Groups with given properties of finite subgroups

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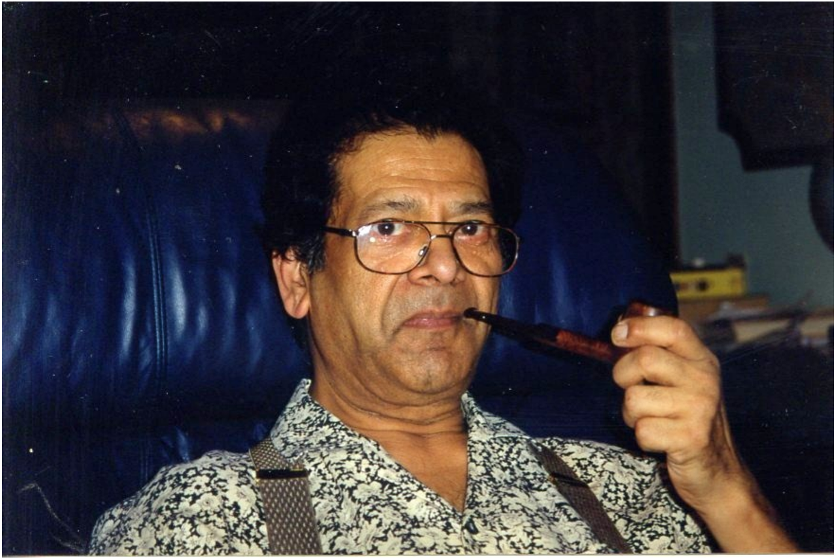
Dedicated to Narain D. Gupta



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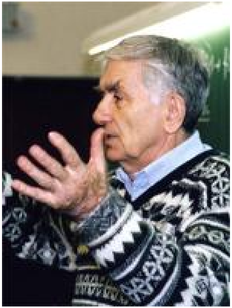
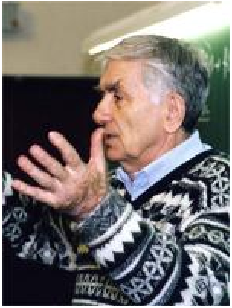
N. D. Gupta, V. D. Mazurov, On groups with small orders of elements, Bull Austral. Math. Soc., **60**, No. 5 (1999), 197–205.

P. S. Novikov, S. I. Adian, 1968

Burnside variety $B(m)$ of groups defined by the law $x^m = 1$, where m is odd and large enough, contains non locally finite groups. Free n-generator group $B(m, n)$ in this variety is infinite for $n > 1$.



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S. I. Adian, 1971

Every finite subgroup of $B(m, n)$, where $m \geqslant 665$ and m is odd, is cyclic.

W. Burnside, 1901

A 2-group whose finite subgroups are cyclic is abelian.

Theorem 1

Suppose that in every finite even order subgroup F of periodic group G every involution u of F and every element x of F satisfy the equality $[u, x]^2 = 1$. Then the subgroup I generated by all involutions of G is locally finite and is a 2-group. Besides, the normal closure in G of every order 2 subgroup is abelian.

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Theorem 2

Suppose that in every finite order subgroup F of a group G every involution u and every element x of F satisfy the equality $[u, x]^2 = 1$. Then the subgroup I generated by all involutions of G is locally finite and is a 2-group.

Corollary 1

Suppose that in a group G the order of the product of every two involutions is finite and every finite even order subgroup of G is nilpotent or has exponent 4. Then the Sylow 2-subgroup T of the group G is normal in G and either is nilpotent of class 2 or has exponent 4. If in addition G is periodic then $G = TC_G(T)$.

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Corollary 2

Suppose that in every finite even order subgroup of a periodic group G the equality $[x, y]^2 = 1$ holds. Then the Sylow 2-subgroup T of a group G is locally finite and normal in G. Moreover, $T' = [T, T]$ is a group of exponent 4, $T'' = [T', T']$ lies in the center of T and every finite subgroup of G/T is abelian.

Proposition

Let G be a group generated by three involutions. If the order of the product of every two involutions of G divides 4, then G is a finite 2-group.

GRAZIE!