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A word $w$ is said to be concise if whenever $G_w$ is finite for a group $G$, it always follows that $w(G)$ is finite. P. Hall asked whether every word is concise, but later Ivanov proved that this problem has a negative solution in its general form. On the other hand, many relevant words are known to be concise. For instance, Turner-Smith showed that the lower central words $\gamma_k$ and the derived words $\delta_k$ are concise. Merzlyakov showed that every word is concise in the class of linear groups. There is an open problem, due to Dan Segal, whether every word is concise in the class of residually finite groups.
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These results remind the situation with coverings of groups.

A covering of a group $G$ is a family $\{S_i\}_{i \in I}$ of subsets of $G$ such that $G = \bigcup_{i \in I} S_i$.

If $\{H_i\}_{i \in I}$ is a covering of $G$ by subgroups, it is natural to ask what information about $G$ can be deduced from properties of the subgroups $H_i$.

In the case where the covering is finite actually quite a lot about the structure of $G$ can be said. The first result in this direction is due to Baer who proved that $G$ admits a finite covering by abelian subgroups if and only if it is central-by-finite. The nontrivial part of this assertion is an immediate consequence of a subsequent result of B.H. Neumann, 1954: if $\{S_i\}$ is a finite covering of $G$ by cosets of subgroups, then $G$ is also covered by the cosets $S_i$ corresponding to subgroups of finite index in $G$. In other words, we can get rid of the cosets of subgroups of infinite index without losing the covering property.

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On Verbal Subgroups in Finite and Profinite Groups
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Theorem

Let $w$ be an outer commutator word and $G$ a profinite group that has finitely many periodic subgroups $G_1, G_2, \ldots, G_s$ whose union contains all $w$-values in $G$. Then $w(G)$ is locally finite.
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It follows from the proof that if under the hypothesis of the above theorem the subgroups $G_1, G_2, \ldots, G_s$ have finite exponent, then $\omega(G)$ has finite exponent as well.
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Using the Lie-theoretic techniques that Zelmanov created in his solution of the restricted Burnside problem, we obtained the following related result.

**Theorem**

Let $e$, $k$, $s$ be positive integers and $G$ a profinite group that has subgroups $G_1, G_2, \ldots, G_s$ whose union contains all $\gamma^k$-values in $G$. Suppose that each of the subgroups $G_1, G_2, \ldots, G_s$ has finite exponent dividing $e$. Then $\gamma^k(G)$ has finite $(e, k, s)$-bounded exponent.
It follows from the proof that if under the hypothesis of the above theorem the subgroups $G_1, G_2, \ldots, G_s$ have finite exponent, then $w(G)$ has finite exponent as well. We address the question whether the exponent of $w(G)$ is bounded in terms of the exponents of $G_1, G_2, \ldots, G_s$ and $s$. We answered the question in the affirmative only in the particular case where $w = \gamma_k$. 

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Sometimes the assumption that the subgroups are countably many is good enough. For example, we have

1. Let $G$ be a profinite group having countably many soluble subgroups whose union contains all $w$-values. Then $w(G)$ is soluble by finite.

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Let $e$ be a positive integer and $w$ a multilinear commutator. Suppose that $G$ is a finite group in which any nilpotent subgroup generated by $w$-values has exponent dividing $e$. Then the exponent of the corresponding verbal subgroup $w(G)$ is bounded in terms of $e$ and $w$ only.
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Let \( w \) be a multilinear commutator, \( G \) a finite group and \( P \) a Sylow \( p \)-subgroup of \( w(G) \). Is \( P \) generated by \( w \)-values?
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Another question addressed in this talk –

Let $e$ be a positive integer and $w$ a word. Suppose that $G$ is a finite group in which any nilpotent subgroup generated by $w$-values has exponent dividing $e$. Is the exponent of the corresponding verbal subgroup $w(G)$ bounded in terms of $e$ and $w$ only?

There are no easy counter-examples to this question. The answer is positive for every non-commutator word (easy to check). Actually I think I know also the proof for Engel words and some other non-multilinear commutator words.

Grazie mille per l’attenzione!
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