Some results regarding outer commutator words

Gustavo A. Fernández-Alcober

(joint work with Cristina Acciarri, Marta Morigi, and Pavel Shumyatsky)

University of the Basque Country, Bilbao

Ischia Group Theory 2018
March 21st, 2018

Outer commutator words

Outer commutator words

Definition

An outer commutator word is a word which is formed by nesting commutators, but using always different indeterminates.

Outer commutator words

Definition

An outer commutator word is a word which is formed by nesting commutators, but using always different indeterminates.

Examples of outer commutator words

Outer commutator words

Definition

An outer commutator word is a word which is formed by nesting commutators, but using always different indeterminates.

Examples of outer commutator words

- The lower central words $\gamma_i = [x_1, x_2, \ldots, x_i]$.

Outer commutator words

Definition

An outer commutator word is a word which is formed by nesting commutators, but using always different indeterminates.

Examples of outer commutator words

- The lower central words $\gamma_i = [x_1, x_2, \ldots, x_i]$.
- The derived words δ_i, defined recursively by $\delta_0 = x_1$ and

$$\delta_i = [\delta_{i-1}(x_1, \ldots, x_{2^{i-1}}), \delta_{i-1}(x_{2^{i-1}+1}, \ldots, x_{2^i})].$$

For example, $\delta_1 = [x_1, x_2] = \gamma_2$ and $\delta_2 = [[x_1, x_2], [x_3, x_4]]$.

Outer commutator words

Definition

An outer commutator word is a word which is formed by nesting commutators, but using always different indeterminates.

Examples of outer commutator words

- The lower central words $\gamma_i = [x_1, x_2, \ldots, x_i]$.
- The derived words δ_i, defined recursively by $\delta_0 = x_1$ and

$$\delta_i = [\delta_{i-1}(x_1, \ldots, x_{2^{i-1}}), \delta_{i-1}(x_{2^{i-1}+1}, \ldots, x_{2^i})].$$

  For example, $\delta_1 = [x_1, x_2] = \gamma_2$ and $\delta_2 = [[x_1, x_2], [x_3, x_4]]$.
- For convenience, we also consider indeterminates as outer commutator words.

Outer commutator words

Definition

An outer commutator word is a word which is formed by nesting commutators, but using always different indeterminates.

Examples of outer commutator words

- The lower central words $\gamma_i = [x_1, x_2, \ldots, x_i]$.
- The derived words δ_i, defined recursively by $\delta_0 = x_1$ and

$$\delta_i = [\delta_{i-1}(x_1, \ldots, x_{2^{i-1}}), \delta_{i-1}(x_{2^{i-1}+1}, \ldots, x_{2^i})].$$

  For example, $\delta_1 = [x_1, x_2] = \gamma_2$ and $\delta_2 = [[x_1, x_2], [x_3, x_4]]$.
- For convenience, we also consider indeterminates as outer commutator words.
- $[[x_1, x_2], [[x_3, x_4], [x_5, x_6]], x_7]$ is an outer commutator word, but the Engel word $[x, y, y, y]$ is not.

Outer commutator words

Outer commutator words

Outer commutators are also known as multilinear commutators, since they represent multilinear words in Lie algebras.

Outer commutator words

Outer commutators are also known as multilinear commutators, since they represent multilinear words in Lie algebras.

But they are not multilinear as group words: the property

$$\omega(g_1,\ldots,g_ih_i,\ldots,g_r)=\omega(g_1,\ldots,g_i,\ldots,g_r)\,\omega(g_1,\ldots,h_i,\ldots,g_r).$$

is not generally true.

Outer commutator words

Outer commutators are also known as multilinear commutators, since they represent multilinear words in Lie algebras.

But they are not multilinear as group words: the property

$$\omega(g_1, \dots, g_i h_i, \dots, g_r) = \omega(g_1, \dots, g_i, \dots, g_r)\, \omega(g_1, \dots, h_i, \dots, g_r).$$

is not generally true.

Similarly, we need not have

$$\omega(g_1, \dots, g_i^n, \dots, g_r) = \omega(g_1, \dots, g_i, \dots, g_r)^n.$$

We can associate a binary tree to every outer commutator word ω by recursion:

Representation of outer commutator words by trees

We can associate a binary tree to every outer commutator word ω by recursion:

- If ω is a single indeterminate, then consider an isolated vertex.

Representation of outer commutator words by trees

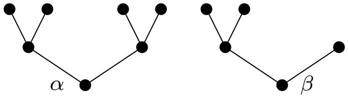
We can associate a binary tree to every outer commutator word ω by recursion:

- If ω is a single indeterminate, then consider an isolated vertex.
- Otherwise, if $\omega = [\alpha, \beta]$, draw the tree of ω by connecting the trees of α and β with a new vertex below them.

Representation of outer commutator words by trees

We can associate a binary tree to every outer commutator word ω by recursion:

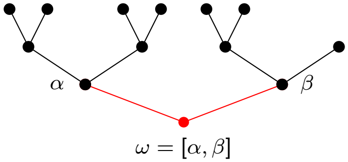
- If ω is a single indeterminate, then consider an isolated vertex.
- Otherwise, if $\omega = [\alpha, \beta]$, draw the tree of ω by connecting the trees of α and β with a new vertex below them.



Representation of outer commutator words by trees

We can associate a binary tree to every outer commutator word ω by recursion:

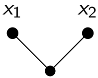
- If ω is a single indeterminate, then consider an isolated vertex.
- Otherwise, if $\omega = [\alpha, \beta]$, draw the tree of ω by connecting the trees of α and β with a new vertex below them.



Some examples

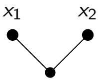
Some examples

The following are the trees of γ_2, γ_3, and γ_4:

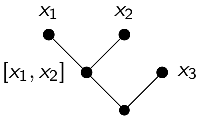


$\gamma_2 = [x_1, x_2]$

The following are the trees of γ_2, γ_3, and γ_4:



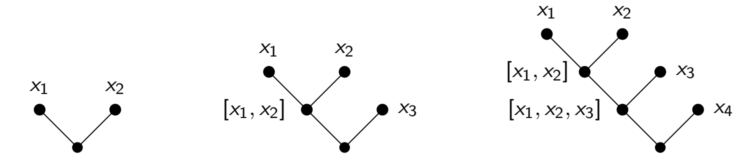
$\gamma_2 = [x_1, x_2]$



$\gamma_3 = [x_1, x_2, x_3]$

Some examples

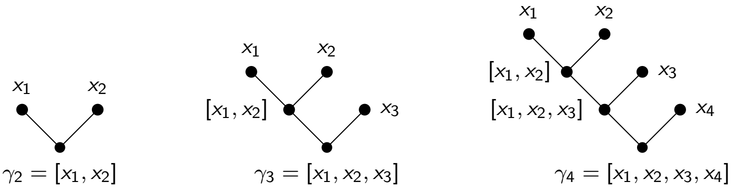
The following are the trees of γ_2, γ_3, and γ_4:



$\gamma_2 = [x_1, x_2]$ $\gamma_3 = [x_1, x_2, x_3]$ $\gamma_4 = [x_1, x_2, x_3, x_4]$

Some examples

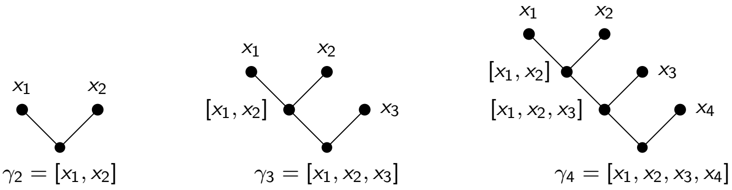
The following are the trees of γ_2, γ_3, and γ_4:



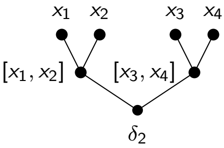
and of δ_2 and δ_3:

Some examples

The following are the trees of γ_2, γ_3, and γ_4:

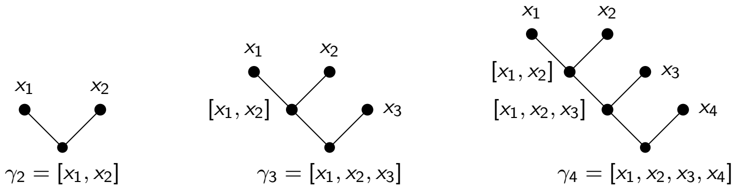


and of δ_2 and δ_3:

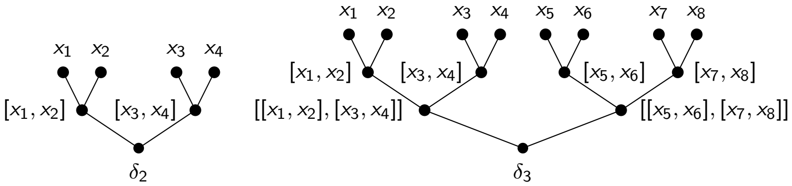


Some examples

The following are the trees of γ_2, γ_3, and γ_4:



and of δ_2 and δ_3:



Height and defect

Definition

Let ω be an outer commutator word. Then:

Height and defect

Definition

Let ω be an outer commutator word. Then:

- The height h of ω is the height of its tree.

Height and defect

Definition

Let ω be an outer commutator word. Then:

- The height h of ω is the height of its tree.

Observe that the derived word δ_h has height h and its tree is the 'complete' binary tree of height h.

Height and defect

Definition

Let ω be an outer commutator word. Then:

- The height h of ω is the height of its tree.

Observe that the derived word δ_h has height h and its tree is the 'complete' binary tree of height h.

- The defect of ω is the number of vertices that we need to add to its tree in order to obtain the tree of δ_h.

Height and defect

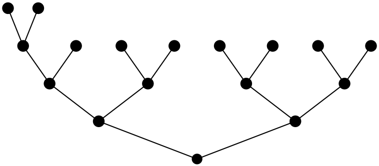
Definition

Let ω be an outer commutator word. Then:

- The height h of ω is the height of its tree.

Observe that the derived word δ_h has height h and its tree is the 'complete' binary tree of height h.

- The defect of ω is the number of vertices that we need to add to its tree in order to obtain the tree of δ_h.



Height and defect

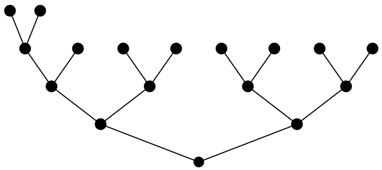
Definition

Let ω be an outer commutator word. Then:

- The height h of ω is the height of its tree.

Observe that the derived word δ_h has height h and its tree is the 'complete' binary tree of height h.

- The defect of ω is the number of vertices that we need to add to its tree in order to obtain the tree of δ_h.



$[[\gamma_3, \gamma_2], \delta_2]$

Height and defect

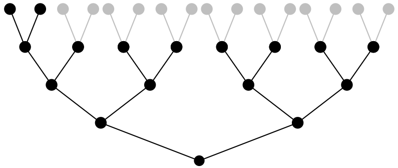
Definition

Let ω be an outer commutator word. Then:

- The height h of ω is the height of its tree.

Observe that the derived word δ_h has height h and its tree is the 'complete' binary tree of height h.

- The defect of ω is the number of vertices that we need to add to its tree in order to obtain the tree of δ_h.



$[[\gamma_3, \gamma_2], \delta_2]$

Height and defect

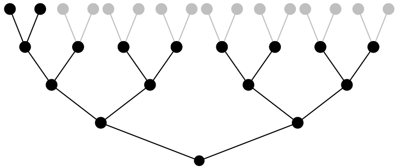
Definition

Let ω be an outer commutator word. Then:

- The height h of ω is the height of its tree.

Observe that the derived word δ_h has height h and its tree is the 'complete' binary tree of height h.

- The defect of ω is the number of vertices that we need to add to its tree in order to obtain the tree of δ_h.



$[[\gamma_3, \gamma_2], \delta_2]$

The word $[[\gamma_3, \gamma_2], \delta_2]$ has height 4 and defect 14.

A general strategy

A general strategy

If we want to prove a result about outer commutator words, we can:

A general strategy

If we want to prove a result about outer commutator words, we can:

- First, prove it by induction on h for the derived words δ_h.

A general strategy

If we want to prove a result about outer commutator words, we can:

- First, prove it by induction on h for the derived words δ_h.
- Then prove it for general outer commutator words by induction on the defect.

Hall's question

Let ω be a group word, and let G be a group. If ω takes finitely many values in G, is the verbal subgroup $\omega(G)$ finite?

A question of Philip Hall: conciseness

Hall's question

Let ω be a group word, and let G be a group. If ω takes finitely many values in G, is the verbal subgroup $\omega(G)$ finite?

Recall that $\omega(G)$ is the subgroup generated by all values of ω in G.

A question of Philip Hall: conciseness

Hall's question

Let ω be a group word, and let G be a group. If ω takes finitely many values in G, is the verbal subgroup $\omega(G)$ finite?

Recall that $\omega(G)$ is the subgroup generated by all values of ω in G.

Definition

We say that ω is concise if the answer to Hall's question is positive for that word.

A question of Philip Hall: conciseness

Hall's question

Let ω be a group word, and let G be a group. If ω takes finitely many values in G, is the verbal subgroup $\omega(G)$ finite?

Recall that $\omega(G)$ is the subgroup generated by all values of ω in G.

Definition

We say that ω is concise if the answer to Hall's question is positive for that word.

So Hall's question amounts to asking: are all words concise?

Some concise words

Some concise words

The following words are concise:

Some concise words

The following words are concise:

- Words lying outside the commutator subgroup of the free group. (P. Hall, 1950's)

Some concise words

The following words are concise:

- Words lying outside the commutator subgroup of the free group. (P. Hall, 1950's)
- The lower central words γ_i. (P. Hall, 1950's)

Some concise words

The following words are concise:

- Words lying outside the commutator subgroup of the free group. (P. Hall, 1950's)
- The lower central words γ_i. (P. Hall, 1950's)
- The derived words δ_i. (Turner-Smith, 1964)

Some concise words

The following words are concise:

- Words lying outside the commutator subgroup of the free group. (P. Hall, 1950's)
- The lower central words γ_i. (P. Hall, 1950's)
- The derived words δ_i. (Turner-Smith, 1964)
- All outer commutator words. (Jeremy Wilson, 1974)

Some concise words

The following words are concise:

- Words lying outside the commutator subgroup of the free group. (P. Hall, 1950's)
- The lower central words γ_i. (P. Hall, 1950's)
- The derived words δ_i. (Turner-Smith, 1964)
- All outer commutator words. (Jeremy Wilson, 1974)

However, not all words are concise.

Some concise words

The following words are concise:

- Words lying outside the commutator subgroup of the free group. (P. Hall, 1950's)
- The lower central words γ_i. (P. Hall, 1950's)
- The derived words δ_i. (Turner-Smith, 1964)
- All outer commutator words. (Jeremy Wilson, 1974)

However, not all words are concise. For n odd, $n > 10^{10}$, and p a prime, $p > 5000$, the word

$$[[x^{pn}, y^{pn}]^n, y^{pn}]^n$$

is not concise. (Ivanov, 1989)

Bounded conciseness via ultraproducts

Bounded conciseness via ultraproducts

Let ω be a concise word. Is there a function f such that, whenever ω takes m values in a group G, we have $|\omega(G)| \leq f(m)$?

Bounded conciseness via ultraproducts

Let ω be a concise word. Is there a function f such that, whenever ω takes m values in a group G, we have $|\omega(G)| \leq f(m)$?

One can see that the answer is positive by using ultraproducts: if there exists a family $\{G_n\}_{n\in\mathbb{N}}$ of groups such that

- ω takes at most m values in every G_n.
- $\lim_{n\to\infty} |\omega(G_n)| = \infty$.

Bounded conciseness via ultraproducts

Let ω be a concise word. Is there a function f such that, whenever ω takes m values in a group G, we have $|\omega(G)| \leq f(m)$?

One can see that the answer is positive by using ultraproducts: if there exists a family $\{G_n\}_{n\in\mathbb{N}}$ of groups such that

- ω takes at most m values in every G_n.
- $\lim_{n\to\infty} |\omega(G_n)| = \infty$.

Then the ultraproduct U of these groups with respect to a non-principal ultrafilter has at most m values of ω, but $|\omega(U)| = \infty$.

Bounded conciseness via ultraproducts

Let ω be a concise word. Is there a function f such that, whenever ω takes m values in a group G, we have $|\omega(G)| \leq f(m)$?

One can see that the answer is positive by using ultraproducts: if there exists a family $\{G_n\}_{n\in\mathbb{N}}$ of groups such that

- ω takes at most m values in every G_n.
- $\lim_{n\to\infty} |\omega(G_n)| = \infty$.

Then the ultraproduct U of these groups with respect to a non-principal ultrafilter has at most m values of ω, but $|\omega(U)| = \infty$.

However, neither the ultraproduct argument nor Jeremy Wilson's proof provide an explicit expression for the order of $\omega(G)$ when ω is an outer commutator word.

Uniform bounded conciseness of outer commutator words

Theorem A (F-A, Morigi, 2010)

Let ω be an outer commutator word and let G be a group in which ω takes m different values. Then:

Theorem A (F-A, Morigi, 2010)

Let ω be an outer commutator word and let G be a group in which ω takes m different values. Then:

- *If G is soluble, $|\omega(G)| \leq 2^{m-1}$.*

Theorem A (F-A, Morigi, 2010)

Let ω be an outer commutator word and let G be a group in which ω takes m different values. Then:

- *If G is soluble, $|\omega(G)| \leq 2^{m-1}$.*
- *If G is not soluble, $|\omega(G)| \leq [(m-1)(m-2)]^{m-1}$.*

Uniform bounded conciseness of outer commutator words

Theorem A (F-A, Morigi, 2010)

Let ω be an outer commutator word and let G be a group in which ω takes m different values. Then:

- *If G is soluble, $|\omega(G)| \leq 2^{m-1}$.*
- *If G is not soluble, $|\omega(G)| \leq [(m-1)(m-2)]^{m-1}$.*

- Observe that the bounds in Theorem A are independent of the outer commutator word ω. This is why we speak of uniform bounded conciseness.

Uniform bounded conciseness of outer commutator words

Theorem A (F-A, Morigi, 2010)

Let ω be an outer commutator word and let G be a group in which ω takes m different values. Then:

- *If G is soluble, $|\omega(G)| \leq 2^{m-1}$.*
- *If G is not soluble, $|\omega(G)| \leq [(m-1)(m-2)]^{m-1}$.*

- Observe that the bounds in Theorem A are independent of the outer commutator word ω. This is why we speak of uniform bounded conciseness.
- Theorem A does not depend on ultraproducts.

Uniform bounded conciseness of outer commutator words

Theorem A (F-A, Morigi, 2010)

Let ω be an outer commutator word and let G be a group in which ω takes m different values. Then:

- *If G is soluble, $|\omega(G)| \leq 2^{m-1}$.*
- *If G is not soluble, $|\omega(G)| \leq [(m-1)(m-2)]^{m-1}$.*

- Observe that the bounds in Theorem A are independent of the outer commutator word ω. This is why we speak of uniform bounded conciseness.
- Theorem A does not depend on ultraproducts.
- Our proof of Theorem A is also independent of Wilson's result about the conciseness of outer commutator words.

Conciseness in a class of groups

Conciseness in a class of groups

Definition

Let w be a word and let $\mathcal{F}$ be a class of groups.

- ω is concise in $\mathcal{F}$ if whenever ω takes finitely many values in a group $G \in \mathcal{F}$, then $\omega(G)$ is finite.
- ω is boundedly concise in $\mathcal{F}$ if there is a function f such that, whenever ω takes m values in $G \in \mathcal{F}$, we have $|\omega(G)| \leq f(m)$.

Conciseness in a class of groups

Definition

Let w be a word and let $\mathcal{F}$ be a class of groups.

- ω is concise in $\mathcal{F}$ if whenever ω takes finitely many values in a group $G \in \mathcal{F}$, then $\omega(G)$ is finite.
- ω is boundedly concise in $\mathcal{F}$ if there is a function f such that, whenever ω takes m values in $G \in \mathcal{F}$, we have $|\omega(G)| \leq f(m)$.

Open question

Are all words concise in the class of residually finite groups?

Conciseness in a class of groups

Definition

Let w be a word and let $\mathcal{F}$ be a class of groups.

- ω is concise in $\mathcal{F}$ if whenever ω takes finitely many values in a group $G \in \mathcal{F}$, then $\omega(G)$ is finite.
- ω is boundedly concise in $\mathcal{F}$ if there is a function f such that, whenever ω takes m values in $G \in \mathcal{F}$, we have $|\omega(G)| \leq f(m)$.

Open question

Are all words concise in the class of residually finite groups?

Open question

If a word is concise in the class of residually finite groups, is it boundedly concise in that class?

Conciseness in a class of groups

Definition

Let w be a word and let $\mathcal{F}$ be a class of groups.

- ω is concise in $\mathcal{F}$ if whenever ω takes finitely many values in a group $G \in \mathcal{F}$, then $\omega(G)$ is finite.
- ω is boundedly concise in $\mathcal{F}$ if there is a function f such that, whenever ω takes m values in $G \in \mathcal{F}$, we have $|\omega(G)| \leq f(m)$.

Open question

Are all words concise in the class of residually finite groups?

Open question

If a word is concise in the class of residually finite groups, is it boundedly concise in that class?

Note that the ultraproduct argument does not apply in this case.

Uniform bounded conciseness of outer commutator words

Theorem B (F-A, Shumyatsky, 2018)

Let ω be an outer commutator word and let q be a prime-power. Then ω^q is boundedly concise in the class of residually finite groups.

Theorem B (F-A, Shumyatsky, 2018)

Let ω be an outer commutator word and let q be a prime-power. Then ω^q is boundedly concise in the class of residually finite groups.

For $q > 1$, it is not known whether these words are concise in the class of all groups.

Theorem B (F-A, Shumyatsky, 2018)

Let ω be an outer commutator word and let q be a prime-power. Then ω^q is boundedly concise in the class of residually finite groups.

For $q > 1$, it is not known whether these words are concise in the class of all groups.

Our proof of Theorem B relies at some point on methods of Zelmanov for the solution of the Restricted Burnside Problem. This explains the restriction to q being a power of a prime.

A hierarchy in the set of outer commutator words

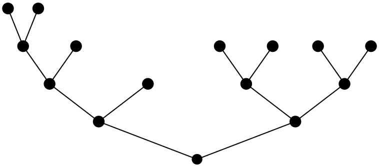
A hierarchy in the set of outer commutator words

Definition

Let φ and ω be two outer commutator words. We say that φ is an extension of ω, if the tree of φ is an upward extension of the tree of ω.

Definition

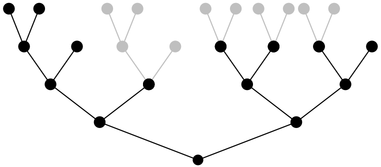
Let φ and ω be two outer commutator words. We say that φ is an extension of ω, if the tree of φ is an upward extension of the tree of ω.



$[\gamma_4, \delta_2]$

Definition

Let φ and ω be two outer commutator words. We say that φ is an extension of ω, if the tree of φ is an upward extension of the tree of ω.



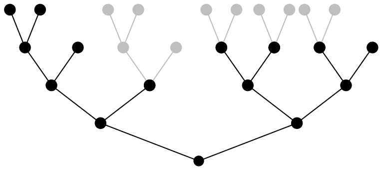
$[\gamma_4, \delta_2]$

An extension of $[\gamma_4, \delta_2]$: $[[\gamma_3, \gamma_3], [\delta_2, \gamma_3]]$.

A hierarchy in the set of outer commutator words

Definition

Let φ and ω be two outer commutator words. We say that φ is an extension of ω, if the tree of φ is an upward extension of the tree of ω.



$[\gamma_4, \delta_2]$

An extension of $[\gamma_4, \delta_2]$: $[[\gamma_3, \gamma_3], [\delta_2, \gamma_3]]$.

Making an extension φ of ω corresponds to replacing some indeterminates of ω by other outer commutator words. Hence every value of φ is also a value of ω.

Definition

Let ω be an outer commutator word, and G a group. An ω-series of G of is a series of verbal subgroups of G,

$$\Phi_{r+1}(G) \leq \cdots \leq \Phi_1(G),$$

where each Φ_i is a finite set of words which are extensions of ω.

Power-closed ω-series

Definition

Let ω be an outer commutator word, and G a group. An ω-series of G of is a series of verbal subgroups of G,

$$\Phi_{r+1}(G) \leq \cdots \leq \Phi_1(G),$$

where each Φ_i is a finite set of words which are extensions of ω.

Every subgroup of an ω-series can be generated by values of ω.

Power-closed ω-series

Definition

Let ω be an outer commutator word, and G a group. An ω-series of G of is a series of verbal subgroups of G,

$$\Phi_{r+1}(G) \leq \cdots \leq \Phi_1(G),$$

where each Φ_i is a finite set of words which are extensions of ω.

Every subgroup of an ω-series can be generated by values of ω.

Definition

An ω-series is power-closed if for every $i = 1, \ldots, r$ and $\varphi \in \Phi_i$, every power of a value of φ in G is a value of ω modulo $\Phi_{i+1}(G)$.

Existence of universal power-closed ω-series

Existence of universal power-closed ω-series

Theorem

Let ω be an outer commutator word of height h. Then there exist sets of words $\Phi_1, \ldots, \Phi_{r+1}$ such that

$$G^{(h)} = \Phi_{r+1}(G) \leq \cdots \leq \Phi_1(G) = \omega(G)$$

is a power-closed ω-series for all groups G.

Theorem

Let ω be an outer commutator word of height h. Then there exist sets of words $\Phi_1, \ldots, \Phi_{r+1}$ such that

$$G^{(h)} = \Phi_{r+1}(G) \leq \cdots \leq \Phi_1(G) = \omega(G)$$

is a power-closed ω-series for all groups G.

Thus the sets of words $\Phi_1, \ldots, \Phi_{r+1}$ are universal, valid for all groups.

Theorem C (Acciari, F-A, Shumyatsky, 2012)

Let G be a finite group, P a Sylow p-subgroup of G, and $m = |G : P|$. Then for every outer commutator word ω, we have

$$P \cap \omega(G) = \langle P \cap S \mid S \text{ is the set of values of } \omega^m \text{ in } G \rangle.$$

A focal subgroup theorem for outer commutator words

Theorem C (Acciarri, F-A, Shumyatsky, 2012)

Let G be a finite group, P a Sylow p-subgroup of G, and $m = |G : P|$. Then for every outer commutator word ω, we have

$$P \cap \omega(G) = \langle P \cap S \mid S \text{ is the set of values of } \omega^m \text{ in } G \rangle.$$

Compare with the usual focal subgroup theorem, which would be the case of $\omega = [x, y]$:

$$P \cap G' = \langle x^{-1}y \mid x, y \in P \text{ are conjugate in } G \rangle.$$

A focal subgroup theorem for outer commutator words

Theorem C (Acciarri, F-A, Shumyatsky, 2012)

Let G be a finite group, P a Sylow p-subgroup of G, and $m = |G : P|$. Then for every outer commutator word ω, we have

$$P \cap \omega(G) = \langle P \cap S \mid S \textit{ is the set of values of } \omega^m \textit{ in } G \rangle.$$

Compare with the usual focal subgroup theorem, which would be the case of $\omega = [x, y]$:

$$P \cap G' = \langle x^{-1}y \mid x, y \in P \text{ are conjugate in } G \rangle.$$

Open question

Under the hypotheses of Theorem C, do we have the following?

$$P \cap \omega(G) = \langle P \cap S \mid S \text{ is the set of values of } \omega \text{ in } G \rangle.$$

Work in progress: cyclic coverings of word values

Work in progress: cyclic coverings of word values

A group that can be covered by cyclic subgroups is finite or cyclic.

Work in progress: cyclic coverings of word values

A group that can be covered by cyclic subgroups is finite or cyclic.

Theorem

Assume that all values of an outer commutator word ω are covered by finitely many cyclic subgroups.

Work in progress: cyclic coverings of word values

A group that can be covered by cyclic subgroups is finite or cyclic.

Theorem

Assume that all values of an outer commutator word ω are covered by finitely many cyclic subgroups.

- *If $\omega = [x, y]$ then G' is either finite or cyclic.*
  (F-A, Shumyatsky, 2007)

Work in progress: cyclic coverings of word values

A group that can be covered by cyclic subgroups is finite or cyclic.

Theorem

Assume that all values of an outer commutator word ω are covered by finitely many cyclic subgroups.

- *If $\omega = [x, y]$ then G' is either finite or cyclic. (F-A, Shumyatsky, 2007)*
- *If $\omega = \gamma_i$ with $i \geq 3$, then $\gamma_i(G)$ is finite-by-cyclic. (Cutolo, Nicotera, 2010)*

Work in progress: cyclic coverings of word values

A group that can be covered by cyclic subgroups is finite or cyclic.

Theorem

Assume that all values of an outer commutator word ω are covered by finitely many cyclic subgroups.

- *If $\omega = [x, y]$ then G' is either finite or cyclic.*
  (F-A, Shumyatsky, 2007)
- *If $\omega = \gamma_i$ with $i \geq 3$, then $\gamma_i(G)$ is finite-by-cyclic.*
  (Cutolo, Nicotera, 2010)
- *If $\omega = \delta_2$ then G'' is finite-by-cyclic.*
  (Acciarri, F-A, Morigi, 2018)

Work in progress: cyclic coverings of word values

A group that can be covered by cyclic subgroups is finite or cyclic.

Theorem

Assume that all values of an outer commutator word ω are covered by finitely many cyclic subgroups.

- *If $\omega = [x, y]$ then G' is either finite or cyclic.*
  (F-A, Shumyatsky, 2007)
- *If $\omega = \gamma_i$ with $i \geq 3$, then $\gamma_i(G)$ is finite-by-cyclic.*
  (Cutolo, Nicotera, 2010)
- *If $\omega = \delta_2$ then G'' is finite-by-cyclic.*
  (Acciarri, F-A, Morigi, 2018)

Open question

In the conditions of the last theorem, is $\omega(G)$ finite-by-cyclic for all outer commutator words?