Ischia Group Theory 2014: Abstracts

RECENT PROGRESS ON THE CONJECTURE OF EXISTENCE OF NONINNER AUTOMORPHISMS OF ORDER p FOR FINITE NONABELIAN p-GROUPS

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Let p be any prime number and let G be an arbitrary finite nonabelian p-group. It is conjectured that G admits a noninner automorphism of order p (see Problem 4.13 of [1]). The validity of the conjecture has been verified for groups G satisfying one of the following properties:

1. G is nilpotent of class 2 or 3,
2. G is of coclass 1 or 2,
3. $G/Z(G)$ is powerful,
4. G is regular,
5. $C_G(Z(\Phi(G))) \neq \Phi(G)$,
6. G' is cyclic

We will talk about recent results on the conjecture.

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ON WORDS THAT ARE CONCISE IN RESIDUALLY FINITE GROUPS

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A group word w is called concise if whenever the word w has only finitely many values in a group G, it always follows that the verbal subgroup $w(G)$ is finite.

In the sixties P. Hall asked whether every word is concise but later Ivanov showed that the answer to Hall's question is negative in its general form [2]. On the other hand, many words are known to be concise. For instance, it was shown in [4] that the multilinear commutator words are concise.

There is an open problem whether every word is concise in the class of residually finite groups (see [3]). In this talk we will present the following result: if w is a multilinear commutator word and q is a prime-power, then the word w^q is concise in the class of residually finite groups.

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** Joint work with Pavel Shumyatsky

THE BURNSIDE PROBLEM ON PERIODIC GROUPS FOR ODD EXPONENTS $n > 100$

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In 1902 (see [1]) W.Burnside formulated the following problem

Is every group generated by finite number of generators and satisfying an identical relation $x^n = 1$ *finite?*

During several decades many mathematicians from different countries studied this problem. Maximal periodic groups $B(r, n)$ with r generators satisfying the identical relation $x^n = 1$ are called *free Burnside groups of exponent* n.

In 1950 W.Magnus formulated a special question on the existence of a maximal finite quotient group of the group $B(r, n)$ for a given pair (r, n). Magnus named this question "*Restricted Burnside Problem*".

A negative solution of the full (nonrestricted) Burnside problem was given in 1968 by P.S. Novikov and S.I. Adian. It was proved that the groups $B(r, n)$ are infinite for any $r > 1$ and odd $n \geq 4381$.

In 1975 the author published a book [2] where he presented an improved and generalized version of Novikov-Adian theory. The infiniteness of the groups $B(r, kn)$ was proved in [2] for odd $n \geq 665$ and any $k \geq 1$. Several other applications of the method for odd exponents $n \geq 665$ were established there as well.

In this talk we introduce a new simplified modification of Novikov-Adian theory that allows to give a shorter proof and stronger results for odd exponents. We prove the following theorem.

Theorem. *The free Burnside groups* $B(m, n)$ *are infinite for any odd exponent* $n > 100$.

A detailed survey of investigations on the Burnside Problem and on the Restricted Burnside problem one can find in authors survey paper [3].

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A MEMORIAL LECTURE TO THE LATE PROFESSOR DAVID CHILLAG (1946-2012)

Zvi Arad

In my lecture I will focus on three topics of my joint research with Professor David Chillag.

Topic 1: A contribution to the final classification of finite groups with CC-subgroups. Notice that a proper subgroup M of a group G is called a CC-subgroup iff centralizer $C_G(m)$ of every $m \in M^\sharp = M \backslash \{1\}$ is contained in M.

Topic 2: Properties of nilpotent injectors of finite solvable groups of odd order.

Topic 3: An analogy between products of conjugacy classes and products of irreducible characters. Generalizations and unifications of these concepts to generalized circulants, semisimple algebras and table algebras.

MODEL THEORETIC ADVANCES FOR GROUPS WITH BOUNDED CHAINS OF CENTRALIZERS *

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A group G has bounded chains of centralizers (G is $\mathfrak{M}_C$) if every chain of centralizers $C_G(A_1) \leq C_G(A_2) \leq \ldots$ is finite. While this class of groups is interesting in its own right, within the field of logic known as model theory, $\mathfrak{M}_C$ groups have been examined because they strictly contain the class of stable groups. Stable groups, which have a rich literature in model theory, robustly extend ideas such as dimension and independence from algebraic groups to a wider setting, including free groups. Stable groups gain much of their strength through a chain condition known as the Baldwin-Saxl chain condition, which implies the $\mathfrak{M}_C$ property as a special case.

Several basic, but key, properties of stable groups have been observed by Wagner [4] and others to follow purely from the $\mathfrak{M}_C$ condition (this builds upon the work of Bludov [2], Khukhro [3], and others). These properties were, as one would expect, purely group-theoretic. From the perspective of logic, the class of $\mathfrak{M}_C$ groups should be unruly, since the $\mathfrak{M}_C$ cannot be captured using first-order axioms (unless one insists on a fixed finite centralizer dimension). Yet the speaker and Tuna Altınel [1] have recently uncovered that $\mathfrak{M}_C$ groups possess a logical property of stable groups as well, namely the abundance of definable nilpotent subgroups. We shall present this result and describe the current investigations for finding an analogue for solvable subgroups.

* This talk is dedicated to Eric Jaligot

** Joint work with Tuna Altınel, Université Lyon 1

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THE LIE MODULE FOR THE SYMMETRIC GROUP

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There is a module Lie(n) for the symmetric group of degree n that occurs naturally in the study of free Lie algebras. I will give an elementary definition of this module over a field F and then describe recent work which seeks to determine the structure of the module in the case where F has positive characteristic p. In this case the non-principal block components of Lie(n) are projective and can be described completely. The non-projective indecomposable summands are in the principal block. We study the problem of describing their vertices and sources and obtain a reduction theorem to the case where n is a power of p. When $n = 9$ and $p = 3$ and when $n = 8$ and $p = 2$ we have a precise solution which suggests a possible general answer. Most of this talk will be accessible to a general audience.

CONJUGACY CLASS SIZES OF FINITE GROUPS

Rachel Camina

Given the conjugacy class sizes of a finite group what does this tell you about the structure of the group? Which sets of natural numbers can occur as class sizes of a finite group? We will consider these two questions.

** Joint work with Susanne Danz, Karin Erdmann and Jürgen Müller

THE GROUP OF INERTIAL AUTOMORPHISMS OF AN ABELIAN GROUP *

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In [2] and [4] an endomorphism φ of an abelian group A is said *inertial* iff $H\varphi/(H \cap H\varphi)$ is finite $\forall H \leq A$. This concept generalizes both multiplication and finitary endomorphism [1] and has been introduced in connection with the study of *inert subgroups* of groups, [3], [5], [6]. A subgroup is said to be inert if it is commensurable with its conjugates, where subgroups H and K are told *commensurable* iff $H \cap K$ has finite index in both H and K.

We consider the group $IAut(A)$ generated by the *inertial automorphisms* (i.e. inertial endomorphism which are bijective) of an abelian group A. From [2], we have that *$IAut(A)$ is formed by products $\gamma_1\gamma_2^{-1}$ where γ_1 and γ_2 are both inertial automorphisms.* Moreover, *if A has finite torsion-free rank, then $IAut(A)$ is formed by inertial automorphisms and is the kernel of the setwise action of $Aut(A)$ on the quotient lattice of subgroups of A wrt commensurability* (which is a lattice congruence, as A is abelian). We have:

Theorem (arXiv:1403.4193)**.** *The group $IAut(A)$ generated by the inertial automorphisms is (locally finite)-by-abelian and metabelian-by-(locally finite).*

Moreover, $IAut(A) = IAut_1(A) \times Q(A)$ where $Q(A)$ is isomorphic to a multiplicative group of rationals and there is $\Gamma_* \lhd IAut_1(A)$ such that:
i) $IAut_1(A)/\Gamma_*$ is locally finite;
ii) Γ_* acts by means of power automorphisms on its derived subgroup, which is a periodic abelian group.

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* This talk is dedicated to Martin Newell
** Joint work with Silvana Rinauro

THE THEOREMS OF SCHUR, BAER AND HALL *

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A well-known theorem of I. Schur states that if G is a group and $G/\zeta(G)$ is finite then G' is finite. In this talk we give a survey of recent work on these theorems and also obtain an analogue of this, and theorems due to R. Baer and P. Hall, for groups G that have subgroups A of $\mathrm{Aut}(G)$ such that $A/\mathrm{Inn}(G)$ is finite. A main result of this recent work is as follows:

Theorem. *Let G be a group and let A be a subgroup of $Aut(G)$. Suppose that $Inn(G) \leq A$ and that $A/Inn(G)$ is finite of order k. If $G/C_G(A)$ is finite of order t, then $[G, A]$ is finite of order at most kt^d, where $d = (1/2)(\log_p t + 1)$ and p is the least prime dividing t.*

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SOME APPLICATIONS OF MAGNUS EMBEDDING

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Let $R \hookrightarrow F_n \twoheadrightarrow G$ be a presentation of a group G. The group F_n/R' is called the *free abelianized extension* of G associated with this presentation.

In 1939 W. Magnus [1] published an extremely useful method for obtaining a certain sort of matrix representation of such free abelianized extensions. His method is now known as 'Magnus embedding' or 'Magnus representation'.

In this talk we'll discuss Magnus embedding, see how easy and natural it is, and prove a few theorems, some old and some new.

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* This talk is dedicated to the memory of Brian Hartley

** Joint work with Leonid A. Kurdachenko and Aleksander A. Pypka

ON THE HAUSDORFF SPECTRUM OF SOME GROUPS ACTING ON THE p-ADIC TREE

Gustavo A. Fernández-Alcober

The Hausdorff spectrum of a profinite group G is the set of Hausdorff dimensions of the closed subgroups of G. If A is the full group of automorphisms of the p-adic tree, we consider a class of subgroups of A which are a generalisation of the well-known Gupta-Sidki group. In this talk we give a survey on recent research about the Hausdorff spectrum of the groups in this class. This is joint work with Amaia Zugadi-Reizabal.

FACTORIZING A GROUP WITH CONJUGATE SUBGROUPS

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Let G be a finite group. We study the minimal number n such that there exist n pairwise conjugated proper subgroups of G whose product, in some order, equals G. We call $\gamma_{cp}(G)$ this number. I will present the following results.

Theorem. *If G is nonsolvable then G is the product of at most 36 conjugates of a proper subgroup. In other words $\gamma_{cp}(G) \leq 36$.*

Theorem. *For all integer $n > 2$ there exists a solvable group G with $\gamma_{cp}(G) = n$.*

** Joint work with Dan Levy, School of Computer Sciences, The Academic College of Tel-Aviv-Yafo - Israel

SOME INVERSE PROBLEMS IN BAUMSLAG-SOLITAR GROUPS

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The Baumslag-Solitar groups are defined as follows:

$$BS(m,n) = \langle a, b \ \mid \ b^{-1}a^m b = a^n \rangle$$

where m, n are integers.

We concentrate on the groups $BS(1,n)$ and their subsets of the type

$$S = \{b^r a^{x_1}, b^r a^{x_2}, \ldots, b^r a^{x_k}\} = b^r a^A$$

where r is a positive integer and $A = \{x_1, x_2, \ldots, x_k\}$ denotes a finite sequence of integer. In particular, $|S| = |A|$.

First we prove that

$$S^2 = \{b^r a^{x_i} b^r a^{x_j} \ \mid \ x_i, x_j \in A\}$$

satisfies the following equality:

$$S^2 = b^{2r} a^{n \star A + A}$$

where

$$n \star A + A = \{nx_i + x_j \ \mid \ x_i, x_j \in A\}.$$

Sets of type $n \star A + A$ are caled sums of dilates.

Moreover, we show that the groups $BS(1,n)$ for $n \geq 2$ are orderable, which implies, in view of our previous result, that $|X^2| \geq 3|X| - 2$ for all nonabelian subsets of such $BS(1,n)$.

Finally we apply known and new results concening the size of sums of dilates in order to prove various results concerning the size of S^2 for $S = b^r a^A \subset BS(1,2)$.

CONJUGACY IN RELATIVELY EXTRA-LARGE ARTIN GROUP *

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Let $M = (m_{i,j})$ be a symmetric $n \times n$ matrix, $n \geq 1$, with non-negative integer entries, different from 1. *The Artin group A defined by M* is presented by $\langle x_1, \dots, x_n \mid R_{ij}\ ,\ 1 \leq i < j \leq n \rangle$, where either R_{ij} is the empty word or $R_{ij} = UV^{-1}$, where U is the initial subword of $(x_i x_j)^{m_{ij}}$ of length m_{ij} and V is the initial subword of $(x_j x_i)^{m_{ij}}$ of length m_{ij}. A is called *large* if $m_{ij} \notin \{1, 2\}$ and *extra-large* if $m_{i,j} \notin \{1, 2, 3\}$. For a subset S of $\{x_1, \dots, x_n\}$ we say that A *is extralarge relative to the (parabolic) subgroup H* generated by S if the following two conditions are satisfied

(i) If $x_i \notin S$ and $x_j \notin S$ then $m_{ij} \notin \{1, 2\}$

(ii) If $x_i \in S$ and $x_j \notin S$ then $m_{ij} \notin \{1, 2, 3\}$

Conjugacy of elements in Artin groups has been studied for special classes of groups: large and extra-large ([1], [2], [5]), finite type (i.e. the corresponding Coexter group is finite), right-angled i.e. $m_{ij} \in \{0, 2\}$ ([4]). For each of these classes of groups different methods were used, which do not seem to have common generalisations which extend the results to groups that are the mixture of them.

In this work we show how different methods may have common extensions which allows detailed description of conjugacy in mixed type groups. Our main results are the following

Theorem. *Let H be a right-angled parabolic subgroup of A and assume that A is extra-large relative to H. Then each of the following holds*

1. *A has solvable conjugacy problem*
2. *If T is a subgroup of H which does not contain x^j, $j \in \mathbb{Z}$, $x \in \{x_1, \dots, x_n\}$ and T is malnormal in H, then T is malnormal in A.*
3. *For every non-cyclic parabolic subgroup of P of H we have $N_G(P) = N_H(P)$ and $C_G(P) = C_H(P)$.*

The Theorem extends to cases when the subgroup H is arbitrary (H generated by $S \subset \{x_1, \dots, x_n\}$) provided that S satisfies some mild combinatorial conditions.

We use small cancellation theory.

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* This talk is dedicated to the memory of my friend David Chillag

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CAPABLE SPECIAL p-GROUPS OF RANK 2: STRUCTURE RESULTS

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A finite p-group G such that $G' = Z(G)$ and G' is an elementary abelian p-group of rank 2 is called special of rank 2. A group G is capable if there exists a group H such that $H/Z(H)$ is isomorphic to G. The goal of this research is to classify up to isomorphism all of the capable special p-groups of rank 2. In this talk we will determine the structure of these groups, give a parameterized presentation for each group and provide a criterion for exactly when a special p-group of rank 2 and exponent p^2 is capable.

FINITE GROUPS ADMITTING FROBENIUS GROUPS OF AUTOMORPHISMS WITH ALMOST FIXED-POINT-FREE KERNEL

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Recently finite groups G admitting a Frobenius group of automorphisms FH with kernel F and complement H were studied under the assumption that the kernel F acts fixed-point-freely, $C_G(F) = 1$. It was proved that then many parameters of G are bounded in terms of the corresponding parameters of the fixed-point subgroup $C_G(H)$ of the complement (sometimes also depending on $|H|$). These parameters are the order, rank, Fitting height, nilpotency class (only for metacyclic FH), exponent (partial results).

A natural and important next step is considering finite groups G with a Frobenius group of automorphisms FH in which the kernel F no longer acts fixed-point-freely. Then it is natural to strive for similar restrictions, in terms of the complement H and its fixed points $C_G(H)$, for a subgroup of index bounded in terms of $|C_G(F)|$ and $|F|$: "almost fixed-point-free" action of F implying that G is "almost as good as" when F acts fixed-point-freely. In this talk we discuss results obtained in this direction.

** Joint work with Hermann Heineken and Robert F. Morse

A GROUP ACTION ON $\mathbb{Z}^2$ EXHIBITING FRACTAL-LIKE PATTERNS

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Let $G := \langle a, b, c \rangle < \mathrm{Sym}(\mathbb{Z}^2)$, where

$$a : (m,n) \longmapsto (m-n, m),$$

$$b : (m,n) \longmapsto \begin{cases} (m, 2n+1) & \text{if } n \equiv 0 \pmod{2}, \\ (m, (n-1)/2) & \text{if } n \equiv 1 \pmod{4}, \\ (m,n) & \text{if } n \equiv 3 \pmod{4}, \end{cases}$$

$$c : (m,n) \longmapsto \begin{cases} (m, 2n+3) & \text{if } n \equiv 0 \pmod{2}, \\ (m, (n-3)/2) & \text{if } n \equiv 3 \pmod{4}, \\ (m,n) & \text{if } n \equiv 1 \pmod{4}. \end{cases}$$

In this 10-minute talk I will show pictures of spheres about $(0,0)$ under the action of G. These exhibit beautiful fractal-like patterns with details resembling e.g. the dragon curve fractal etc. once the radius is larger than about 20, or in other words, once they are too large to fit entirely on a typical computer screen. If time permits, I will show further related examples.

In the notation of [2], we have $G < \mathrm{RCWA}(\mathbb{Z}^2)$, where $\mathrm{RCWA}(\mathbb{Z}^2)$ is the group of permutations of $\mathbb{Z}^2$ which are affine on residue classes modulo lattices in $\mathbb{Z}^2$. The GAP package RCWA [3] provides basic routines for computation in this group, and notably more elaborate routines for computation in its "counterpart" in one dimension which is discussed in [1].

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HOMOGENOUS FINITARY SYMMETRIC GROUPS *

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Let $\xi = (p_1, p_2, \ldots)$ be a given infinite sequence of not necessarily distinct primes. In 1976, the structure of locally finite groups $S(\xi)$ (respectively $A(\xi)$) which are obtained as a direct limit of finite symmetric (finite alternating) groups are investigated in [2]. The countable locally finite groups $A(\xi)$ gives an important class in the theory of infinite simple locally finite groups. The classification of these groups using the lattice of Steinitz numbers is completed by Kroshko-Sushchansky in 1998, see [3]. We extend the results on the structure of centralizers of elements to centralizers of arbitrary finite subgroups. Moreover we construct for each infinite cardinal κ, a new class of uncountably many simple locally finite groups of cardinality κ as a direct limit of finitary symmetric groups. We investigate the centralizers of elements and finite subgroups in this new class of simple locally finite groups.

In particular we will mention the proof of the following results.

Theorem. *(Güven, Kegel, Kuzucuoğlu) Let ξ be an infinite sequence of not necessarily distinct primes. Let F be a finite subgroup of $FSym(\kappa)(\xi)$ and $\Gamma_1, \ldots, \Gamma_k$ be the set of orbits of F such that the action of F on any two orbits in Γ_i is equivalent. Let the type of F be $t(F) = ((n_{j_1}, r_1), (n_{j_2}, r_2), \ldots, (n_{j_k}, r_k))$. Then $C_{FSym(\kappa)(\xi)}(F) \cong (\mathop{Dr}\limits_{i=1}^{k} C_{Sym(\Omega_i)}(F)(C_{Sym(\Omega_i)}(F)\bar{\wr}\, S(\xi_i)) \times FSym(\kappa)(\xi')$ where $Char(\xi_i) = \frac{Char(\xi)}{n_{j_i}} r_i$ and $Char(\xi') = \frac{Char(\xi)}{n_{j_1}}$ and Ω_i is a representative of an orbit in the equivalence class Γ_i for $i = 1, \ldots, k$.*

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** Joint work with Ü. B. Güven , O. H. Kegel

ANOTHER PRESENTATION FOR SYMPLECTIC STEINBERG GROUPS

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We show that kernel of the natural projection from the symplectic Steinberg group $\mathrm{StSp}(2l, R)$ onto the elementary symplectic group $\mathrm{Ep}(2l, R)$ is contained in center of $\mathrm{StSp}(2l, R)$. This allows to conclude that different definitions of the symplectic K-theory coincide. We also give another presentation for $\mathrm{StSp}(2l, R)$.

IS IT POSSIBLE TO CLASSIFY ALL GROUPS OF PRIME POWER ORDER UP TO ISOMORPHISM?

Charles Leedham-Green

This question can only have a precise answer if the question is made precise. So I ask: is it possible to divide the class of all groups of prime power order into infinitely many infinite classes in such a way that

1. Any two groups of the same order and nilpotency class belong to the same class, and
2. The number of isomorphism classes of groups of a given order in each class may be determined in polynomial time (whatever that may mean)?

CODEGREES AND NILPOTENCE CLASS OF p-GROUPS

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If χ is an irreducible character of a finite group G, then the codegree of χ is $|G : \ker \chi|/\chi(1)$. We show that if G is a p-group, then the nilpotence class of G is bounded in terms of the largest codegree for an irreducible character of G.

DEFORMATION THEORY AND FINITE SIMPLE QUOTIENTS OF TRIANGLE GROUPS

Alex Lubotzky

Many works have been dedicated to the question: given a hyperbolic triangle group $T = T(a, b, c)$, which finite (simple) groups are quotients of T? The case $o(a, b, c) = (2, 3, 7)$ being of special interest. Many positive and negative results have been proven by either explicit or random methods. We will present a new method to study this problem using deformation of group representations. This will enable us to 1) prove a conjecture of Marion which gives a guiding rule for the wealth of results 2) adding many new groups to the list of quotients of T.

This includes the case of (2.3.7), for which we saw that simple groups of type E_8 are quotients of it (answering a question of Guralnick).

This is a joint work with Michael Larsen and Claude Marion (to appear in JEMS).

** Joint work with Ni Du

BIAS OF GROUP GENERATORS IN THE SOLVABLE CASE

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Let $\Phi_G(t)$ be the set of all generating t-tuples of a finite group G and let $Q_{G,t}$ be the probability distribution on G of the first components of t-tuples chosen uniformly from $\Phi_G(t)$. The distribution $Q_{G,t}$ appears as the limiting distribution of the "product replacement algorithm". For the product replacement algorithm to generate "random" group elements, it is necessary that $Q_{G,t}$ be close to U_G, the uniform distribution on G. Babai and Pak demonstrated a defect in the product replacement algorithm: for certain groups, $Q_{G,t}$ is far from U_G. Their result can reformulate in the context of profinite groups. Indeed a profinite group G is the inverse limit of its finite epimorphic images G/N, where N runs in the set $\mathcal{N}$ of the open normal subgroups of G and for every choice of $N \in \mathcal{N}$ two probability distributions $Q_{G/N,t}$ and $U_{G/N}$ are defined on the quotient group G/N: this allows us to consider G as a measure space obtained as an inverse system of finite probability spaces in two different ways. One of the two measures obtained in this way is the usual normalized Haar measure μ_G. The other measure $\kappa_{G,t}$ has the property that $\kappa_{G,t}(X) = \inf_{N\in\mathcal{N}} Q_{G/N,t}(XN/N)$ for every closed subset X of G. We estimate the bias of the measure $\kappa_{G,t}$ considering

$$\|\kappa_{G,t} - \mu_G\|_{\rm tv} = \sup_{B\in\mathcal{B}(G)} |\kappa_{G,t}(B) - \mu_G(B)|$$

where $\mathcal{B}(G)$ is the set of the measurable subsets of G. The result of Babai and Pak implies that if $\hat{F}_2$ is the profinite completion of the free group of rank 2 and $t \geq 4$, then $\|\kappa_{\hat{F}_2,t} - \mu_{\hat{F}_2}\|_{\rm tv} = 1$.

Pak proposed the following open problem: *can one exhibit the bias for a sequence of finite solvable groups?* In other words does there exist a t-generated prosolvable group G with $\|\kappa_{G,t} - \mu_G\|_{\rm tv} = 1$? It is not particularly difficult to give an affirmative answer in the particular case when $t = d(G)$, the smallest cardinality of a generating set for G. A more important and intriguing question is whether we can find a finitely generated prosolvable group G with the property that $\|\kappa_{G,t} - \mu_G\|_{\rm tv} = 1$ for some integer t significatively larger than $d(G)$. We give the following answer:

Theorem. *Let $d \in \mathbb{N}$ with $d \geq 3$. There exists a finitely generated prosolvable group G such that $d(G) = d$ and $\|\kappa_{G,d(G)+k} - \mu_G\|_{tv} = 1$ for every k such that $2k \leq d - 3$.*

** Joint work with Eleonora Crestani

ONE CONSTRUCTION OF INTEGRAL REPRESENTATIONS OF p-GROUPS

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We consider the arithmetic background of integral representations of finite groups over the maximal orders of local and algebraic number fields.

Some infinite series of integral pairwise inequivalent absolutely irreducible representations of finite p-groups with the extra congruence conditions are constructed. Certain problems concerning integral two-dimensional representations over number rings are discussed.

CONJUGACY CLASS SIZES AND COMMUTATIVITY INVARIANTS IN FINITE p-GROUPS - OLD PROBLEMS AND NEW ANSWERS

Avinoam Mann

CYCLIC AND PRO-CYCLIC COVERINGS OF COMMUTATORS

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We study the groups G in which the set of commutators of G is covered by finitely many subgroups $H_1, \ldots, H_m$; that is, the set of commutators of G is contained in the union of the subgroups H_i, for $i = 1, \ldots, n$. Fernández-Alcober and Shumyatsky proved that if G is a group in which the set of all commutators is covered by finitely many cyclic subgroups, then the derived group G' is either finite or cyclic [2].

We deal with profinite groups in which all commutators are covered by finitely many procyclic subgroups. Our main results is the following:

Theorem A. *Let G be a profinite group such that all commutators in G are covered by finitely many procyclic subgroups. Then G' is finite-by-procyclic.*

Moreover, G' can be infinite and not procyclic.

If we concentrate on pro-p groups, the result is sharper:

Theorem B. *Let p be a prime and let G be a pro-p group such that all commutators in G are covered by m procyclic subgroups. Then G' is finite of m-bounded order of or procyclic.*

Moreover, the main result in [2] can be enriched with the following additional information.

Theorem C. *Let G be a group that possesses m cyclic subgroups whose union contains all commutators of G. Then G has a characteristic subgroup M such that the order of M is m-bounded and G'/M is cyclic.*

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** Joint work with Pavel Shumyatsky, Gustavo A. Fernández-Alcober

CAPABLE SPECIAL p-GROUPS OF RANK 2: THE ISOMORPHISM PROBLEM

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A finite p-group G such that $G' = Z(G)$ and G' is an elementary abelian p-group of rank 2 is called special of rank 2. A group G is capable if there exists a group H such that $H/Z(H)$ is isomorphic to G. A result of H. Heineken shows the capable special p-groups of rank 2 have order at most p^7. Of such groups of exponent p we know from published classifications that there is a constant number of isomorphism classes. The number of isomorphism classes of special p-groups of rank 2 and exponent p^2 grows with p for groups of order p^5, p^6, and p^7. However, our use of the small groups library in GAP gives evidence that the number of capable special groups of each order is constant. Hence, the capable special p-groups of rank 2 and exponent p^2 cannot only be characterized by a structure description but can actually be classified. We will show that for odd p there are three isomorphism classes each for capable special p-groups of rank 2 exponent p^2 and order p^5 and p^6 and one such class for order p^7.

ON THE COVERING NUMBER OF SMALL SYMMETRIC GROUPS AND SOME SPORADIC SIMPLE GROUPS

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We say that a group G has a finite covering if G is a set theoretical union of finitely many proper subgroups. The minimal number of subgroups needed for such a covering is called the covering number of G denoted by $\sigma(G)$.

Let S_n be the symmetric group on n letters. For odd n Maroti determined $\sigma(S_n)$ with the exception of $n = 9$, and gave estimates for n even showing that $\sigma(S_n) = 2^{n-2}$. Using GAP calculations, as well as incidence matrices and linear programming, we show that $\sigma(S_8) = 64$, $\sigma(S_10) = 221$, $\sigma(S_12) = 761$. We also show that Maroti s result for odd n holds without exception proving that $\sigma(S_9) = 256$.

We establish in addition that the Mathieu group M_12 has covering number 208, and improve the estimate for the Janko group J_1 given by P.E. Holmes.

WORD MAPS ON FINITE SIMPLE GROUPS

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The theory of word maps on finite non-abelian simple groups has attracted much recent attention over recent years. For a given nontrivial word w, every element of every sufficiently large finite simple group G can be expressed as a product of at most 2 values of w in G, and word maps for certain words have been proved surjective on all finite simple groups. We will survey this topic, illustrating techniques of proof.

* Joint work with Hermann Heineken and Luise-Charlotte Kappe
** Joint work with Luise-Charlotte Kappe, and Eric Swartz

ABSOLUTE GALOIS PRO-p GROUPS WITH CONSTANT GENERATING NUMBER ON OPEN SUBGROUPS

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Let p be a prime number. Answering a question posed by Iwasawa in the early '80s, we characterize all the finitely generated pro-p groups satisfying the formula

$$d(U) = d(G) \quad \text{for all open } U \leq G \tag{1}$$

which can be realized as absolute Galois groups of fields. (Here $d(U)$ denotes the minimal number of topological generators for the subgroup U)

Theorem. *Let G be a finitely generated pro-p group that can be realized as absolute Galois group of a field. Then (1) holds for G if, and only if, G is locally powerful, i.e., if G has a presentation*

$$G = \langle \sigma, \tau_1, \ldots, \tau_m \,|\, [\tau_i, \tau_j] = 1, [\sigma, \tau_i] = \tau_i^q \;\forall\, i, j \,\rangle \tag{2}$$

with q a p-th power (possibly $q = 0$).

(Recall that G is powerful if $[G, G] \leq p$ and it is locally powerful if every closed subgroup is powerful.) The proof requires techniques from pro-p groups theory and from Galois cohomology. Note that the groups with a presentation (2) are meta-abelian (abelian if $q = 0$).

Formula (1) is the specialization of

$$d(U) - n = |G : U|(d(G) - n) \quad \text{for all open } U \leq G, n \geq 1 \text{ fixed} \tag{3}$$

in the case $n = d(G)$ (if $n = 1$ then (3) is the Schreier formula). A similar result was obtained – indipendently and with a different approach – in [2]. This result has also interesting arithmetic implications.

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GROUP ALGEBRAS WITH THE BOUNDED SPLITTING PROPERTY

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Let R be a ring with identity and M a left R-module. The *singular submodule* $Z(M)$ of M is the subset of elements whose annihilator in R is an essential left ideal of R. If $R = \mathbb{Z}$, this is just the torsion subgroup of M. The ring R has the *bounded splitting property* (BSP) if, for any left R-module M, whenever $Z(M)$ is bounded, i.e., it embeds in a left R-module generated by elements whose annihilators are essential left ideals, it is a direct summand of M. For example $\mathbb{Z}$, and more generally any principal ideal domain, has the BSP.

The problem addressed here is to find necessary and sufficient conditions on a group G and a field K for the group algebra KG to have the BSP. When G is finite, the answer is known: KG has to be semisimple, i.e., $\mathrm{char}(K)$ must not divide the order of G. If G is infinite, a necessary condition is that every subnormal subgroup of G is finite or of finite index. Groups with the latter property are investigated in the case where there is an ascending series with finite or locally nilpotent factors. As a result, necessary and sufficient conditions on a field K and a group G with an ascending series with finite or locally nilpotent factors are obtained for KG to have the BSP.

GRÖBNER BASIS COMPUTATION IN GROUP RINGS AND APPLICATIONS FOR GROUPS

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We describe some possibilities to solve efficiently problems for finitely presented groups using Gröbner Basis Computations. Here we discuss briefly the order of a group element, the Tits Alternative and the growth in groups. For the latter our special focus lies on the Hilbert-Dehn functions for the main presentation $< s,t|s^2 = (st)^q = 1 >$ of a Hecke group $H(q)$, i.e. the growth functions of the number of shortest words representing group elements. Rather than referring to the abstract theory of Fuchsian groups, we provide explicit recursive formulars based on the computation of the Gröbner bases of the defining ideals of the corresponding group rings. As a consequence, we show that the generating series of the functions are rational power series. The next topic is the growth of the number of subgroups of a given finite index in the Hecke group $H(q)$. We also discuss the number of subgroups with additional properties like normal subgroups of genus one. In particular we here get some interesting connections with special functions in analytic number theory.

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* Joint work with Blas Torrecillas of the University of Almería, Spain
** Joint work with Martin Kreuzer, University of Passa

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ON THE UNIT GROUP OF COMMUTATIVE GROUP RINGS

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Let $V(\mathbb{Z}_{p^e}G)$ be the group of normalized units of the group algebra $\mathbb{Z}_{p^e}G$ of a finite abelian p-group G over the ring $\mathbb{Z}_{p^e}$ of residues modulo p^e with $e \geq 1$. The abelian p-group $V(\mathbb{Z}_{p^e}G)$ and the ring $\mathbb{Z}_{p^e}G$ are applicable in coding theory, cryptography and threshold logic (see [1, 4, 5, 7]).

In the case when $e = 1$, the structure of $V(\mathbb{Z}_pG)$ has been studied by several authors (see the survey [2]). The invariants and the basis of $V(\mathbb{Z}_pG)$ has been given by B. Sandling (see [6]). In general, $V(\mathbb{Z}_{p^e}G) = 1 + \omega(\mathbb{Z}_{p^e}G)$, where $\omega(\mathbb{Z}_{p^e}G)$ is the augmentation ideal of $\mathbb{Z}_{p^e}G$. Clearly, if $z \in \omega(\mathbb{Z}_{p^e}G)$ and $c \in G$ is of order p, then $c + p^{e-1}z$ is a nontrivial unit of order p in $\mathbb{Z}_{p^e}G$. We may raise the question whether the converse is true, namely does every $u \in V(\mathbb{Z}_{p^e}G)$ of order p have the form $u = c + p^{e-1}z$, where $z \in \omega(\mathbb{Z}_{p^e}G)$ and $c \in G$ of order p?

We obtained a positive answer to this question and applied it for the description of the group $V(\mathbb{Z}_{p^e}G)$ (see [3]). Our research can be considered as a natural continuation of Sandling's results.

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** Joint work with V. Bovdi

THE UNITARY COVER OF A FINITE GROUP AND THE EXPONENT OF THE SCHUR MULTIPLIER *

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For a finite group we introduce a particular central extension, the unitary cover, having minimal exponent among those satisfying the projective lifting property. We obtain new bounds for the exponent of the Schur multiplier relating to subnormal series, and we discover new families for which the bound is the exponent of the group. Finally, we show that unitary covers are controlled by the Zel'manov solution of the restricted Burnside problem for 2-generator groups.

EXPONENT OF LOCALLY FINITE GROUPS WITH SMALL CENTRALIZERS

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In the theory of locally finite groups centralizers play an important role. In particular the following family of problems has attracted great deal of attention in the past.

Let G be a locally finite group containing a finite subgroup whose centralizer is small in some sense. What can be said about the structure of G?

In some situations quite a significant information about G can be deduced. We will talk about relatively recent results in which the centralizers have finite exponent. In particular we will talk about the following theorem.

Theorem. *If a locally finite group G contains a non-cyclic subgroup A of order p^2 for a prime p such that $C_G(A)$ is finite and $C_G(a)$ has finite exponent for all nontrivial elements $a \in A$, then G is almost locally soluble and has finite exponent.*

The proof depends on the classification of finite simple groups and the solution of the restricted Burnside problem.

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ON GROUPS WITH CUBIC POLYNOMIAL CONDITIONS

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We provide a finiteness rank condition for finitely generated subgroups of a ring satisfying a finite number of cubic polynomials over the center of the ring. Then, with the help of computational techniques, we derive structural results for groups defined by a finite number of cubic unipotent conditions.

PARABOLIC FACTORIZATIONS OF STEINBERG GROUPS

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Let R be a commutative ring with identity and Φ a reduced irreducible root system of rank l. Let $\mathrm{St}(\Phi, R)$, StP_i and $\widehat{\mathrm{U}}_i^-$ denote the Steinberg group of type Φ over R, its parabolic subgroup corresponding to the i-th simple root α_i and the opposite unipotent radical subgroup, respectively.

For a classical Φ R. K. Dennis, A. Suslin, M. Tulenbayev and M. R. Stein have shown that $\mathrm{St}(\Phi, R)$ admits so-called "parabolic factorization" under certain restrictions on the stable rank of R, namely

$$\mathrm{St}(\Phi, R) = \mathrm{StP}_1 \cdot \left(\widehat{\mathrm{U}}_1^- \cap \widehat{\mathrm{U}}_l^-\right) \cdot \mathrm{StP}_l.$$

Injective stability for K_1-functors modeled on Chevalley groups and surjective stability for K_2 follows from this decomposition as a corollary.

Denote by $d(i, j)$ the distance between simple roots α_i and α_j on the Dynkin diagram of Φ. We find sufficient conditions on R which imply similar factorizations for a more general choice of pairs of parabolic subgroups StP_i, StP_j. Our main result is the following theorem.

Theorem. *Let $1 \leq i < j \leq l$. Assume that either Φ is classical and $\mathrm{sr}(R) \leq d(i, j)$ or Φ is exceptional and one of the following conditions is satisfied:*

Φ	$\{i, j\}$	*condition on* R
E_l, $l = 6, 7, 8$	$\{2, l\}$	$\mathrm{sr}(R) \leq d(2, l) = l - 3$
E_l, $l = 6, 7, 8$	$\{1, l\}$	$\mathrm{asr}(R) \leq d(1, l) = l - 2$
F_4	$\{1, 4\}$	$\mathrm{sr}(R) \leq 2$
G_2	$\{1, 2\}$	$\mathrm{sr}(R) \leq 1$

Then $\mathrm{St}(\Phi, R) = \mathrm{StP}_i \cdot \left(\widehat{\mathrm{U}}_1^- \cap \widehat{\mathrm{U}}_l^-\right) \cdot \mathrm{StP}_l.$

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COMMUTATOR WIDTH OF CHEVALLEY GROUPS

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The famous Ore conjecture, proved in 2010, states that every element of a non-abelian finite simple group is a commutator. On the other hand, R. Ree proved that the same statement is true for any semi-simple connected algebraic group over an algebraically closed field. The natural problem is to investigate what happens for algebraic and algebraic-like groups over other types or rings.

It turns out, that over rings of small dimension (rings of stable rank 1, Dedekind of arithmetic type) every element of a classical group can be expressed as a product of a small number of commutators. For Bak's hyperbolic unitary groups (which include GL_n, symplectic and even-dimensional orthogonal groups) this was done by L. Vaserstein and his co-authors.

Their methods can be carried to the setting of Chevalley groups, thus giving good estimates of the commutator width for exceptional groups.

SANDWICH CLASSIFICATION THEOREM

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Let D be a subgroup of a group G. We study the lattice $\mathcal{L} = L(D, G)$ of subgroups of G, containing D, and the lattice $\mathcal{L}'$ of subgroups of G, normalized by D. In some examples this lattices break into disjoint union of sandwiches $L(F_i, N_i)$, where i ranges over some index set and F_i is normal in N_i. In this situation one can list all subgroups from $\mathcal{L}$ or $\mathcal{L}'$ computing all the quotients N_i/F_i. Of course, this knowledges can be trivial if there are too much sandwiches, e. g. if the index set coinside with the lattice itself and $N_i = F_i$. Therefore Z. I. Borewich [1] introduced the following restriction on the lower layers of sandwiches.

Definition. *For a subgroup $H \leq G$ denote by D^H the smallest subgroup, containing D and normalized by H. A subgroup $H \in \mathcal{L}$ is called D-full if $D^H = H$. We say that the lattice $\mathcal{L}$ satisfies* sandwich classification theorem *(SCT) if D^H is D-full for each subgroup $H \in \mathcal{L}$.*

It follows from the definition, that $\mathcal{L} = \bigsqcup L(F_i, N_i)$, where F_i ranges over all D-full subgroups of G and N_i is the normalizer of F_i in G. A similar notion for the lattice $\mathcal{L}'$ was introduced by the author in [2]

Definition. *For a subgroup $H \leq G$ let $[D, H]$ be the subgroup of G, generated by all commutators $[d, h] = d^{-1}h^{-1}dh$, $d \in D$, $h \in H$. A subgroup $H \leq G$ is called D-perfect if $[D, H] = H$. We say that the lattice $\mathcal{L}'$ satisfies SCT if $[D, H]$ is D-perfect for each subgroup $H \in \mathcal{L}'$.*

It is easy to see that SCT for $\mathcal{L}'$ implies SCT for $\mathcal{L}$. It turns out the the converse is also true if D is a perfect subgroup, i. e. $[D, D] = D$.

Theorem. *Suppose that D is perfect and $\mathcal{L}$ satisfies SCT. Then $\mathcal{L}'$ also satisfies SCT. Moreover, a subgroup P is D-perfect iff it is an F-perfect subgroup in some D-full subgroup F.*

Further we discuss examples of SCT in Chevalley groups over rings.

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GENERALIZING THE CONCEPT OF QUASINORMALITY

Stewart Stonehewer

Quasinormal subgroups have been studied for nearly 80 years. In finite groups, questions concerning them invariably reduce to p-groups, and here they have the added interest of being invariant under projectivities, unlike normal subgroups. However, it has been shown recently that certain groups, constructed by Berger and Gross in 1982, of an important 'universal' nature with regard to the existence of core-free quasinormal subgroups, have remarkably few such subgroups. Therefore in order to overcome this misfortune, a generalization of the concept of quasinormality will be defined. It could be seen as the beginning of a lengthy undertaking. But some of the initial findings are encouraging, in particular the fact that this 'larger' class of subgroups remains invariant under projectivities in finite p-groups, thus connecting group and subgroup lattice structures.

CLASSICAL HURWITZ GROUPS OF LOW RANK*

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A finite group is called a Hurwitz group if it can be generated by two elements of respective orders 2 and 3, whose product has order 7 (e.g., see [1] or [4]). Let $\mathbb{F}$ be an algebraically closed field of characteristic $p \geq 0$. All the $(2,3,7)$ irreducible triples of $\mathrm{GL_n}(\mathbb{F})$, $n \leq 7$, were parametrized in [5] and [6] (except two non-rigid cases in dimension 6, still unpublished). Also [3] considers a similar problem. This classification led to exclude that many finite classical groups can be Hurwitz (as done in [8] for intermediate ranks) and to discover new Hurwitz groups, like $\mathrm{PSL}_6(p^a)$, $\mathrm{PSU}_6(p^b)$, $\mathrm{PSL}_7(p^c)$, $\mathrm{PSU}_7(p^d)$, for at most one value of a, b, c, d, made precise in each case. On the other hand the symplectic groups $\mathrm{PSp}_6(q)$ are Hurwitz for all $q = p^m \geq 5$, p odd. In particular $n = 6$ turns out to be the smallest degree for which a family of simple groups of Lie type of degree n over $\mathbb{F}_{p^m}$ contains Hurwitz groups for infinitely many values of m. This fact is related to the rigidity of irreducible Hurwitz triples of small rank (see [2] for a discussion).

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* This talk is dedicated to David Chillag

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ON LEFT 3-ENGEL ELEMENTS

Gunnar Traustason

It is still an open question whether a left 3-Engel element of a group G is always contained in the locally nilpotent radical. We will discuss this question and present some new results.

DECOMPOSITION OF UNIPOTENTS, REVISITED

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The talk is devoted to the recent versions and new applications of decomposition of unipotents. Let Φ be a root system, R be a commutative ring, and let $G(\Phi, R)$ be a Chevalley group of type Φ over R. Further, denote by $x_\alpha(\xi)$, where $\alpha \in \Phi$ and $\xi \in R$, the corresponding elementary root element.

For the group $\mathrm{SL}(n, R)$ this method was initially proposed in 1987 by Alexei Stepanov, to give a simplified proof of Suslin's normality theorem. Soon thereafter I generalised it to other split classical groups, and then together with Eugine Plotkin we generalised it to exceptional Chevalley groups.

In the simplest form, this method gives explicit polynomial factorisations of root type elements $gx_\alpha(\xi)g^{-1}$, where $g \in G(\Phi, R)$, in terms of factors sitting in proper parabolic subgroups, and eventually in terms of elementary generators. Among other things, this allows to give extremely short and straightforward proofs of the main structure theorems for such groups.

However, for exceptional groups the early versions of the method relied on the presence of huge classical embeddings, such as $\mathrm{A}_5 \leq \mathrm{E}_6$, $\mathrm{A}_7 \leq \mathrm{E}_7$ and $\mathrm{D}_8 \leq \mathrm{E}_8$. Also, even for some classical groups, the method would not give explicit bounds on the length of elementary decompositions of root elements.

Inspired by the A_2-proof of structure theorems for Chevalley groups, proposed by myself, Mikhail Gavrilovich, Sergei Nikolenko, and Alexander Luzgarev, recently we succeeded in developing new powerful versions of the method, that only depend on the presence of small rank classical embeddings. I plan to describe some of these recent advances.

- A_3-proof, D_4-proof, etc. for Chevalley groups of types E_6 and E_7.
- Explicit linear length factorisation of transvections for $\mathrm{Sp}(2l, R)$.
- Applications to description of overgroups of subsystem subgroups, in particular, for the case $\mathrm{A}_7 \leq E_7$ (joint with Alexander Shchegolev).
- Description of subnormal subgroups of Chevalley groups of types E_6 and E_7 (joint with Zuhong Zhang).

Finally, I plan to discuss possible generalisations to isotropic (but non necessarily split) reductive groups. In this generality, Victor Petrov and Anastasia Stavrova initiated the use of localisation methods, further developed by Stavrova, Luzgarev, Stepanov, and others. Decomposition of unipotents would constitute an viable alternative to and in some cases enhancement of localisation methods, and might have serious impact on the structure theory of such groups.

STALLINGS' DECOMPOSITION THEOREM FOR PRO-p GROUPS

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Stallings' decomposition theorem asserts that a finitely generated (discrete) group with infinitely many ends decomposes non-trivially as a free product with amalgamation over a finite subgroup or as an HNN-extension with finite base group.

For a pro-p group G it is not possible to define a *space of ends* as in the discrete case. Instead, following a suggestion of O. Mel'nikov, one defines ad hoc the number of ends by $\mathbf{e}(G) = \mathrm{rk}(H^1(G, \mathbb{Z}_p[\![G]\!]))$ (or $\mathbf{E}(G) = \dim(H^1(G, \mathbb{F}_p[\![G]\!]))$. In joint work with P. Zalesskiĭ we proved the following analogue of Stallings' decomposition theorem for pro-p groups.

Theorem. *Let G be a finitely generated pro-p group satisfying $\mathbf{e}(G) = \infty$. Then either G decomposes non-trivially as a free pro-p product with amalgamation over a finite subgroup, or G is an HNN-extension with finite base group.*

The proof of this theorem requires rather sophisticated techniques in the theory of pro-p groups (cf. [1], [2]). In the talk I will present some ideas of the proof.

References

[1] B. Torrecillas and Th. Weigel, *Lattices and cohomological Mackey functors for finite cyclic p-groups*, Adv. Math. **244** (2013), (20C11, 20J06), 533–569.

[2] Th. Weigel, *The projective dimension of profinite modules for pro-p groups* (preprint, available at `http://arxiv.org/abs/1303.5872`).

[3] Th. Weigel and P. Zalesskii, *Stallings' decomposition theorem for finitely generated pro-p groups* (preprint, available at `http://arxiv.org/abs/1305.4887`).

[**] Joint work with Pavel Zalesskii

INFINITE DIMENSIONAL LIE ALGEBRAS AND BEYOND

Efim Zelmanov

We will discuss infinite dimensional slow growing Lie algebras and superalgebras with connections to Grigorchuk groups, Ring Theory and Physics.

LARGE ODD ORDER GROUPS ACTING ON RIEMANN SURFACES

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Let G be a finite group of odd order. The *symmetric genus* $\sigma(G)$ is the minimum genus of any Riemann surface on which G acts. Suppose that G acts on a Riemann surface of genus $g \geq 2$. For an odd order group, the analog of Hurwitz Theorem is that $|G| \leq 15(g-1)$. If $|G| > 8(g-1)$, then $|G| = K(g-1)$, where K is 15, $\frac{21}{2}$, 9 or $\frac{33}{4}$, and we say these groups have large order. We call these four types of groups LO-1 through LO-4 groups, respectively. The genus of groups in each of these families is dependent only on the order of the group. Previously, we have shown that there are infinite families of each type in a non-constructivist way [1]. A number of new examples of such groups are constructed. We also show that the possible orders of groups in each family are restricted. For example, if a prime p divides the order of an LO-1 group G, then p^3 must divide $|G|$. Previously, nilpotent LO-3 groups were studied by Zomorrodian [2]. We study metabelian LO-3 groups. Every metabelian LO-3 group is a quotient of a finite metabelian LO-3 group with a well defined structure. Suppose that G is a metabelian LO-3 group, p is a prime $p \equiv 2$ $(mod\ 3)$ and $|G| = p^k m$ where $(p, m) = 1$, then k is even. Also, if $p \equiv 2$ $(mod\ 3)$, all even powers of p occur in the order of some LO-3 group G. If $p \equiv 1$ $(mod\ 3)$, then all powers of p occur in the order of some LO-3 group G. This gives us the genus spectrum of all metabelian LO-3 groups. Finally, the integers that occur as the symmetric genus of groups in these classes have density zero in the positive integers.

References

[1] C.L. May, and J. Zimmerman, *The Symmetric Genus of Groups of Odd Order*, Houston J. Math. **34**, No. 2 (2008), 319-338.

[2] R. Zomorrodian, *Classification of p-groups of automorphisms of Riemann surfaces and their lower central series*, Glasgow Math. J. **29** (1987), 237-244.

** Joint work with Coy L. May