Parabolic subgroups of Chevalley groups over Hasse domains

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Notations

R – ring

Φ – root system

$G(\Phi, R)$ – Chevalley group of type Φ

$E(\Phi, R)$ – elementary group of type Φ

$E(\Phi, R, I)$ – relative elementary group

$$E(\Phi, R) = \langle x_\alpha(\xi) \mid \alpha \in \Phi,\ \xi \in R \rangle$$

$$E(\Phi, R, I) = \langle x_\alpha(\xi) \mid \alpha \in \Phi,\ \xi \in I \rangle^{E(\Phi, R)}$$

Intermediate subgroups

Let G be a Chevalley group and H a subgroup of G, containing standart Borel subgroup, i.e.

$$B \leqslant H \leqslant G.$$

Then we call H an *intermediate* subgroup.

The Problem

Classify intermediate subgroups of Chevalley group over some good classes of rings, i.e.

$$B(\Phi, R) \leqslant H \leqslant G(\Phi, R)$$

Classify H.

Known results

- R is a field — Jacques Tits
- R is a semilocal ring — Borewicz, Dybkova, Suzuki, Vavilov.

Hasse Domains

Let k be a global field, i.e. finite algebraic extension of $\mathbb{Q}$ or a field of algebraic functions over a finite field.

$$S = \{\text{finite set of valuations of } k\}$$

For valuation p of field k, we denote by v_p the corresponding exponent.

$$R_S = \{x \in k \mid v_p(x) \geq 0 \text{ for all } p \notin S\}$$

Key Theorem

Let H be an intermediate subgroup of Chevalley group G. Then the following alternative holds:

- either H contains a relative elementary subgroup E_I
- or H is contained in a proper parabolic subgroup

Theorem (Alexandrov, Vavilov)

Let H be an intermediate subgroup of $SL(n,R)$ or $Sp(2n,R)$. Then the following alternative holds:

- either H contains a relative elementary subgroup E_I
- or H is contained in a proper parabolic subgroup

Theorem (Batalkin, Vavilov)

Let H be an intermediate subgroup of $SO(2n, R)$. Then the following alternative holds:

- either H contains a relative elementary subgroup E_I
- or H containing in a proper parabolic subgroup

For using classification theorem we need two technical assumptions:

1. $R = \mathbb{Z}[R^*]$
2. The ideal of R generated by $\varepsilon^2 - 1$, $\varepsilon \in R^*$ coincides with R.

- N. Vavilov, *Parabolic subgroups of Chevalley groups over commutative ring*, 1982

Thank you!