma_k(G)$ is locally nilpotent [1, 2]. This suggests the following:

Conjecture. *Let w be a group-word and n a positive integer. Assume that G is a residually finite group in which all w-values are n-Engel. Then the corresponding verbal subgroup $w(G)$ is locally nilpotent.*

In this talk, we answer the question positively for some particular words and examine whether this holds in the case where G is locally graded rather than residually finite.

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* Joint work with R. Bastos, P. Shumyatsky and M. Tota

SYMPLECTIC ALTERNATING ALGEBRAS

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Symplectic alternating algebras are algebraic structures that have arisen from the study of 2-Engel groups but seem also to be of interest in their own right with some beautiful properties. In this talk I will give an introduction to these structures.

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* Joint work with Antonio Tortora and Maria Tota

STEINBERG-LIKE REDUCIBLE CHARACTERS FOR CHEVALLEY GROUPS

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The Steinberg character plays outstanding role in the character theory of Chevalley groups. This inspires attempts to study, and possibly determine, the characters that have some, but not all, Steinberg character properties.

Let G be a simple Chevalley group in characteristic p, $|G|_p$ the p-part of the order of G. Then St, the Steinberg character of G, is of degree $|G|_p$ and vanishes at all elements of order divisible by p. It is well known that St is a unique irreducible character of G with these properties.

Problem. Determine reducible characters χ of G of degree $|G|_p$ that vanish at all elements of order divisible by p.

We call characters in the above problem Steinberg-like characters. The characters of projective modules of degree $|G|_p$ are Steinberg-like. Malle and Weigel [1] determined them under assumption that the trivial character 1_G is a constituent of χ. It turns out that the latter restriction can be dropped:

Theorem. [3] *Let χ be the character of a reducible projective module of degree $|G|_p$. Then 1_G is a constituent of χ, and therefore χ belongs to the list provided in* [1].

Further results have been obtained jointly with M. Pellegrini.

Theorem. [2] *Let G be a simple Chevalley group in characteristic p.*

(1) *Steinberg-like characters do not exist if $G \in \{E_6(q), E_7(q), E_8(q)$, $PSU_n(q)$, $n > 3$ or $PSU_3(q)$ for $3 \nmid q+1$, $PSpin^-_{2n}(q), n > 3$, $PSL_n(q), n > 5$ or $n = 4, 5$ and $3 \nmid q+1$, $PSp_{2n}(q), n > 5$ or $2 \le n \le 5$ and $7 \nmid q+1$, $PSpin^+_{2n}(q), n > 5\}$.*

(2) *If $G \in \{PSL_2(q)$, ${}^2B_2(q)$, ${}^2G_2(q)$, $PSL_3(q)$, $PSL_n(q)$ for $n = 4, 5$ and $3|q + 1$, $PSU_3(q)$ for $3|q + 1$, $PSp_4(q)$ for $7|q + 1\}$ then G has Steinberg-like characters.*

References

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