ON NON ABELIAN TENSOR ANALOGUES OF 4-ENGEL GROUPS

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Abstract

In this paper we study some properties of 4⊗-Engel elements of groups. In particular, we prove that G is a 4⊗-Engel group if and only if the normal closer of every element in G is a 3⊗-Engel group.

1. Introduction.

For any group G, the nonabelian tensor square $G \otimes G$ is a group generated by the symbols g ⊗ h, subject to the relations:

$gg \otimes h = (g^g \otimes h^g)(g \otimes h)$ **and**

$g \otimes hh = (g \otimes h)(g^h \otimes h^h)$, **where g, g ,h, h∈G and** $g^h = h^{-1}gh$.

The more general concept of nonabelian tensor product of groups acting on each other in certain compatible way was introduced by R. Brown and J.-L. Loday in [5], following the ideas of R. K. Dennis [6]. Also, tensor analogues of right n-Engel elements have been defined. Recall that the set of right n-Engel elements of a group G is defined by $R_n(G) = \{a \in G: [a, nx] = 1, for\ all\ x \in G\}$. **Here** $[a, nx]$ **stands for the commutator** $[\cdots[[a, x], x], \cdots]$ **with n copies of x. It is well-known that** $R_1(G) = Z(G)$ **and that** $R_2(G)$ **is a subgroup of G . The set of right (left)** $n_\otimes$**-Engel elements of a group G is then defined as** $R_n^{\otimes}(G) = \{a \in G: [a, n-1x] \otimes x = 1_\otimes\ for\ all\ x \in G\}$.**and** $Ln^{\otimes}(G) = \{a \in G: [x, n-1a] \otimes a = 1_\otimes for\ all\ x \in G\}$ **respectivly.**

2. Results.

Lemma2.1 *Let g, g', h, h' ∈G. The following relations hold in G⊗G*

a) $(g^{-1} \otimes h)^g = (g \otimes h)^{-1} = (g \otimes h^{-1})^h$.
b) $(g' \otimes h')^{g \otimes h} = (g' \otimes h')^{[g,h]}$.
c) $[g, h] \otimes g' = (g \otimes h)^{-1}(g \otimes h)^{g'}$.
d) $g' \otimes [g, h] = (g \otimes h)^{-g'}(g \otimes h)$.
e) $[g, h] \otimes [g', h'] = [g \otimes h, g' \otimes h']$.

Note here that G acts on $G \otimes G$ **by** $(g \otimes h)^{g'} = g^{g'} \otimes h^{g'}$ **.**

Proposition 2.2 *For a given group G there exists a homomorphism* $k: G \otimes G \to G$ *such that* $k: g \otimes h \to [g, h]$ *Moreover,* $ker\ k \leq Z(G \otimes G)$ *and G acts trivially on ker k*

Theorem 2.3. *Let G be any group, then we have* $R_4^{\otimes}(G) \subseteq R_4(G)$ *and* $L_4^{\otimes}(G) \subseteq L_4(G)$. **More generally** $R_n^{\otimes}(G) \subseteq R_n(G)$ *and* $L_n^{\otimes}(G) \subseteq L_n(G)$ **for all** $n \in \mathbb{N}^*$

Theorem 2.4. *A group G is a* $3_\otimes$*-Engel groupe if and only if the normal closer of every element in G is a* $2_\otimes$*-Engel group.*

Theorem 2.5. *A group G is a* $4_\otimes$*-Engel groupe if and only if the normal closer of every element in G is a* $3_\otimes$*-Engel group.*