Dedicated to the memory of

Valeria Fedri

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Sugli amalgami di p-gruppi finiti non immergibili in un p-gruppo finito

Rend. Sem. Mat. Univ. Padova 37, 98-103 (1967)

Valeria Fedri

Su un criterio di classificazione dei p-gruppi finiti

Matematiche (Catania) 24, 240-248 (1969)

Valeria Fedri - Umberto Tiberio

The finite goups whose proper local subgroups are supersolvable

Boll. Un. Mat. It. A (5) 17 no.1, 73-78 (1980)

Valeria Fedri - Umberto Tiberio

A characterization of minimal non-F finite solvable groups

Boll. Un. Mat. Ital. Suppl. no.2, 173-180 (1980)

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On groups with F-projectors
Boll. Un. Mat. It. A (6) 1 no.2, 253-260 (1982)

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Boll. Un. Mat. Ital. A (6) 2 no.2, 155-162 (1983)

Valeria Fedri - Luigi Serena

Finite soluble groups with supersoluble Sylow normalizers

Archiv Math. 50 no.1, 11-18 (1988)

Valeria Fedri - Luigi Serena

Symmetric and general linear groups with supersoluble Sylow normalizers

Ann. Mat. Pura Appl. 154(4), 359-370 (1989)

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BOUNDS ON THE FITTING LENGTH OF FINITE SOLUBLE GROUPS WITH SUPERSOLUBLE SYLOW NORMALISERS

R.A. BRYCE, V. FEDRI AND L. SERENA

We prove that, in a finite soluble group, all of whose Sylow normalisers are supersoluble, the Fitting length is at most $2m+2$, where p^m is the highest power of the smallest prime p dividing $|G/G^S|$: here G^S is the supersoluble residual of G. The bound $2m+2$ is best possible. However under certain structural constraints on G/G^S, typical of the small examples one makes by way of experimentation, the bound is sharply reduced. More precisely let p be the smallest, and r the largest, prime dividing the order of a group G in the class under consideration. If a Sylow p-subgroup of G/G^S acts faithfully on every r-chief factor of G/G^S, then G has Fitting length at most 3.

1. INTRODUCTION

We denote by N^S the class of finite groups in which the normalisers of all Sylow subgroups are supersoluble. The motivation for the study of the class N^S is a result of Bianchi et al. [1] which says that only nilpotent groups, among all finite groups, have *nilpotent* Sylow normalisers.

The study of soluble groups in N^S was begun in [3]. Several of the results proved there indicate a close connection between the structure of a soluble group G in N^S and that of its supersoluble co-radical $\overline{G} = G/G^S$. The aim of the present paper is to investigate this connection more closely. More precisely we investigate bounds on the Fitting length of a soluble group in N^S in terms of the structure of $\overline{G}$. While there are groups G in N^S of arbitrarily large Fitting length with given isomorphism type of $\overline{G}$ (see Theorem 3.3 of [3], for example) we show that such groups necessarily involve just two primes in their order. For soluble groups in N^S involving at least three primes we show that Fitting length is bounded by a linear function of a certain invariant.

THEOREM 1.1. *Let G be a soluble group in N^S involving at least three distinct primes in its order. Let p^m be the highest power of the smallest prime p dividing $|\overline{G}|$. Then the Fitting length of G is at most $2m+2$, and this bound is best possible.*

Other more technical results show that under certain restrictions on $\overline{G}$ this bound can be sharply reduced (see Theorem 4.1).

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Bounds on the Fitting length of finite soluble groups with supersoluble Sylow normalisers

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ON A CONJECTURE CONCERNING CARTAN MATRICES OF FINITE SIMPLE GROUPS

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0. Introduction

The following conjecture posed by R.Brandl appears in the Kourovskaia tetrad [10] : Is every finite simple group characterized by its Cartan matrix over an algebraically closed field of characteristic 2? A study of this question was begun in [2], where the conjecture was proved for some groups of the class PSL(2,r). The purpose of the following paper is to prove the conjecture for the class of simple CIT-groups, where a finite group of even order is called a CIT-group if the centralizer of any involution is a 2-group. In [12] Suzuki proved that the class of simple CIT-groups coincides with the class of simple CN-groups that is the class of finite simple groups in which the centralizer of any nonidentity element is nilpotent. Moreover, in [13], Suzuki proved that the class of simple CIT-groups consists of the following groups:

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On a conjecture concerning Cartan matrices of finite simple groups

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A HUGHES-LIKE PROPERTY FOR FINITE GROUPS

by R. A. BRYCE, V. FEDRI and L. SERENA

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Several structure theorems are proved for groups G having the following property. There is a prime p and a collection of subgroups of G such that the elements of G which lie in the complement of every subgroup of the collection all have order p.

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1. Introduction

Let p be a prime, G a finite group and $\mathscr{A}$ a union of subgroups of G. We say that $\mathscr{A}$ has the *Hughes property for exponent p* if the following two conditions hold: firstly $G \neq \mathscr{A}$, and secondly every element of $G \backslash \mathscr{A}$ has order p. (The term 'union' is used here in the sense of set theory: $\mathscr{A}$ need not be a subgroup of G.) The following well-known result describes the structure of a finite group which has a subgroup with the Hughes property.

Theorem 1.1. (Hughes and Thompson **[2]**, Kegel **[4]**). *Let p be a prime, let G be a finite group and let H be a subgroup of G with the Hughes property for exponent p. Then H is nilpotent and, if G is not a p-group, the index of H in G is p.*

A more familiar statement of this is that in a non-nilpotent finite group the Hughes subgroup, that generated by the elements whose order is not p, if not the whole group, is nilpotent and of index p in G.

We will denote by $\mathscr{H}_n(p)$ the class of all finite groups which have a union of n subgroups with the Hughes property for exponent p. Theorem 1.1 says, among other things, that the groups of the class $\mathscr{H}_1(p)$ have a nilpotent normal p-complement. The aim of this article is to prove results like this about more general classes $\mathscr{H}_n(p)$.

Theorem 1.2. *Let n be a positive integer and p a prime greater than n. Each group in the class $\mathscr{H}_n(p)$ has a nilpotent normal p-complement.*

Theorem 1.3. *Let G be a group in the class $\mathscr{H}_n(p)$, and let π be the set of primes other than p, which are greater than or equal to n. Then $O_\pi(G)$ is nilpotent and $G/O_\pi(G)$ is a π'-group.*

It is of interest to examine what these theorems say about $\mathscr{H}_n(p)$-groups when n is small. When $n=1$ Theorem 1.2 follows from Theorem 1.1. When $n=2$ all odd primes

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A Hughes-like property for finite groups

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On self-centralizing Sylow subgroups of order four

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On self-centralizing Sylow subgroups of order four

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COVERING GROUPS WITH SUBGROUPS

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A group is *covered* by a collection of subgroups if it is the union of the collection. The intersection of an irredundant cover of n subgroups is known to have index bounded by a function of n, though in general the precise bound is not known. Here we confirm a claim of Tompkinson that the correct bound is 16 when n is 5. The proof depends on determining all the 'minimal' groups with an irredundant cover of five maximal subgroups.

1. Introduction

A *covering* or *cover* of a group G is a collection of subgroups of G whose union is G. We use the term n-cover for a cover with n members. The cover is *irredundant* if no proper sub-collection is also a cover. Neumann [5] obtained a uniform bound for the index of the intersection of an irredundant n-cover; see Tompkinson [7] for an improved bound. We shall write $f(n)$ for the largest index $|G : D|$ over all groups G with an irredundant n-cover with intersection D. An immediate consequence is that such a group G has a permutation representation of degree at most $f(n)$, with kernel $\mathrm{core}_G(D)$. In particular $G/\mathrm{core}_G(D)$ is a finite group with an irredundant n-cover whose intersection is core-free.

The groups with an irredundant core-free intersection covering are known precisely when $n = 3$ (Scorza [6]) and when $n = 4$ (Greco [4, p.58]): see Propositions 2.3 and 2.4 below. Partial results are known for $n = 5$: Greco [3] lists all groups with an irredundant 5-cover in which all pairwise intersections are the same; and Tompkinson [7] claims that $f(5) = 16$.

The aim of the present article is to fill in some of the missing detail when $n = 5$. We are concerned with irredundant, core-free intersection 5-covers in which all five subgroups of the cover are maximal. A cover in which all subgroups are maximal we shall call *maximal*.

Theorem 1.1. *Let G be a group with a maximal irredundant cover of five subgroups with core-free intersection D. Then either*

(a) $D = 1$ *and G is elementary Abelian of order 16; or*
(b) $D = 1$ *and* $G \cong \mathrm{Alt}_4$; *or*
(c) $|D| = 3$, $|G| = 48$ *and G embeds in* $\mathrm{Alt}_4 \times \mathrm{Alt}_4$.

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Covering groups with subgroups

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SUBGROUP COVERINGS OF SOME LINEAR GROUPS

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Dedicated to Bernhard Neumann on his 90th birthday.

A *cover* for a group is a collection of proper subgroups whose union is the whole group. A cover is *minimal* if no other cover contains fewer members. We term *minimised* a minimal cover with the property that substituting for a member of the cover by a proper subgroup of that member produces a collection which is no longer a cover. We here describe the minimised covers for the groups $\mathrm{GL}_2(q)$, $\mathrm{SL}_2(q)$, $\mathrm{PSL}_2(q)$ and $\mathrm{PGL}_2(q)$.

1. INTRODUCTION

Let G be a group. A *cover* of G is a collection $\mathcal{A} = \{A_i : 1 \leqslant i \leqslant n\}$ of proper subgroups of G whose union is G. The cover $\mathcal{A}$ is *irredundant* if no proper sub-collection is also a cover; and *minimal* if no cover of G has fewer than n members. In this minimal case we write $\sigma(G) = n$.

Covers of groups have been studied by many authors. For example Neumann [5] shows that the intersection of the members of an irredundant cover with n members has index bounded by a function of n. Tomkinson [8] improved this bound. Minimal covers seem to have been introduced by Cohn [1]; and Tomkinson [9] showed that, for a finite soluble group G, $\sigma(G)$ is $p^\alpha + 1$ where p^α is the size of the smallest chief factor of G with multiple complements. He confirmed a conjecture of Cohn [1] that $\sigma(G) = 7$ for no group G. His proof suggests that investigating minimal covers of insoluble groups might be of interest. Here we make a small beginning by looking at the groups $\mathrm{GL}_2(q)$, $\mathrm{SL}_2(q)$, $\mathrm{PSL}_2(q)$ and $\mathrm{PGL}_2(q)$ (Theorem 3.5). We find σ for these groups and, more to the point, we give a description of all the minimal covers which are minimised in a sense to be described below (Theorem 4.4).

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A Generalized Hughes Property of Finite Groups

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