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Abstracts

GRAPHS ENCODING THE GENERATING PROPERTIES OF A FINITE GROUP

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The generating graph $\Gamma(G)$ of a finite group G is the graph defined on the non-identity elements of G in such a way that two distinct vertices are connected by an edge if and only if they generate G. The graph $\Gamma(G)$ gives interesting information only if G is 2-generated.

We introduce an alternative definition that works in a more general setting and encodes the generating properties of a d-generated finite group, for any positive integer d. In the talk we will present some results and questions related to the study of these graphs.

GROUP GRADINGS ON FINITE DIMENSIONAL DIVISION ALGEBRAS

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Finite group grading plays a key role in the study of finite dimensional k-central division algebras and more generally in the study of finite dimensional k-central simple algebras and Brauer groups. The group grading (via the crossed product construction) provides the bridge between Brauer groups and Galois cohomology. Also, group grading (via the the structure of symbol algebras) provides the bridge between Brauer groups and K-theory. The main question we are interested here, roughly speaking, is "what are all possible (finite) group gradings on finite

*Joint work with Andrea Lucchini.

**Joint work with Darrell Haile and Yakov Karasik.

dimensional division algebras?". While this seems to be a difficult problem (in its full generality), it turns out that any group grading on a finite dimensional division algebra is obtained by combining suitable extensions of the two gradings mentioned above (crossed products and symbol algebras). One of the obstacles for finding a full classification is to determine which finite groups G admit a field k and a 2-cocycle $H^2(G, k^*)$ such that the twisted group algebra $k^\alpha G$ is a division algebra.

LEFT BRACES AND SOLUTIONS OF THE QUANTUM YANG-BAXTER EQUATION

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Braces were introduced by Rump in 2007 as a useful tool in the study of the set-theoretic solutions of the quantum Yang-Baxter equation. In fact, several aspects of the theory of finite left braces and their applications in the context of the Yang-Baxter equation have been extensively investigated recently.

In this talk we will introduce two finite brace theoretical properties associated to nilpotency and analyse their impact in the finite solutions of the Yang-Baxter equation.

ALGEBRAS WHOSE GROUP OF UNITS IS HYPERBOLIC

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In the articles [3, 4, 5] the structure of a group G for which the group of units $U(KG)$ of the group ring KG over a commutative ring K is a hyperbolic group (in the sense of M. Gromov [2]) was considered. A complete description of the structure was given only in [1]. We try to extend the results and technique of [1] to certain classes of algebras whose group of units is hyperbolic.

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*Joint work with Ramón Esteban-Romero and Hangyang Meng.

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ASYMPTOTIC LINEAR BOUNDS FOR THE NORMAL COVERING NUMBER OF THE SYMMETRIC GROUPS

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Let $\gamma(S_n)$ be the minimum number of proper subgroups $H_i,\ i = 1, \dots, k$ of the symmetric group S_n such that each element in S_n lies in some conjugate of one of the H_i. Using some deep number theoretic results on the theory of partitions, we have proved in [1] that there exists a positive constant c with $\gamma(S_n) \geq cn$. This method has allowed for the first time to get a lower bound for $\gamma(S_n)$ linear in n. However, number theory on its own provides estimates on c that are unrealistically small when compared to computational data.

Recently, in [2], finite primitive groups containing a permutation having a small number of cycles have been classified. In this talk, we show how this classification together with some further number theoretic results can be used to derive some expressive asymptotic linear lower bounds for $\gamma(S_n)$.

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*Joint work with Cheryl E. Praeger and Pablo Spiga.

INVOLUTIONS IN MULTIPLE HOLOMORPHS OF GROUPS

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The holomorph of a group G can be defined as the normalizer

$$\mathrm{Hol}(G) = N_{S(G)}(\rho(G)) = \mathrm{Aut}(G)\rho(G)$$

of the image of the right regular representation $\rho : G \to S(G)$, where $S(G)$ is the group of permutations on the set G. If N is any regular subgroup of $S(G)$, then the normalizer $N_{S(G)}(N)$ is isomorphic to the holomorph of N.

If the regular subgroup N is isomorphic to G, and N and G have the same holomorph, in the sense that $N_{S(G)}(N) = \mathrm{Hol}(G)$, then $\rho(G)$ and N are conjugate under an element of the multiple holomorph

$$N_{S(G)}(\mathrm{Hol}(G)) = N_{S(G)}(N_{S(G)}(\rho(G)))$$

of G, and the group

$$T(G) = N_{S(G)}(\mathrm{Hol}(G))/\mathrm{Hol}(G)$$

acts regularly on the set of such N. If G is non-abelian, inversion yields an involution in $T(G)$.

We mention the connections of these topics with right skew braces and Hopf Galois extensions. We describe some questions, involving commutative rings and quasisimple groups, that arise from the study of multiple holomorphs. We survey some classes of groups G for which $T(G)$ is an elementary abelian 2-group, and provide examples to show that this does not hold true in general.

GROUPS WITH RESTRICTIONS ON SUBGROUPS WHICH ARE NOT COMMENSURABLE WITH A NORMAL SUBGROUP

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A subgroup H is called a *cn*-subgroup if it is commensurable with a normal subgroup. i.e. there is $N \lhd G$ such that $|HN/H \cap N| < \infty$.
This concept generalizes both the classical notions of nearly-normal subgroup, i.e. $|H^G : H| < \infty$, and normal-by-finite (or core-finite) subgroup, i.e. $|H : H_G| < \infty$.

Recently it has been shown that, under assumption of generalized solubility of the group, if in a group all subgroups are cn, then the group is finite-by-soluble-by-finite.

I will discuss how to generalize other statements that hold for both properties *nearly-normal* and *core-finite* subgroups to analogue ones for the property of being a cn-subgroup.

LARGE SUBGROUPS IN INFINITE GROUPS

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The aim of this talk is to describe the influence of large subgroups on the structure of uncountable groups. Moreover, sufficient conditions for the existence of large characteristic subgroups with a given property will be considered.

* Joint work with Silvana Rinauro.

THE NUMBER OF MAXIMAL SUBGROUPS AND PROBABILISTIC GENERATION OF FINITE GROUPS

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In this talk we present some significant bounds for the number of maximal subgroups of a given index of a finite group. As a consequence, new bounds for the number of random generators needed to generate a finite d-generated group with high probability which are significantly tighter than the ones obtained in the paper of Jaikin-Zapirain and Pyber (Random generation of finite and profinite groups and group enumeration, *Ann. Math.*, **183**:769–814, 2011) are obtained. The results of Jaikin-Zapirain and Pyber, as well as other results of Lubotzky, Detomi, and Lucchini, appear as particular cases of our theorems.

SOME RESULTS REGARDING OUTER COMMUTATOR WORDS

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Outer commutator words, also known as multilinear commutators, are group words that are obtained by nesting commutators, always using different variables. Thus the terms of the lower central series and the derived series can be defined as verbal subgroups of outer commutator words, the word $[[x_1, x_2], [x_3, x_4, x_5], x_6]$ is an outer commutator, and the Engel word $[x_1, x_2, x_2, x_2]$ is not. In this talk, we survey on results obtained by the author in collaboration with C. Acciarri, M. Morigi, and P. Shumyatsky, regarding bounded conciseness of outer commutator words and their powers, and a variation of the focal subgroup theorem for outer commutator words. These results strongly depend on the existence of a special series of subgroups in the corresponding verbal subgroup, which is obtained by introducing a hierarchy in the set of outer commutators via a graphical representation of the words. The essence of this method will also be explained in the talk.

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*Joint work with Adolfo Ballester-Bolinches, Paz Jiménez Seral and Hangyang Meng.

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+Joint work with Cristina Acciarri, Marta Morigi and Pavel Shumyatsky.

ON THE NUMBER OF CYCLIC SUBGROUPS OF A FINITE GROUP

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In this talk I will present recent developements of the study of the number of cyclic subgroups of a finite group G, denote it by $c(G)$. In joint works with Massimiliano Patassini and Igor Lima we showed that a finite group G of order n verifies $c(G) \geq c(C_n)$ with equality if and only if $G \cong C_n$ (where C_n is the cyclic group of order n) and we studied the "limit values" of the invariant $\alpha(G) := c(G)/|G|$, more precisely:

- if $\alpha(G) > \alpha(D_8) = 7/8$ then G is abelian;
- if $\alpha(G) > \alpha(S_3) = 5/6$ then G is nilpotent;
- if $\alpha(G) > \alpha(S_5) = 67/120$ then G is solvable;
- if $\alpha(G) > \alpha(S_4) = 17/24$ then G is supersolvable.

We also studied the interesting problem of determining all the group extensions $N \to G \to G/N$ with the property that $\alpha(G) = \alpha(G/N)$.

In our study we use results of Wall on groups with many involutions and results of Guralnick and Robinson on the probability of commutation.

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*Joint work with Massimiliano Patassini and Igor Lima.

SYMMETRIC GROUPS AND FIXED POINTS ON MODULES

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Suppose p is a prime and S is a non-identity Sylow p-subgroup of a finite group G. In local analysis, we try to derive global information about G from information about normalizers of non-identity subgroups of S. It is especially useful when we can derive information from the normalizer of only one subgroup of S. In such cases, usually p is odd and counterexamples show that the result cannot be extended to the prime 2.

I plan to discuss a surprising exception, in which G is a group of automorphisms of an abelian p-group D and we wish to determine the fixed points of G on D. Although there exists an easy counterexample for $p = 2$, we are still able to derive useful information for $p = 2$. (This is part of joint work with Justin Lynd that removes the classification of finite simple groups from the proof of the Martino-Priddy Conjecture in topology.)

INTEGRAL FORMS IN VERTEX OPERATOR ALGEBRAS

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An integral form in an algebra is the integral span of a basis which is closed under the product. For an integral form in a vertex operator algebra (VOA), we require closure under the given countably many products, plus a few additional conditions. Reference for VOA theory: Vertex Operator Algebras and the Monster, by Frenkel, Lepowsky, Meurman; see definition p.244. Of particular interest are G-invariant integral forms where G is a finite subgroup of the automorphism group. We mention a subset of recent results on (1) integral forms in lattice type VOA; (2) infinite dimensional graded representations of Chevalley-Steinberg groups (over any commutative ring) on vertex algebras which extend their natural action on the adjoint module; (3) the case of a G-invariant inte- gral form in the Moonshine VOA where G is the full automorphism group, isomorphic to the Monster sporadic simple group; (4) maximal G-invariant integral forms in degree 2 summands of dihedral VOAs.

REGULAR BIPARTITE DIVISOR GRAPHS

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Given a finite group G, it is an area of research to convey nontrivial information about the structure of G through some sets of invariants associated to G such as the set of degrees of the irreducible complex characters of G and would be interesting to distinguish the group structure of G influenced by these sets. In [1], the author considered the cases where $B(G)$ is a path or a cycle and discussed some properties of G. In particular she proved that $B(G)$ is a cycle if and only if G is solvable and $B(G)$ is either a cycle of length four or six. As cycles are special types of regular graphs, in this talk we consider the case where $B(G)$ is an n-regular graph for $n \in \{1, 2, 3\}$.

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FINITE GROUPS AND THEIR BREADTH

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The n-breadth $b_n(G)$ of a finite group G is defined for divisors n of $exp(G)$ as $n^{-1}|x \in G|x^n = 1|$, an integer. This was first conjectured by Frobenius and in the meantime proved. The (global) breadth $B(G)$ of G is the maximum of all $b_n(G)$ defined. Recently some classes of groups G satisfying $B(G) = n$ for a fixed integer have been considered by Meng and Shi. These classes always consist of infinitely many different isomorphism classes. It would be desirable to have a small subset of theses isomorphism classes allowing the construction of all isomorphism classes. This can be done by considering all groups G satisfying $B(G) = n > B(G/N)$ for all nontrivial normal subgroups $N \subset G$. The procedure how to construct the full set of isomorphism classes is outlined, the main example will be $n = 8$.

*Joint work with Francesco G. Russo.

FREE PROFINITE PRODUCTS OF FINITE SOLUBLE GROUPS

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Let $G := \coprod_{i\in I} G_i$ be the free profinite product of prosouble groups G_i. We contribute to the description of prosoluble subgroups of such G. In particular, the question of existence of a prosoluble retract for the projection onto the maximal prosoluble factor group is discussed.

TWO CRITERIA FOR SOLVABILITY OF FINITE GROUPS

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In this talk I shall discuss the following two theorems, which were proved in a joint paper with Patrizia Longobardi and Mercede Maj.

Theorem A. *Let G be a finite group of order n containing a subgroup A of prime power index p^r. If A contains a cyclic subgroup B of index less or equal two, then G is a solvable group.*

Theorem B. *Let G be a finite group of order n and suppose that $\psi(G) \geq \frac{1}{6.68}\psi(C_n)$, where $\psi(G)$ denotes the sum of orders of all elements of G and C_n denotes the cyclic group of order n. Then G is a solvable group.*

*Joint work with Kivanç Ersoy.

A GAP-CONJECTURE AND ITS SOLUTION: ISOMORPHISM CLASSES OF CAPABLE SPECIAL p-GROUPS OF RANK 2

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A group is said to be capable if it is a central quotient group and a p-group is special of rank 2 if its center is elementary abelian of rank 2 and equal to its commutator subgroup. In 1990, Heineken showed that if G is a capable special p-group of rank 2, then $p^5 \leq |G| \leq p^7$. Over a decade ago we asked GAP to determine the number of isomorphism classes of capable special p-groups of rank 2 for small primes p. GAP told us that in these cases, the number of isomorphism classes of special p-groups of rank 2 grows with p. However, for the capable among them the number of isomorphism classes is independent of the prime p. Finally, we were able to show that what GAP conjectured is true for all primes p.

O. ORE'S 1951 QUESTION: IS EVERY ELEMENT OF A NON-ABELIAN SIMPLE GROUP A COMMUTATOR?

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Theorem. *Every* (*countable*) *group embeds into a simple* (*countable*) *group S in which any two elements of the same order are conjugate. Every element of S may be written as a commutator of two elements of S.*

*Joint work with Hermann Heineken and Robert F. Morse.

ALMOST ENGEL COMPACT GROUPS

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We say that a group G is almost Engel if for every $g \in G$ there is a finite set $\mathscr{E}(g)$ such that for every $x \in G$ all sufficiently long commutators $[...[[x, g], g], \dots, g]$ belong to $\mathscr{E}(g)$, that is, for every $x \in G$ there is a positive integer $n(x, g)$ such that $[...[[x, g], g], \dots, g] \in \mathscr{E}(g)$ if g is repeated at least $n(x, g)$ times. (Thus, Engel groups are precisely the almost Engel groups for which we can choose $\mathscr{E}(g) = \{1\}$ for all $g \in G$)

We prove that if a compact (Hausdorff) group G is almost Engel, then G has a finite normal subgroup N such that G/N is locally nilpotent. If in addition there is a uniform bound $|\mathscr{E}(g)| \leq m$ for the orders of the corresponding sets, then the subgroup N can be chosen of order bounded in terms of m. The proofs use the Wilson–Zelmanov theorem saying that Engel profinite groups are locally nilpotent.

HAUSDORFF SPECTRA OF FINITELY GENERATED PRO-p GROUPS

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Every finitely generated pro-p group G comes equipped with a range of translation invariant metrics that are naturally induced by filtration series such as the p-power filtration. Given such a metric, the distribution of closed subgroup in G gives rise to a corresponding Hausdorff spectrum. It is a long-standing open question whether the finiteness of the Hausdorff spectrum, with respect to the p-power filtration, say, implies that the pro-p group G is p-adic analytic. I will report on joint work with Anitha Thillaisundaram and Amaia Zugadi-Reizabal in this context, and highlight some open problems arising from our results. Time permitting, I will also talk about ongoing work with Alejandra Garrido and Oihana Garaialde-Ocana concerning the Hausdorff spectra of free pro-p groups.

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[*]Joint work with Pavel Shumyatsky.

INFINITE-DIMENSIONAL LEIBNIZ ALGEBRAS IN THE SPIRIT OF INFINITE GROUP THEORY

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In the theory of Lie algebras, there is a large part, in which problems like those that arise in group theory are considered. More precisely, we are talking about problems, approaches, setting tasks. This part of the theory of Lie algebras developed very intensively, there is a huge array of articles and several books. An essential generalization of Lie algebras is the Leibniz algebras. The Lie algebras are exactly the anticommutative Leibniz algebras. The theory of Leibniz algebras has recently been developing very intensely. It should be noted, however, that in most of the articles on Leibniz algebras finite-dimensional algebras are considered, moreover, in most of these articles Leibniz algebras were considered over a field of characteristic 0. This is like the situation that existed in group theory, as the theory of groups first developed as a theory of finite groups. Thus, the question of the theory of infinite dimensional Leibniz algebra naturally arises. Since in the Lie algebras the approach "in the spirit of group theory" turned out to be successful and fruitful, it is natural to apply it also to Leibniz algebras. There are logical questions about Leibniz algebras whose subalgebras are ideals, about Leibniz algebras in which a relation "to be an ideal" is transitive, and on analogues of some properties of nilpotent and locally nilpotent groups, and so on. In this paper, we will consider some analogues of this kind.

REGULAR DIRECT LIMITS OF SYMMETRIC GROUPS

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Let $Sym(n)$ denote the finite symmetric group on n letters. Starting with a finite group of order $n \geq 3$ and embedding this group by right regular representation into $Sym(n)$ and continuing the embedding of $Sym(n)$ by right regular representation into $Sym(Sym(n))$ and continuing like this, we obtain a direct limit group which is called Hall universal group U. The structure of centralizers of elements and centralizers of finite abelian subgroups in U is studied by Hartley in [1]. The structure of centralizers of arbitrary finite subgoups in Hall's universal group is determined in [3]. Namely we prove the following:

Theorem. *Let U be the Hall's universal group and F be a finite subgroup of U. Then the centralizer $C_U(F)$ is an extention of $Z(F)$ by U.*

Hall's universal group is a locally finite group, so it does not contain elements of infinite order. But if we start with an infinite symmetric group and embed it by right regular representation into its symmetric group and continue with transfinite induction sufficiently large cardinal number of times and on the limit ordinals we take the union of the preceeding obtained subgroups, we obtain a direct limit group. This group has some interesting properties in particular they are κ-existentially closed. We mention basic properties of these groups for details see [2]. Moreover we determine the the structure of centralizers of some subgroups; namely we prove the following:

Theorem. *Let κ be a limit cardinal of cofinality $|I|$. Let G be the direct limit of symmetric groups G_i obtained by right regular representation of G_i into G_{i+1}, where $i \in I$ and for limit ordinals we take the union. Let F be a subgroup of G contained in G_i for some $i \in I$. Then the centralizer $C_G(F)$ is isomorphic to an extension of $Z(F)$ by a group isomorphic to G.*

Corollary. *Let G be the κ-existentially group of order inaccessible cardinal κ and F be any proper subgroup of G. Then $C_G(F)$ is isomorphic to an extension of $Z(F)$ by a subgroup isomorphic to G.*

In particular if $Z(F) = \{1\}$, then $C_G(F)$ is isomorphic to G.

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*Joint work with Otto H. Kegel

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SEMI-EXTRASPECIAL GROUPS WITH AN ABELIAN SUBGROUP OF MAXIMAL POSSIBLE ORDER

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Let p be a prime. A p-group G is defined to be semi-extraspecial if for every maximal subgroup N in $Z(G)$ the quotient G/N is a an extraspecial group. In addition, we say that G is ultraspecial if G is semi-extraspecial and $|G : G'| = |G'|^2$. In this paper, we prove that every p-group of nilpotence class 2 is isomorphic to a subgroup of some ultraspecial group. Given a prime p and a positive integer n, we provide a framework to construct of all the ultraspecial groups order p^{3n} that contain an abelian subgroup of order p^{2n}. In the literature, it has been proved that every ultraspecial group G order p^{3n} with at least two abelian subgroups of order p^{2n} can be associated to a semifield. We provide a generalization of semifield, and then we show that every semi-extraspecial group G that is the product of two abelian subgroups can be associated with this generalization of semifield.

RANDOM WALKS ON VIRTUALLY SOLVABLE MINIMAX GROUPS

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A *virtually solvable minimax group* is a group that can be built up from finite groups, cyclic groups, and quasicyclic groups by forming finitely many extensions. The finitely generated, virtually solvable minimax groups are important because they comprise all the finitely generated, virtually solvable groups that fail to have any sections isomorphic to $C_p \wr C_\infty$ for some prime p (proved by Peter Kropholler in 1984).

In this talk, we discuss the following new result about these groups.

Theorem A. (Kropholler and Lorensen) *Every finitely generated, virtually solvable minimax group can be expressed as a homomorphic image of a virtually torsion-free, virtually solvable minimax group.*

This theorem has the potential to reduce many questions about these groups to the case where the group is virtually torsion-free. This case is often much simpler to handle than the general case because the group is then linear over $\mathbb{Q}$. One illustration of Theorem A's utility is the following corollary generalizing a theorem of Ch. Pittet and L. Saloff-Coste about random walks from 2003.

Corollary. *Let G be a finitely generated, virtually solvable minimax group. Let $p(t)$ be the probability of returning to one's starting point after $2t$ steps of a random walk on the Cayley graph of G with respect to some finite symmetric generating set of G. Then*

$$p(t) \gtrsim \exp(-t^{\frac{1}{3}}).$$

Pittet and Saloff-Coste's original result treats the case where G is virtually torsion-free. Since forming a quotient of a group always increases the return probability, Theorem A allows us to deduce the general case immediately from the virtually torsion-free case.

Theorem A is proved by embedding the minimax group densely in a locally compact group with an open normal, locally pro-p nilpotent subgroup that gives rise to a virtually abelian quotient. This is a new approach to studying minimax groups, one that we hope will yield further applications in the future.

*Joint work with Peter H. Kropholler

THE EXPECTED NUMBER OF RANDOM ELEMENTS TO GENERATE A FINITE GROUP

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The expected number $e(G)$ of elements of a finite group G which have to be drawn at random, with replacement, before a set of generators is found, can be determined using the Möbius function defined on the subgroup lattice of G. We will discuss several applications of this result. If all the Sylow subgroups of G can be generated by d elements, then $e(G) \leq d + \kappa$ where κ is an absolute constant that is explicitly described in terms of the Riemann zeta function and best possible in this context. Approximately, κ equals 2.752394. If G is a permutation group of degree n, then either $G = S_3$ and $e(G) = 2.9$ or $e(G) \leq \lfloor n/2 \rfloor + \kappa^*$ with $\kappa^* \sim 1.606695$.

ON THE ARITHMETIC OF INTEGRAL REPRESENTATIONS OF FINITE GROUPS

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Let R be a Dedekind domain with quotient field K, let H be an R-order, L – absolutely irreducible H-representation module (i.e., a finitely generated H-module that is torsion free as an R-module and such that $K \otimes_R L$ is an absolutely irreducible $K \otimes_R H$-module). The most interesting case is the case where $H = RG$, G a finite group and $char K \nmid |G|$, it was treated in many classical papers by K. Roggenkamp, D. K. Faddeev, I. Reiner, A. V. Roiter, L. A. Nazarova, W. Plesken, G. Nebe and many others.

The classical Jordan-Zassenhaus theorem states that every isomorphism class of KG-representation modules splits in a finite number of isomorphism classes of RG-representation modules if the ideal class group of R is finite. The most interesting questions are:

1. Is it possible to find a reasonable estimate of the number of isomorphism classes?
2. What happens if $cl(R)$, the ideal class group of R, is infinite?
3. Can we describe the representations explicitly?
4. Let R be the maximal order in a number field K. Is it possible to find out whether a representation over K can be realized over R?

Given a character ρ of G, does there exist an integral representation $G \to GL_n(K)$ realizing ρ with a number field K of a minimal degree over $\mathbb{Q}$; is there a representation $G \to GL_n(K)$ with K realizing ρ which cannot be made integral, and whether there is a finite number or infinitely many such fields K? We are interested in the above question and some representations arising from Galois actions on torsion points of elliptic curves.

PROBABILISTIC IDENTITIES IN FINITE GROUPS

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Given a word $w(x_1, ..., x_n)$ and a finite group G, $w = 1$ is a *probabilistic identity* in G, if that equation holds for many r-tuples of G. Sample results:

1. If more than a half of the pairs of elements in G commute, then G is nilpotent (P. Lescot, 1987).

2. If at least 7/9 of the elements of G have order 3, then G has exponent 3 (T. J. Laffey, 1976).

We will survey some recent results of a similar nature.

ON THE NILPOTENCY OF THE VERBAL SUBGROUP CORRESPONDING TO THE ENGEL WORD

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Let $w = w(x_1, \ldots, x_s)$ be a group-word, that is a nontrivial element of the free group F on free generators $x_1, \ldots, x_s$. Given a group G, the subgroup of G generated by all w-values is denoted by $w(G)$ and it is called the *verbal subgroup of* G *corresponding to* w.

In [1] B. Baumslag and J. Wiegold established that the following property characterizes the nilpotency of a finite group G when $w = x$:

P: "If a and b are w-values of coprime orders $|a|$ and $|b|$, respectively, then the order of ab is the product of $|a|$ and $|b|$".

Later in 2017 R. Bastos, C. Monetta and P. Shumyatsky proved that P characterizes the nilpotency of $w(G)$ when w is a lower central word and G is a finite group (see [2]). Going further, one could ask what happens for other group-words.

The aim of this talk is to show that the property P characterizes the nilpotency of $w(G)$ when w is the Engel word and G is a residually finite group.

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*Joint work with Antonio Tortora.

POLYNOMIALLY-BOUNDED DEHN FUNCTIONS OF GROUPS

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The minimal non-decreasing function $d(n)\colon \mathbf{N} \to \mathbf{N}$ such that every word w vanishing in a group $G = \langle A \mid R\rangle$ and having length $||w|| \leq n$ is freely equal to a product of at most $d(n)$ conjugates of relators from $R^{\pm 1}$, is called the *Dehn function* of the presentation $G = \langle A \mid R\rangle$. In other words, the Dehn function $d(n)$ of the presentation is the smallest function that bounds from above the areas of loops of length $\leq n$ in the Cayley complex $Cay(G)$,

On the one hand, it is well known that, up to equivalence, the only subquadratic Dehn function of finitely presented groups is the linear one. On the other hand there is a huge class of Dehn functions $d(n)$ with growth at least n^4 (essentially all possible such Dehn functions) constructed by M. V. Sapir, J. C. Birget and E. Rips, in [Isoperimetric and isodiametric functions of groups, Annals of Mathematics, 157, 2(2002), 345-466] and based on the time functions of Turing machines and S-machines. The class of Dehn functions n^α with $\alpha \in (2;4)$ remained more mysterious even though it has attracted quite a bit of attention (for example, [N. Brady and M. Bridson, There is only one gap in the isoperimetric spectrum, Geometric and Functional Analysis, 10 (2000), 1053-1070]).

I fill the gap obtaining Dehn functions of the form n^α (and much more) for all real $\alpha \geq 2$ computable in reasonable time, for example, $\alpha = \pi$ or $\alpha = e$, or α is any algebraic number.

IDENTIFYING GROUPS WITH A LARGE p-SUBGROUP

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The determination of groups with a large p-subgroup, p a prime, is a significant step in an alternative approach to the classification of some of the finite simple groups. In this talk, I will present examples of large p-subgroups in the finite simple groups. I will then explain a strategy for understanding groups which possess such a subgroup with special emphasis on the final identification of the simple groups.

KOSZUL PROPERTY IN GALOIS COHOMOLOGY

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Absolute Galois groups of fields are a main object of interest in algebraic number theory and related subjects, but we are very far from a satisfactory understanding of their structure. Yet, recently a great breakthrough has been obtained with the proof of Bloch-Kato conjecture. This gave mathematicians the first substantial insight on the rather mysterious Galois cohomology of an absolute Galois group (and of its pro-p quotients), an important invariant of a field. Its most significant consequence is that, in case a field contains a primitive p-th root of unity, the Galois cohomology of its absolute Galois group with coefficients in $\mathbb{F}_p$ is a quadratic algebra. There is a class of quadratic algebras with an uncommonly good homological behaviour and endowed with a useful duality functor: the class of Koszul algebras. L. Positselski conjectured that in the above situation the Galois cohomology of absolute Galois groups is always Koszul, and proved this for various classes of fields, e. g. for algebraic number fields.

We discuss the meaning of Koszulity in the framework of Galois theory and we show some new ways to prove Koszulity of Galois cohomology for other significant classes of fields.

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*Joint work with Gernot Stroth.

**Joint work with Jan Minac, Marina Palaisti, Claudio Quadrelli and Tan Nguyen Duy.

COMPOSITION LENGTHS OF FINITE GROUPS

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The composition length $c(G)$ of a finite group G is the number of its composition factors (counting multiplicities), and is sometimes viewed as a measure of its size or complexity. Various upper bounds on $c(G)$ have been used to bound the size of a minimum generating set for G, or an invariable generating set for G, and there are various links with graph isomorphism. Often it is useful to have bounds in terms of parameters for the way the group is represented: for example, in terms of the degree of a permutation group, or the dimension and field size of a matrix group over a finite field.

My colleagues Stephen Glasby, Kyle Rosa and Gabriel Verret and I have found explicit, upper bounds for $c(G)$, both for permutation groups and matrix groups G. The bounds are sharp and we have been able to describe all the examples which attain the bounds. Our bounds improve various results in the literature dating back to 1974. In particular, for primitive permutation groups G of degree n, where Laci Pyber had obtained an upper bound for $c(G)$ of the form '$c \log n$' with $c \approx 4.244$, we show that the optimal value for the constant c is $8/3 \approx 2.667$.

METANILPOTENT GROUPS SATISFYING THE MINIMAL CONDITION FOR NORMAL SUBGROUPS

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First we will review the research on this class of groups that was carried out in the 1970's by D. McDougall, B. Hartley and H. Silcock. Then we will discuss new results relating to factorizations, Frattini properties, Sylow structure and the number of maximal subgroups.

*Joint work with Stephen Glasby, Kyle Rosa and Gabriel Verret.

EXTENSIONS OF THE GROUP $C(n)$ BY MEANS OF THE DIRECT PRODUCT OF TWO CYCLIC GROUPS OF ORDER p

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This talk reports the investigation of extensions of the group $C(n) = C_{p_\infty} \oplus C_{p_\infty} \oplus \cdots \oplus C_{p_\infty}$, by means of the direct product of two cyclic groups $A = \langle \bar{a} \rangle_p \times \langle \bar{b} \rangle_p$, where p is a prime number. It has been concluded from this work that all non-isomorphic extensions of the group $C(n)$ by means of the group $A = \langle \bar{a} \rangle_p \times \langle \bar{b} \rangle_p$, that correspond to Z_p-irreducible representations of the group $A = \langle \bar{a} \rangle_p \times \langle \bar{b} \rangle_p$, are:

1. $G(T, 0, 0, 0), G(T, 0, 0, a_0)$;
2. $G(U, 0, 0, 0)$, $G(U, 0, , 0)$, $G(U, 0, 0, c_0)$, $G(U, 0, \beta, c_0)$ where $c_0 = (a_0, 0, ..., 0) \in C(p-1)$, $\beta = (a_0, 2a_0, ..., (p-1)a_0) \in C(p-1)$.

ON GROUPS OF FINITE UPPER RANK

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The *upper rank* of a group is the supremum of the Prüfer ranks of its finite quotient groups. The upper p-rank, for a prime p, is defined analogously, using the Sylow p-subgroups of finite quotients. I'll discuss to what extent the upper rank of a finitely generated group is controlled by its upper p-ranks.

HALF AXES IN POWER ASSOCIATIVE ALGEBRAS

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Let A be a commutative, non-associative algebra over a field $\mathbb{F}$ of characteristic $\neq 2$. A half-axis in A is an idempotent $e \in A$ such that e satisfies the Peirce multiplication rules in a Jordan algebra, and, in addition, the 1-eigenspace of ad_e (multiplication by e) is one dimensional.

In this paper we consider the identities
$(*)$ $x^2x^2 = x^4$ and $x^3x^2 = xx^4$.

We show that if identities $(*)$ hold strictly in A, then one gets (very) interesting identities between elements in the eigenspaces of ad_e (note that if $|\mathbb{F}| > 3$ and the identities $(*)$ hold in A, then they hold strictly in A). Furthermore we prove that if A is a primitive axial algebra of Jordan type half (i.e., A is generated by half-axes), and the identities $(*)$ hold strictly in A, then A is a Jordan algebra. We use *linearization techniques.*

Axial algebras were recently introduced by J. I. Hall, F. Rehren and S. Shpectorov. These algebras have intimate relationships with the Monster algebra used by R. L. Griess to construct the Monster group.

ON CONCISENESS OF WORDS IN RESIDUALLY FINITE GROUPS

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A group-word w is called concise if whenever the set of w-values in a group G is finite it always follows that the verbal subgroup $w(G)$ is finite. More generally, a word w is said to be concise in a class of groups X if whenever the set of w-values is finite for a group $G \in X$, it always follows that $w(G)$ is finite. P. Hall asked whether every word is concise. Due to Ivanov the answer to this problem is known to be negative. On the other hand, for residually finite groups the problem remains wide open. We recently discovered that the Lie-theoretic techniques created by Zelmanov in his solution of the restricted Burnside problem can be used to prove conciseness in residually finite groups of many words. Our talk will be about recent developments with respect to that problem.

A CANONICAL FORM AND EXCHANGE LAWS FOR THE SYMMETRIC GROUP S_n

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Consider the symmetric group S_n, which is a finite simply-laced Coxeter group, generated by $n-1$ involutions $s_1, s_2, \ldots s_{n-1}$, where $(s_i \cdot s_{i+1})^3 = 1$, and $(s_i \cdot s_j)^2 = 1$, for $|i-j| \geq 2$. Let $t_2 = s_1$, $t_3 = s_1 \cdot s_2$, and etc. $t_i = s_1 \cdot s_2 \cdots s_{i-1}$, for every $2 \leq i \leq n$. It is easy to show that every element has a unique presentation in a canonical form $t_2^{i_2} \cdot t_3^{i_3} \cdots t_n^{i_n}$, where $0 \leq i_j < j$, for every $2 \leq j \leq n$. There is an easy and a very surprising algorithm to compute the Coxeter length of the elements of S_n written in the described canonical form. Moreover, the canonical form of $t_q^{r_q} \cdot t_p^{r_p}$, where $q > p$ is surprisingly, a product of at most three powers of the generators t_j, where one of them is t_q, i.e, $t_q^{r_q} \cdot t_p^{r_p} = t_a^{i_a} \cdot t_b^{i_b} \cdot t_q^{i_q}$, or $t_q^{r_q} \cdot t_p^{r_p} = t_a^{i_a} \cdot t_q^{i_q}$, where $a < b < q$, and a, b, i_a, i_b, i_q are determined easily by p, q, r_p and r_q. Therefore, there is an easy algorithm to find the canonical form for a product of two elements which are already written in that form.

SMALL 4-QUASINORMAL SUBGROUPS

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Generalising the concept of quasinormality, a subgroup H of a group G is said to be 4-quasinormal in G if $\langle H, K \rangle = HKHK$ for all cyclic subgroups K of G. (An intermediate concept would be 3-quasinormality, but in finite p-groups - our main concern - this is equivalent to quasinormality.) Quasinormal subgroups have many interesting properties and it has been shown that some of these can be extended to 4-quasinormal subgroups, particularly in finite p-groups. However, even in the smallest case, when H is a 4-quasinormal subgroup of order p in a finite p-group G, precisely how H is embedded in G is not immediately obvious. We shall consider one of these questions regarding the commutator subgroup $[H, G]$.

LEFT 3-ENGEL ELEMENTS IN GROUPS

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Let G be a group. An element $x \in G$ is said to be a left 3-Engel element if $[[[g, x], x], x] = 1$ for all $g \in G$. It is an open question whether x must be in the locally nilpotent radical. In this lecture we give some background to this question and present some recent progress.

CHARACTERS OF FINITE GROUPS AND p-ADIC NUMBERS

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It is standard to consider that characters of finite groups assume their values in the field $\mathbf{C}$ of complex numbers. However, for some questions, such as modular representation theory for some prime p, it is more convenient and also standard to assume that characters of finite groups assume their values in some algebraic closure of the field $\mathbf{Q}_p$ of p-adic numbers. While $\mathbf{Q}_p$ is isomorphic to a subfield of $\mathbf{C}$, there is no preferred such isomorphism. This yields that, in many cases, an irreducible character of a finite group G taking values in $\mathbf{Q}_p$, for example, may correspond to multiple irreducible complex characters, depending on the chosen isomorphism of fields. This has some consequences for the systematic study of the rationality properties of the ordinary characters of finite groups.

*Joint work with Enrico Jabara (Venice) and Gareth Tracey (Bath).

SUBNORMAL SUBGROUPS OF CHEVALLEY GROUPS, REVISITED

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In 1972 John Wilson published a pathbreaking paper which, among other things, featured description of subnormal subgroups of $GL(n, R)$, over an arbitrary commutative ring R. Technically, this description is an immediate corollary of the description of subgroups in $GL(n, R)$ normalised by a relative elementary subgroup $E(n, R, A)$, for an ideal A in R. The answer to this last problem is that for any such subgroup H there exists an ideal I such that $E(n, R, A^m I) \leq H \leq C(n, R, I)$, where $C(n, R, I)$ is the full congruence subgroup of level I. Here, the ideal I is not unique as such, only up to a certain equivalence relation. Initially in Wilson's paper, the bound was $m = 7$ for $n \geq 4$. Later it was several times improved (and also deteriorated) in the works by Vaserstein, myself, and others, and, as of today, $m = 4$ for $n \geq 3$.

Subsequently, in 1986–2016, such similar results were obtained for other classical groups as well, with decisive contributions by Leonid Vaserstein, Gerhard Habdank, Zhang Zuhong, and You Hong. There, the result is more complicated (especially when 2 is not invertible in R) and the best published bound for m is $m = 8$ (and there are additional complications in the presence of form parameter).

However, for exceptional groups, similar results were not available until very recently. Initially, using slightly fancier versions of decompositions of unipotents (initially proposed by Alexei Stepanov and myself) with Zhang Zuhong we were able to prove similar results for the two easier exceptional cases, those of Chevalley groups of types E_6 and E_7, with the bound $m = 7$.

However, in 2017 something truly remarkable happened. Namely, Raimund Preusser noticed that for classical groups the method of decomposition of unipotents works also the other way, and gives very strong constructive forms of Brenner's lemma, expressing elementary unipotents as products of elementary conjugates of an arbitrary non-central matrix g. He himself used this to obtain very explicit description of normal subgroups in those cases.

We immediately observed, that the same method works also for exceptional groups, and later were able to apply Preusser's idea also at the relative level. This new proof works uniformly for all simply laced Chevalley groups, those of types A_l, D_l, E_6, E_7 and E_8, and in January 2018 we observed, that it even works with the same bound $m = 4$, as for A_l above.

* Joint work with Zhang Zuhong.

MODEL THEORY FOR FINITE GROUPS

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It is explained why certain types of subgroups are determined in all finite groups by first-order formulae.

STEINBERG-LIKE CHARACTERS FOR NON-ABELIAN SIMPLE GROUPS

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Let G be a finite group, $|G|$ the order of G and p a prime dividing $|G|$. Let S be a Sylow p-subgroups of G. We define a Steinberg-like character to be a proper character, possible reducible, of degree $|S|$ and vanishing at all p-singular elements of G. This is a generalization of the notion of the Steinberg character for a simple group of Lie type in defining charateristic p; this is irreducible and plays a prominent role in the character theory of finite groups of Lie type.

One of motivations is an application to projective modules. Malle and Weigel [1] raised the question on describing projective indecomposable mo-dules for simple groups G of dimension $|S|$. More precisely, determine the pairs (G, p), such that G is a non-abelian simple group with $S \neq 1$ and G has a projective indecomposable FG-module M, say, of dimention $|S|$. Here F is either an algebraically closed field of characteristic p or the ring of integers of a finite extension of the p-adic number field containing a primitive $|G|$-root of unity. These two cases are known to be equivalent, so we assume F to be the ring of integers. In this case it is meaningful to speak on the character χ of M; this vanishes at all p-singular elements of G (again well known).

The problem was solved in [1] under an additional assumption that $(\chi, 1_G) = 1$. A special case where p is the defining characteristic of a group G of Lie type was completed in [4]. In a recent joint paper with G. Malle [2] the problem raised in [1] has been finally solved.

The case where M is irreducible easily follows from earlier known results so below we ignore this case.

Theorem 1. [1] Let G be a non-abelian simple group with Sylow p-subgroup S. If there exists a reducible projective FG-module of dimension $|S|$ then one of the following holds:

*Joint work with Gunter Malle.

(1) $G = PSL_2(q)$, $q > 4$ even, $|S| = q + 1$;
(2) $G = PSL_2(p)$, $|S| = p > 5$;
(3) $G = PSL_2(q)$, $q + 1$ is a 2-power and $p = 2$;
(4) $G = PSL_n(q)$, n is an odd prime, $n \not| (q - 1)$, $|S| = (q^n - 1)/(q - 1)$;
(5) $G = A_p$, $|S| = p \geq 5$;
(6) $G = M_{11}$, $|S| = 11$;
(7) $G = M_{23}$, $|S| = 23$.

A significant part of the reasoning in [2] and in the earlier paper [4] is based on the analysis of irreducible constituents of χ and depends only on the assumption that χ is Steinberg-like. This encourges one to state the problem on description of the Steinberg-like characters for finite simple groups. This problem was first studied in a joint paper with M. Pellegrini [3] for groups of Lie type in defining characteristic p. The Steinberg-like characters that are not the characters of projective modules has been found for groups $PSL_n(q)$, $n = 2, 3, 4, 5$, $PSU_3(q)$, $PSp_4(q)$ and in twisted groups ${}^2B_2(q)$ (Suzuki groups) and ${}^2G_2(q)$ (sometimes with some restriction on q). All other groups was proven to have no Steinberg-like character, with possible exception B_n, $n = 3, 4, 5$ and D_n, $n = 4, 5$. These exceptions are now ruled out in [2], and for other cases the following result holds:

Theorem 2. [2] Let G be an alternating group and $p > 2$, or a groups of Lie type for p different of the defining characterisitic, or a sporadic group. Suppose that Sylow p-subgroups of G are not cyclic. Then G has no reducible Steinberg-like character for p, except when $G = PSL_2(q)$, $p = 2$ and $q + 1$ is a 2-power.

For alternating groups and $p = 2$ there are partial results but in general the problem remains open.There is a conjecture that the alternating groups A_n have no Steinberg-like character unless $n = 2^k$ and $2^k + 1$. For these values of n some reducible Steinberg-like characters have been constructed in [2].

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REPRESENTATION GROWTH OF SPECIAL LINEAR GROUPS

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One way of studying the complex representations of the special linear groups over the p-adic integers is to determine the convergence of their representation zeta function. More in general, let Γ be a topological group such that the number $r_n(\Gamma)$ of its irreducible continuous complex characters of degree n is finite for all $n \in \mathbb{N}$. We define the *representation zeta function* of Γ to be the Dirichlet generating function

$$\zeta_\Gamma(s) = \sum_{n \geq 1} r_n(\Gamma) n^{-s} \quad (s \in \mathbb{C}).$$

It is known that the convergence domain of such function (when not empty) is an open complex half-plane. The *abscissa of convergence* of ζ_Γ is the real number α_Γ giving this half-plane of convergence and it determines the degree of polynomial growth of the sequence $R_N(\Gamma) = \sum_{n \leq N} r_n(\Gamma)$. Indeed, $\log(R_N(\Gamma))/\log N$ tends to α_Γ as N tends to infinity.

Recently Avni and Aizenbud [AA] have given a method that relates special values of the representation zeta function at even integers $2g-2$ with the singularities of the representation variety of the fundamental group of a Riemann surface of genus g into the special linear group. Using this method, they determine that $\alpha_{\mathrm{SL}_d(\mathbb{Z}_p)} < 22$. In this talk I shall report on a recent result [BZ] obtained in collaboration with Budur pushing down this bound to 2.

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*Joint work with Prof. Nero Budur.