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# Representation Growth of Special Linear Groups (with N. Budur)

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March 23, 2018

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# Representation zeta function

## Definition

Let $G$ be a topological group.

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# Representation zeta function

## Definition

Let $G$ be a topological group. For $n \in \mathbb{N}$, we denote by $r_n(G)$ the number of isomorphism classes of continuous $n$-dimensional irreducible complex representations of $G$.

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## Definition

Let $G$ be a topological group. For $n \in \mathbb{N}$, we denote by $r_n(G)$ the number of isomorphism classes of continuous $n$-dimensional irreducible complex representations of $G$.

Assume $r_n(G)$ is finite for all $n \in \mathbb{N}$.

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## Definition

Let $G$ be a topological group. For $n \in \mathbb{N}$, we denote by $r_n(G)$ the number of isomorphism classes of continuous $n$-dimensional irreducible complex representations of $G$.

Assume $r_n(G)$ is finite for all $n \in \mathbb{N}$. We define the *representation zeta function* of $G$ as,

$$\zeta_G(s) = \sum_{n=1}^{\infty} r_n(G) n^{-s} \quad (s \in \mathbb{C}).$$

# Asymptotic behaviour

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**Definition**

The *abscissa of convergence* $\alpha_G$ of the series $\zeta_G(s)$ is the infimum of all $\alpha \in \mathbb{R}$ such that $\zeta_G(s)$ converges on the complex half-plane $\{s \in \mathbb{C} \mid \Re(s) > \alpha\}$.

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# Asymptotic behaviour

## Definition

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We define

$$R_N(G) = \sum_{n=1}^{N} r_n(G) \text{ for } N \in \mathbb{N},$$

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# Asymptotic behaviour

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We define

$$R_N(G) = \sum_{n=1}^{N} r_n(G) \text{ for } N \in \mathbb{N},$$

The abscissa of convergence is such that

$$\lim_{N\to\infty} \frac{\log R_N(G)}{\log N} = \alpha_G.$$

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We shall consider the following situation:

- $\mathbf{G}$ is a semisimple algebraic group defined over $\mathbb{Q}$.

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We shall consider the following situation:

- $\mathbf{G}$ is a semisimple algebraic group defined over $\mathbb{Q}$.
- $K$ is a non-archimedean local field containing $\mathbb{Q}$.

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We shall consider the following situation:

- $\mathbf{G}$ is a semisimple algebraic group defined over $\mathbb{Q}$.
- $K$ is a non-archimedean local field containing $\mathbb{Q}$.
- $\Gamma \leq \mathbf{G}(K)$ compact and open.

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For instance $\Gamma = \mathrm{SL}_d(\mathbb{Z}_p)$.

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$\geq 1/15$ Larsen, Lubotzky

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Let $d$ be a positive integer. Let $K$ be a non-archimedean local field containing $\mathbb{Q}$. Let $\Gamma$ be a compact open subgroup of $\mathrm{SL}_d(K)$.

**Theorem (Aizenbud, Avni, 2013)**

$$\alpha_\Gamma < 22.$$

# Bounds for special linear groups

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**Theorem (Aizenbud, Avni, 2013)**

$$\alpha_\Gamma < 22.$$

**Theorem (Budur, Z. 2017)**

$$\alpha_\Gamma < 2.$$

# Representation varieties

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Let

$$\pi_n = \langle g_1, \ldots, g_n, h_1, \ldots, h_n \mid g_1 h_1 g_1^{-1} h_1^{-1} \cdots g_n h_n g_n^{-1} h_n^{-1} = 1 \rangle$$

be the fundamental group of a compact Riemann surface of genus $n$.

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# Representation varieties

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$$\pi_n = \langle g_1, \ldots, g_n, h_1, \ldots, h_n \mid g_1 h_1 g_1^{-1} h_1^{-1} \cdots g_n h_n g_n^{-1} h_n^{-1} = 1 \rangle$$

be the fundamental group of a compact Riemann surface of genus $n$.

## Definition

Let $A$ be a $\mathbb{Q}$-algebra. We define the $\mathbf{G}$-*representation variety* $R(n, \mathbf{G})$ of $\pi_n$ to be the $\mathbb{Q}$-scheme defined by

$$R(n, \mathbf{G})(A) = \mathrm{Hom}(\pi_n, \mathbf{G}(A)).$$

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# Aizenbud's and Avni's bound

## Theorem (Aizenbud, Avni)

*The following are equivalent:*

1. $\alpha_\Gamma < 2n - 2$.

# Aizenbud's and Avni's bound


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**Theorem (Aizenbud, Avni)**

*The following are equivalent:*

1. $\alpha_\Gamma < 2n - 2$.
2. *The representation variety* $R(n, \mathbf{G})$ *has rational singularities.*

# Aizenbud's and Avni's bound


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## Theorem (Aizenbud, Avni)

*The following are equivalent:*

1. $\alpha_\Gamma < 2n - 2$.
2. *The representation variety* $R(n, \mathbf{G})$ *has rational singularities.*

## Proposition

*Let* $X$ *have rational singularities. Let* $X^{sm}$ *be the smooth locus of* $X$ *and and let* $\omega$ *be a top-dimensional differential form on* $X^{sm}$*. Then for any* $A \subset X$*, the integral* $m(A) = \int_{A \cap X^{sm}} |\omega|$ *defines a Radon measure (so finite on compact subsets).*

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We define the set of *open* $\Gamma$-representations of $\pi_n$ as

$$R(n,\Gamma)^o = \left\{ \rho \in \mathrm{Hom}(\pi_n,\Gamma) \mid \overline{\rho(\pi_n)} \text{ open in } \Gamma \right\}.$$

# Open representations

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**Remark**

The $\Gamma$-orbit space $R(n,\Gamma)^o/\Gamma$ is a $K$-analytic manifold and admits a volume form $v_\Gamma$.

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**Theorem (Aizenbud , Avni)**

*Let* $\mathbf{G}$ *be a connected, simply connected, semi-simple algebraic group over* $\mathbb{Q}$*. Let* $K$ *be a non-archimedean local field containing* $\mathbb{Q}$*. Let* $\Gamma$ *be a compact open subgroup of* $\mathbf{G}(K)$*. Then there exists a non-zero constant* $c_\Gamma$ *such that*

$$\int_{R(n,\Gamma)^o/\Gamma} |v_\Gamma| = c_\Gamma \cdot \zeta_\Gamma(2n-2)$$

*for* $n \geq 2$*, if any the two sides of the equation is finite.*

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How can we use this?

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*for* $n \geq 2$*, if any the two sides of the equation is finite.*

How can we use this? We need to “view” $R(n,\Gamma)^o/\Gamma$ “inside” a compact subset of a variety with rational singularities.

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## Definition

The **G**-character variety is

$$M(n, \mathbf{G}) = R(n, \mathbf{G}) /\!\!/ \mathbf{G}.$$

($/\!\!/$ is the G.I.T. quotient).

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The **G**-character variety is

$$M(n, \mathbf{G}) = R(n, \mathbf{G}) /\!\!/ \mathbf{G}.$$

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## Theorem (Bellamy, Schedler)

*$M(n, \mathrm{SL_d})_{\mathbb{C}}$ is a complex variety with rational singularities if $n \geq 1$.*

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## Definition

The **G**-character variety is

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($/\!\!/$ is the G.I.T. quotient).

## Theorem (Bellamy, Schedler)

*$M(n, \mathrm{SL_d})_{\mathbb{C}}$ is a complex variety with rational singularities if $n \geq 1$.*

## Lemma (Budur, Z.)

*If $n \geq 1$, then $M(n, \mathrm{SL}_d)$ has rational singularities over $\mathbb{Q}$.*

# $R(n,\Gamma)^o/\Gamma$ inside $M(n,\mathbf{G})(K)$

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Define

$$R(n,\mathrm{SL}_d)^o$$

to be the Zariski open set of $R(n,\mathrm{SL}_d)$ such that $R(n,\mathrm{SL}_d)^o(\overline{K})$ consists of representation with Zariski dense image in $\mathrm{SL}_d(\overline{K})$.

# $R(n,\Gamma)^o/\Gamma$ inside $M(n,\mathbf{G})(K)$

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to be the Zariski open set of $R(n,\mathrm{SL}_d)$ such that $R(n,\mathrm{SL}_d)^o(\overline{K})$ consists of representation with Zariski dense image in $\mathrm{SL}_d(\overline{K})$. We define

$$M(n,\mathrm{SL}_d)^o(K) = R(n,\mathrm{SL}_d)^o(K)/\mathrm{SL}_d(K)$$

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- $M(n, \mathrm{SL}_d)^o(K)$ is contained in the smooth locus of $M(n, \mathrm{SL}_d)(K)$.

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- $M(n, \mathrm{SL}_d)^o(K)$ is contained in the smooth locus of $M(n, \mathrm{SL}_d)(K)$.
- Representation growth does not change when passing to a finite index subgroup.

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# Proof of main result

- $M(n, \mathrm{SL}_d)^o(K)$ is contained in the smooth locus of $M(n, \mathrm{SL}_d)(K)$.
- Representation growth does not change when passing to a finite index subgroup. So we assume $\Gamma$ uniform.

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- $M(n, \mathrm{SL}_d)^o(K)$ is contained in the smooth locus of $M(n, \mathrm{SL}_d)(K)$.
- Representation growth does not change when passing to a finite index subgroup. So we assume $\Gamma$ uniform.
- There is a natural map

$$q : R^o(n, \Gamma)/\Gamma \to M^o(n, \mathrm{SL}_d)(K).$$

# Proof of main result


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with fibres of bounded size if $\Gamma$ is uniform

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with fibres of bounded size if $\Gamma$ is uniform and étale onto its image.

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- The quotient $M^o(n, \mathrm{SL}_d)(K)$, has a $K$ analytic structure and admits a volume form $v_{\mathrm{SL}_d}$.

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- The quotient $M^o(n, \mathrm{SL}_d)(K)$, has a $K$ analytic structure and admits a volume form $v_{\mathrm{SL}_d}$.
- $v_\Gamma$ is the pull-back of $v_{\mathrm{SL}_d}$ through $q$.

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- The quotient $M^o(n, \mathrm{SL}_d)(K)$, has a $K$ analytic structure and admits a volume form $v_{\mathrm{SL}_d}$.
- $v_\Gamma$ is the pull-back of $v_{\mathrm{SL}_d}$ through $q$.
- The image $q(R^o(n, \Gamma)/\Gamma)$ is contained in a compact subset of $M^o(n, \mathrm{SL}_d)(K)$ and therefore has finite volume.

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- The quotient $M^o(n, \mathrm{SL}_d)(K)$, has a $K$ analytic structure and admits a volume form $v_{\mathrm{SL}_d}$.
- $v_\Gamma$ is the pull-back of $v_{\mathrm{SL}_d}$ through $q$.
- The image $q(R^o(n,\Gamma)/\Gamma)$ is contained in a compact subset of $M^o(n, \mathrm{SL}_d)(K)$ and therefore has finite volume. So $R^o(n,\Gamma)/\Gamma$ has too (for $n \geq 2$).