On Verbal Subgroups in Finite and Profinite Groups

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This result depends on the classification of finite simple groups. Though this is not as good as we would like to get, the result is sufficient to prove that if G is a finite group having subgroups $G_1, G_2, \dots, G_s$, each of exponent e, whose union contains all γ_k-values, then the exponent of $\gamma_k(G)$ is (e, k, s)-bounded.

Indeed, let P be a Sylow p-subgroup of $\gamma_k(G)$ and $Q = \gamma_k(P)$. Then Q is the product $Q_1 Q_2 \dots Q_s$, where $Q_i = G_i \cap Q$. Using Zelmanov's Lie-theoretic techniques one can show that Q has (e, k, s)-bounded exponent. We also know that P is generated by powers of γ_k-values – elements of order dividing e. Hence P has (e, k, s)-bounded exponent. This holds for every Sylow subgroup of $\gamma_k(G)$.

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