Prosolvable Retracts in Free Profinite Products

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Dedication

To Zvi Arad.

Be strong and of a good courage; be not afraid, neither be thou dismayed: for the Lord thy God is with thee whithersoever thou goest.

Josh. 1,9

A Question

Question

Given a free profinite product $G := A \amalg B$ of finite solvable groups A and B. Is there a closed subgroup H of G isomorphic to the prosolvable free product $A \amalg_s B$?

Subgroups of Free Profinite Products

1971,1973 Binz-Neukirch-Wenzel and more generally Gildenhuys-Ribes provide a Kurosh Subgroup Theorem for open subgroups.

1987 D. Haran and independently O.V. Mel'nikov introduce free products over a continuously indexed space of groups and describe topologically finitely generated subgroups.

1988 Mel'nikov-Zalesskii classify solvable subgroups of profinite groups acting on profinite trees.

1989 Herfort-Ribes embed dicyclic Frobenius groups in the prosolvable case.

1990,1995 P. A. Zalesskii provides a profinite version of the Kurosh Subgroup Theorem for open subgroups and for normal subgroups.

From Where to Go?

Theorem (Pop, 1995)

Let $G = \coprod_{i \in I} G_i$ be free profinite product of profinite groups and F be prosolvable subgroup. Then one of the following is true:

(i) *For a prime p and all $i \in I$ and $g \in G$ the intersection $F \cap G_i^g$ is pro-p;*

(ii) *All nontrivial intersections $F \cap G_i^g$ for some $i \in I$ and $g \in G$ are finite and conjugate in F;*

(iii) *F, up to conjugacy, is contained in some G_i.*

Immediate Remarks

(R.1) (i) $\Rightarrow$ If $A \amalg_s B$ appears as a section of $A \amalg B$ then A and B are both p-groups.

(R.2) The fact that (ii) *can* appear, was proved by Guralnick-Haran 2011, using CFSG in order to embed dicyclic profinite Frobenius groups in $A \amalg B$.
Is this the only possibility?

Free Products of Pro-p Groups

Theorem

The free profinite product $G = \coprod_{i \in I} G_i$ of pro-p groups G_i admits a prosolvable retract $\coprod_{i \in I, s} G_i$.

Question

Is every prosolvable subgroup of $A \amalg B$ isomorphic to a closed subgroup of $A \amalg_s B$?

We believe the answer is 'YES'!

A and B *not* Pro-p

Theorem

Let A be a finite p-group and $B \cong \hat{Z}$. Then $G = A \amalg B$ admits a prosolvable retract $A \amalg_s \hat{Z}$.

Question

Can B be replaced by an arbitrary free profinite group?

Limiting Case (ii) in Pop's Theorem

Theorem

Let $G = \coprod_{i \in I} G_i$ be free profinite product of profinite groups and F an infinite closed prosolvable subgroup of G satisfying (ii) in Pop's Theorem. Assume $\pi(F \cap G_i^g) \geq 2$.
Then F is a profinite Frobenius group with procyclic kernel and finite cyclic complement.

Danke – Çok teşekkürler – grazie

A Technicality

Theorem

Let p be a fixed prime and the profinite group G be generated by a family $\mathcal{A}$ of pro-p subgroups. Assume that for a normal subgroup K and all $x \in G$ the equality $AK \cap B^x K = K$ is valid whenever $A \neq B$ both belong to $\mathcal{A}$.
Then there is a function $g : \mathcal{A} \to G$ such that $H(g) := \langle A^{g(A)} : A \in \mathcal{A} \rangle$ is prosolvable and $G = H(g) R_s(G)$.


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