A GROUP ACTION ON $\mathbb{Z}^2$ EXHIBITING FRACTAL-LIKE PATTERNS

Stefan Kohl

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Our group

Let $G := \langle a, b, c \rangle < \mathsf{Sym}(\mathbb{Z}^2)$, where

$a : (m, n) \longmapsto (m - n, m),$

$b : (m, n) \longmapsto \begin{cases} (m, 2n + 1) & \text{if } n \in 0(2), \\ (m, (n - 1)/2) & \text{if } n \in 1(4), \\ (m, n) & \text{if } n \in 3(4), \end{cases}$

$c : (m, n) \longmapsto \begin{cases} (m, 2n + 3) & \text{if } n \in 0(2), \\ (m, (n - 3)/2) & \text{if } n \in 3(4), \\ (m, n) & \text{if } n \in 1(4). \end{cases}$

By $r(m)$ we denote the residue class $r + m\mathbb{Z}$.

Spheres about $(0,0)$

For sufficiently large radii r, the spheres about $(0,0)$ under the action of G on $\mathbb{Z}^2$ show fractal-like structures.

The patterns start to become visible once r is larger than about 20, where the picture still fits on a typical screen. They quickly get richer structure once r gets larger.

The pictures show the spheres of radii $\leqslant 45$ and parts thereof, respectively. The *entire* sphere of radius 45 contains 109839303 points, and fills a picture of about 3 gigapixel.

The pictures shown in this talk (and more!) can be downloaded from

`http://www.gap-system.org/DevelopersPages/`
`StefanKohl/tmp/Ischia_2014.zip`

References

Stefan Kohl, *A simple group generated by involutions interchanging residue classes of the integers*, Math. Z. **264** (2010), no. 4, 927–938, DOI: 10.1007/s00209-009-0497-8.

Stefan Kohl, *Simple groups generated by involutions interchanging residue classes modulo lattices in $\mathbb{Z}^d$*, J. Group Theory **16** (2013), no. 1, 81–86, DOI: 10.1515/jgt-2012-0031.

Stefan Kohl, *RCWA - Residue-Class-Wise Affine Groups; Version 3.6.1*, 2013, GAP package, http://www.gap-system.org/ Packages/rcwa.html.

Stefan Kohl, *Fractal-like structures arising from the action of a group on $\mathbb{Z}^2$*, http://mathoverflow.net/questions/160071/, March 2014.