Finitary automorphisms of groups

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Groups $\mathrm{FAut}\, G$ and $\mathrm{FGL}(V)$

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An automorphism φ of a group G is said to be *finitary* if

$$|G : \mathrm{C}_G(\varphi)| < \infty$$

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Note

If V is a vector space over $\mathrm{GF}(p)$, then $\mathrm{FAut}\,V = \mathrm{FGL}(V)$.

$\mathrm{FAut}\, G$ is locally finite "up to an abelian group"

Theorem 1

For an arbitrary G, $\mathrm{FAut}\, G$ is an extension of an abelian group by a locally finite group.

Theorem 2

For an arbitrary G, $[\mathrm{FAut}\, G, \mathrm{FAut}\, G]$ is locally finite.

$\mathrm{FOut}\, G$ is always locally finite

Definition 3

The groups of *inner* and *outer* finitary automorphisms of G:

$$\begin{aligned} \mathrm{FInn}\, G &= \mathrm{FAut}\, G \cap \mathrm{Inn}\, G \\ \mathrm{FOut}\, G &= \mathrm{FAut}\, G / \mathrm{FInn}\, G \end{aligned}$$

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For an arbitrary G, $\mathrm{FOut}\, G$ is locally finite.

Corollary

For an arbitrary G, $\mathrm{FAut}\, \mathrm{FAut}\, G$ is locally finite.

$\mathrm{FAut}\,G$ for two special types of groups

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If G is semisimple, then $\mathrm{FAut}\,G$ has a faithful finitary permutation representation.

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Theorem 5

If G is an abelian p-group, then $\mathrm{FAut}\, G$ has a normal subgroup N such that:

(i) N is a ZA-group;

(ii) $(\mathrm{FAut}\, G)/N$ has a faithful finitary linear representation over $\mathrm{GF}(p)$.

Structure theorem

Theorem 6

For an arbitrary group G there is a normal series in $\mathrm{FAut}\, G$:

$$1 \leqslant H_1 \leqslant H_2 \leqslant H_3 \leqslant H_4 = \mathrm{FAut}\, G$$

such that

(i) H_1 is nilpotent of class $\leqslant 4$;

(ii) H_2/H_1 is abelian;

(iii) H_3/H_2 is a ZA-group;

(iv) H_4/H_3 can be embedded into the restricted direct product of groups K_i, such that each K_i is a subgroup of $\mathrm{FGL}(V_i)$, where V_i is a vector space over a prime field.

No new infinite simple sections

Corollary

Let G be an arbitrary group, and let A/B be an infinite simple section of $\mathrm{FAut}\, G$. Then there exist a prime p and a vector space V over $\mathrm{GF}(p)$ such that A/B is isomorphic to a section of $\mathrm{FGL}(V)$.