Regular Bipartite Divisor Graph

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Outline

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- Preliminary Results on Finite Groups

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- Groups whose bipartite divisor graphs are cycles

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- Groups whose bipartite divisor graphs are regular

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- Bipartite divisor graph $B(G)$ is an undirected bipartite graph with vertex set $\rho(X) \cup (cd(X) \setminus \{1\})$; there is an edge between vertices p of $\rho(G)$ and m of $cd(G) \setminus \{1\}$ if p divides m.

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Theorem

Let G be a solvable group and assume that G' is the unique minimal normal subgroup of G. Then all nonlinear irreducible characters of G have equal degree f and one of the following situations obtains:

(1) G *is a* p*-group,* $Z(G)$ *is cyclic and* $\frac{G}{Z(G)}$ *is elementray abelian group of order* f^2.

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Let $K \lhd G$ such that $\frac{G}{K}$ is a Frobenius group with Frobenius kernel $\frac{N}{K}$, an elementary abelian p-group. Let $\psi \in \mathrm{Irr}(N)$. Then one of the following holds:

(1) $[G : N]\psi(1) \in \mathrm{cd}(G)$;

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Lemma

Let G *be a finite group whose* $B(G)$ *is a cycle of length* $n \geqslant 6$*. Then both* $\Delta(G)$ *and* $\Gamma(G)$ *are cycles.*

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(ii) *If G is solvable, then either it is a group of type one mentioned by Lewis or for a prime p either $G \simeq P \times A$, where P is a nonabelian p-group and A is abelian, or G has an abelian normal subgroup of index a power of p.*

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Let G *be a solvable group whose* $B(G)$ *is a 3-regular graph. If at least one of* $\Delta(G)$ *or* $\Gamma(G)$ *is not complete, then* $\Delta(G)$ *is neither 2-regular, nor 3-regular.*

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Let G be a solvable group whose $B(G)$ is a 3-regular graph. If $\Delta(G)$ is regular, then it is a complete graph. Furthermore, if $\Gamma(G)$ is not complete, then $\Delta(G)$ is isomorphic with K_n, for $n \geqslant 5$.

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- $n(B(G)) = 1$.
- $\Delta(G)$ is a non-complete regular graph, $\Rightarrow G \simeq \prod M_i$, where for each i, $M_i = P_i Q_i$ with $P_i \in Syl_{p_i}(G)$ is normal nonabelian, and $Q_i \in Syl_{q_i}(G)$ is not normal in G, (D. M. Kasyoki, 2013).

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Let G be a solvable group whose $B(G)$ is a 3-regular graph. If $\Delta(G)$ is regular, then it is a complete graph. Furthermore, if $\Gamma(G)$ is not complete, then $\Delta(G)$ is isomorphic with K_n, for $n \geqslant 5$.

- $n(B(G)) = 1$.
- $\Delta(G)$ is a non-complete regular graph, $\Rightarrow G \simeq \prod M_i$, where for each i, $M_i = P_i Q_i$ with $P_i \in Syl_{p_i}(G)$ is normal nonabelian, and $Q_i \in Syl_{q_i}(G)$ is not normal in G, (D. M. Kasyoki, 2013). Contradicts three regularity of $B(G)$.$\Rightarrow \Delta(G)$ is a complete graph

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bibliography

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