Abstracts

COPRIME AUTOMORPHISMS OF FINITE GROUPS

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Let G be a finite group admitting a coprime automorphism α. There is a plenty of results in literature illustrating the strong influence that the centralizer $C_G(\alpha)$ has over the structure of G, including Thompson's Theorem that if α has prime order and $C_G(\alpha) = 1$, then G is nilpotent.

Denote by $I_G(\alpha)$ the set of commutators $g^{-1}g^{\alpha}$, where $g \in G$, and by $[G, \alpha]$ the subgroup generated by $I_G(\alpha)$. Since $|G| = |C_G(\alpha)||I_G(\alpha)|$, there is a kind of vague duality between $C_G(\alpha)$ and $I_G(\alpha)$. In the talk we will discuss some results with the intent of showing that also properties of $I_G(\alpha)$ strongly impact the structure of G.
Based on a joint work with R. M. Guralnick and P. Shumyatsky.

References

[1] C. Acciarri, R. M. Guralnick and P. Shumyatsky, Coprime automorphisms of finite groups, *Transactions of the American Mathematical Society*, Vol. 375 N. 7 (2022), 4549–4565.

ELEMENTARY ABELIAN REGULAR SUBGROUPS ON $\mathrm{Sym}(\mathbf{2^n})$

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In a recent paper [1], motivated by a variant of the differential cryptanalysis on block ciphers exploiting conjugacy class of the translation subgroup of $\mathrm{Sym}(V)$ [2], where $V = \mathbb{F}_2^n$, we studied the families of second-maximal-intersection subgroups of $\mathrm{Sym}(V)$, i.e. the families of elementary abelian regular subgroups of $\mathrm{Sym}(V)$ that intersect the image σ_V of the right regular representation σ in a second-maximal subgroup of σ_V. In this talk, we show that each second-maximal-intersection subgroup is contained in $N_{\mathrm{Sym}(V)}(\sigma_V) = \mathrm{AGL}(V)$. Moreover, every Sylow 2-subgroup of $\mathrm{AGL}(V)$ contains one and only one second-maximal-intersection subgroup as a normal subgroup. As a consequence, we conclude that $|N_{\mathrm{Sym}(V)}(\Sigma) : \Sigma| = 2$, where Σ is a Sylow 2-subgroup of $\mathrm{AGL}(V)$.

References

[1] R. Aragona, R. Civino, N. Gavioli, C. M. Scoppola, *Regular subgroups with large intersection*, Annali di Matematica Pura ed Applicata 198(6), pp. 2043–2057, 2019.

[2] R. Civino, C. Blondeau, M. Sala, *Differential attacks: using alternative operations*, Designs Codes Cryptography 87(2–3), pp. 225–247, 2019.

*Joint work with Roberto Civino, Norberto Gavioli, Carlo Maria Scoppola.

GRADING SWITCHING FOR MODULAR NON-ASSOCIATIVE ALGEBRAS

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In the talk we describe a *grading switching* for arbitrary non-associative algebras of prime characteristic p, aimed at producing a new grading of an algebra from a given one. This is inspired by a fundamental tool in the classification theory of modular Lie algebras known as *toral switching*, which relies on a suitable adaptation of the exponential of a derivation. We trace the development of grading switching, from an early version (due to S. Mattarei) based on taking the Artin-Hasse exponential of a nilpotent derivation, to a more general version which uses certain generalized Laguerre polynomials playing the role of generalized exponentials. Both versions depend on the existence of appropriate analogues of the functional equation $e^x \cdot e^y = e^{x+y}$ for the classical exponential.

*Joint work with Sandro Mattarei.

LARGE CHARACTERISTICALLY SIMPLE SECTIONS OF FINITE GROUPS

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The main aim of this talk is to show that if G is a finite group for which there are k non-Frattini chief factors isomorphic to a characteristically simple group A, then G has a normal section C/R that is the direct product of k minimal normal subgroups of G/R isomorphic to A. This is a significant extension of the notion of crown for isomorphic chief factors.

New bounds for the number of maximal subgroups of a given index of a finite group are also exhibited.

*Joint work with Ramón Esteban-Romero and Paz Jiménez-Seral.

ON ELEMENTS IN ALGEBRAS HAVING FINITE NUMBER OF CONJUGATES

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Let $U(R)$ be the group of units of an associative ring R with unity. Let

$$\Delta U = \{a \in U(R) \mid [U(R) : C_{U(R)}(a)] < \infty\}$$

be the FC-radical of $U(R)$ and let

$$\nabla(R) = \{a \in R \mid [U(R) : C_{U(R)}(a)] < \infty\}$$

be the FC-subring of R.

The investigation of the FC-radical ΔU and the FC-subring $\bigtriangledown(R)$ has been initiated by *H. Zassenhaus* (see [5, 6]). In [5, 6] the FC-subring of $\mathbb{Z}$-order as a unital ring with a finite $\mathbb{Z}$-basis and a semisimple quotient ring has been described.

An infinite subgroup H of $U(R)$ is said to be an ω-subgroup if the left annihilator of each nonzero Lie commmutator $[x, y]$ in R contains only a finite number of elements of the form $1 - h$, where $x, y \in R$ and $h \in H$.

In the case when R is an algebra over a field F and $U(R)$ contains an ω-subgroup, we describe its FC-subalgebra and its FC-radical (see [2]). This is a generalization of results in [1, 3, 4, 5, 6].

References

[1] V. Bovdi. Twisted group rings whose units form an FC-group. *Canad. J. Math.*, 47(2):274–289, 1995.

[2] V. Bovdi. On elements in algebras having finite number of conjugates. *Publ. Math. Debrecen*, 57(1-2):231–239, 2000.

[3] G. H. Cliff and S. K. Sehgal. Group rings whose units form an FC-group. *Math. Z.*, 161(2):163–168, 1978.

[4] M. A. Dokuchaev, S. O. Juriaans, C. Polcino Milies, and M. L. S. Singer. Finite conjugacy in algebras and orders. *Proc. Edinb. Math. Soc.* (2), 44(1):201–213, 2001.

[5] S. K. Sehgal and H. Zassenhaus. On the supercentre of a group and its ring theoretic generalization. In *Integral representations and applications* (*Oberwolfach*, 1980), volume 882 of *Lecture Notes in Math.*, pages 117–144. Springer, Berlin, 1981.

[6] S. K. Sehgal and H. J. Zassenhaus. Group rings whose units form an FC-group. *Math. Z.*, 153(1):29–35, 1977.

COVERINGS OF FINITE p-GROUPS BY CONJUGACY CLASSES OF CYCLIC SUBGROUPS

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A covering of a group G is a set of proper subgroups whose union is G. In 2017 I was asked a question about coverings of finite p-groups by conjugacy classes of cyclic subgroups. In particular, does the number of conjugacy classes in such a covering necessarily grow with the order of the group? Clearly this is not true when $p = 2$, consider the dihedral 2-groups. We can now answer the question for odd p. I will talk about how we reached the answer and how we were led to the infinite world of pro-p groups.

*Joint work with Yiftach Barnea, Mariagrazia Bianchi, Mikhail Ershov, Mark L. Lewis & Emanuele Pacifici.

USING GALOIS FIELDS TO CONSTUCT FINITE p-GROUPS

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I hope to describe how Galois fields can be used to construct:
first Camina p-groups P (with $[P, x] = P'$ for every element $x \in P - P'$) [1];
then semidirect products of an extraspecial normal subgroup by a cyclic group [2];
and finally (in response to a question asked by D. MacHale, and prompted by an example due to R. Heffernan) finite soluble groups G with $|\text{Aut } G| < |G|$.

References

[1] R. Dark and C.M. Scoppola, *On Camina groups of prime power order*, J. Algebra 181, pp. 787–802, 1996.

[2] R. Dark, A.D. Feldman and M.D. Pérez-Ramos, *Extraspecially irreducible groups*, Adv. Group Theory and Appl. 2, pp. 31–65, 2016.

*Joint work with A.D. Feldman and M.D. Pérez-Ramos.

SOME TOPICS IN THE THEORY OF f-SUBNORMAL SUBGROUPS

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A subgroup H of a group G is called *f-subnormal* in G if there is a finite chain of subgroups

$$H = H_0 \leq H_1 \leq \ldots \leq H_n = G \tag{1}$$

such that either $|H_{i+1} : H_i|$ is finite or H_i is normal in H_{i+1}, for $0 \leq i \leq n-1$. In this case we call the chain of subgroups (1) an *f-subnormal series*. If $n = 1$, then we say H is *f-normal* in G.

This generalization of subnormality was introduced by Phillips [1].

In this talk I will introduce some of the basic theory concerning such subgroups and will also give some recent results concerning them that have been obtained in the past few years, presenting joint work with Maria Ferrara and Marco Trombetti.

References

[1] R. E. Phillips, *Some generalizations of normal series in infinite groups*, J. Austral. Math. Soc. **14** (1972), 496–502.

*Joint work with Maria Ferrara and Marco Trombetti.

EMBEDDING FINITE SIMPLE GROUPS INTO LARGER ONES AS SUBGROUPS WITH TRIVIAL CENTRALIZERS

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The following problem is asked by B. Hartley in [2, Problem 3.15].

Problem. Let $F \cong PSL_m(q)$ be a finite simple group and suppose that $F \leq G \cong PSL_n(q)$ where q is a power of the prime p. If $C_G(F) = 1$, does it follow that n is bounded in terms of m?

In [2], several versions of the problem are discussed. In this talk, we answer them negatively. In particular we construct a counter example and prove that n can be only bounded by p.

Theorem. [1] ***(E.-Falcone)*** *For any odd n and for any $p \geq n$ with $q = p^k$ for some k, there is an embedding $\varphi_n : PSL_2(p) \longrightarrow PSL_n(q)$ such that $C_{PSL_n(q)}(\varphi_n(PSL_2(p))) = 1$.*

In our talk we also discuss some other versions and some consequences of this theorem about other problems on centralizers in simple locally finite groups.

This is a joint work with G. Falcone from University of Palermo. The speaker thanks Alexander von Humboldt Stiftung PSI Program for the support.

References

[1] K. Ersoy, G. Falcone, “Simple groups embedded as subgroups with trivial centralizer”, in preparation.

[2] B. Hartley, “Simple Locally Finite Groups”, in *Finite and Locally Finite Groups* , Kluwer Academic, Dordrecht, 1995, 1-44.

HAUSDORFF DIMENSION OF SOME GROUPS OF AUTOMORPHISMS OF ROOTED TREES

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Hausdorff dimension is a way of measuring the relative size of a subset in a metric space that has been applied, for example, to measure fractal sets. More recently, it has also been considered in the setting of (countably based) profinite groups. In this talk, we survey results about the Hausdorff dimension of (the topological closure of) some distinguished families of subgroups of the group of automorphisms of a rooted tree.

References

[1] G.A. Fernández-Alcober and A. Zugadi-Reizabal, GGS-groups: order of congruence quotients and Hausdorff dimension, *Transactions of the American Mathematical Society* **366** (2014), pp. 1993–2017.

[2] G.A. Fernández-Alcober, S. Gül, and A. Thillaisundaram, The congruence quotients of multi-EGS groups, preprint.

*Joint work with S. Gül, A. Thillaisundaram, and A. Zugadi-Reizabal.

A CHAIN OF NORMALIZERS AND PARTITIONS AND A MODULAR IDEALIZER CHAIN

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In recent papers [1, 2, 3] we studied the conjugacy class of a regular elementary abelian subgroup T of the symmetric group $\mathrm{Sym}(2^n)$. In particular we computed, via GAP software package, the chain N_i of normalizers in a Sylow 2-subgroup Σ_n of $\mathrm{Sym}(2^2)$ defined iteratively starting from T. We noticed that the indices $|N_i : N_{i-1}|$ are equal to 2^{b_i}, where $b_i = \sum_{j\leq i} a_j$ is the partial sum of the sequence a_j of the number partitions of j in at least two distinct parts. In this talk we present some techniques developed in order to prove this result, including the notion of a special family of elements of Σ_n called rigid commutators. Finally, some generalizations to Lie algebras are given.

References

[1] Riccardo Aragona, Roberto Civino, Norberto Gavioli, Carlo Maria Scoppola (2022), *Unrefinable partitions into distinct parts in a normalizer chain*, Discrete Mathematics Letters, vol. 8, p. 72-77.

[2] Riccardo Aragona, Roberto Civino, Norberto Gavioli, Carlo Maria Scoppola (2021), *A Chain of Normalizers in the Sylow 2-subgroups of the symmetric group on 2^n letters*, Indian Journal of Pures & Applied Mathematics, vol. 52, p. 735-746.

[3] Riccardo Aragona, Roberto Civino, Norberto Gavioli, Carlo Maria Scoppola (2021), *Rigid commutators and a normalizer chain*, Monatshefte für Mathematik, vol. 196, p. 431-455.

*Joint work with R. Aragona, R. Civino, C. M. Scoppola.

HALL CLASSES OF GROUPS

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If N is a nilpotent normal subgroup of a group G, the structure of the factor group G/N' has a strong influence on the whole group G. The first evidence of this phenomenon can be traced back to a classical theorem of Philip Hall which states that if G/N' is nilpotent, then G itself is nilpotent. A group class $\mathfrak{X}$ is called a *Hall class* if it contains every group G admitting a nilpotent normal subgroup N such that G/N' belongs to $\mathfrak{X}$, so that Hall's nilpotency criterion just says that nilpotent groups form a Hall class. The aim of this talk is to describe further relevant group properties determining Hall classes.

SYLOW SUBGROUPS OF FINITE PERMUTATION GROUPS

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We say that a finite group G acting on a set Ω has *Property* $(*)_p$ for a prime p if P_ω is a Sylow p-subgroup of G_ω for all $\omega \in \Omega$ and Sylow p-subgroups P of G, that is the operations of taking point stabilisers and finding Sylow p-subgroups commute. Property $(*)_p$ arose in the recent work of Tornier (2018) on local Sylow p-subgroups of Burger-Mozes groups. I will discuss recent joint work with John Bamberg, Alexander Bors, Alice Devillers, Cheryl E. Praeger and Gordon F. Royle that studies the permutation groups with Property $(*)_p$ and which includes a complete classification of the finite 2-transitive groups with this property.

THE STORY OF A CURIOUS CLASS OF GROUPS

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The genesis of this talk was a group theoretic question posed by Tim Boykett (Johannes Kepler University) that arose from a problem concerning near-rings and state automata. This is not a question that group theorists would think to ask; however, we found that it led to deep results.

We call G a *J-group* if there exists a fixed element $k \in G$ and a function $f\colon G \to G$ satisfying

$$f(xk) = xf(x) \qquad \text{for all } x \in G. \tag{2}$$

Is the class of J-groups theoretically classifiable? It turns out that all (finite) J-groups have odd order, but not all odd order groups are J-groups. Whether all finite nilpotent groups are J-groups seems to be a difficult question. Our research has a number of twists and turns.

*Joint work with Dominik Bernhardt, Tim Boykett, Alice Devillers and Johannes Flake.

MORSE SYSTEM AND INTERMEDIATE GROWTH

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I will explain how to gain a group of intermediate growth between polynomial and exponential from the Thue-Morse substitution. This will require a discussion about full topological groups and Schreier dynamical systems.

ON CONJUGACY CLASSES IN GROUPS

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Let x be an element of a group G and let x^G denote the conjugacy class of x. Clearly $\langle x\rangle \leq C_G(x)$. We shall call a non-trivial element x and its conjugacy class **deficient** if $\langle x\rangle < C_G(x)$. All the results which will be mentioned as "proved by us" were obtained jointly with Patrizia Longobardi and Mercede Maj.

Let j denote a non-negative integer. We shall say that a group G belongs to $D(j)$ if **exactly** j of its non-trivial conjugacy classes are deficient. We proved that a non-trivial finite group G belongs to $D(0)$ (i.e. has no deficient classes) if and only if either G is of prime order or it is a non-abelian group of order a product of two distinct primes. We also determined all finite groups in $D(1)$.

A group G is called locally graded if every non-trivial finitely generated subgroup of G has a proper normal subgroup of finite index. In 2014, C. Delizia, U. Jezernik, P. Moravec and C. Nicotera proved that if $G \in D(0)$ is locally graded, then it is finite. We proved several similar results concerning groups in $D(1)$. For example, we proved that if G in $D(1)$ is locally finite, then it is finite and the same holds if G in $D(1)$ is a periodic locally graded group.

GROUPS AND GROUP RINGS ARE SIMPLY THE BEST! FOR CODING AT LEAST

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Here $[n, r]$ signifies a code of length n and dimension r; the *rate* is $\frac{r}{n}$. It corresponds to a system of matrices $GH^T = 0$, usually over some field, where G (generator matrix) is an $r \times n$ matrix and H (check matrix) is an $(n - r) \times n$ matrix. A code has an associated (minimum) distance d and is then denoted by $[n, r, d]$. A $[n, r, d]$ code can correct $\lfloor \frac{d}{2} \rfloor$ errors and the biggest d can be is $(n - r + 1)$.

We would like the code (i) to have a specified large rate, (ii) be capable of correcting as many errors as possible, (if possible to have $d = (n - r + 1)$), (iii) have efficient encoding and decoding algorithms.

We could also require a code to be (i) over a field of characteristics p, ($p = 2$ is special of course) or (ii) over a field of prime order; in a field of prime order the arithmetic is simply modular arithmetic.

Special *types* of codes may be required for specific purposes. For example a code containing its dual may be required – for the code to be a *dual-containing* code. (The dual code is obtained by switching the functions of the generator and check matrices.) Why dual-containing? From a dual-containing code, a QECC (Quantum Error Correcting Code) may be constructed – and everyone knows that by saying *quantum*, attention is immediate, importance is assumed but entanglement and confusion follow!

Another type requirement may be to construct an LCD (Linear complementary dual) code, one such that the intersection with its dual is zero, $\mathcal{C}^{\perp} \cap \mathcal{C} = 0$. Such LCD codes are useful in security areas. "LCD codes have been studied amongst other things for improving the security of information on sensitive devices against *side-channel attacks* (SCA) and *fault non-invasive attacks*, and have found use in *data storage* and *communications' systems*."

Another requirement could be that the check matrix of the code has a small number of non-zero elements (relative to its length). Such a code is called an LDPC, *Low Density Parity Check*, code.

Using essentially methods derived from groups and group ring structures, codes to given requirements on rate, distance and type as above may be constructed. This talk will give the background to this and show how it works with some examples. The main paper is [1].

References

[1] "Linear block and convolutional codes to required rate, distance and type", to appear, available on ArXiv.

APPLIED GROUP THEORY IN THE QUANTUM AND ARTIFICIAL INTELLIGENCE ERA

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In this talk I present an overview of the current state-of-the-art in post-quantum group-based cryptography. I describe several families of groups that have been proposed as platforms, with special emphasis in polycyclic groups and graph groups, dealing in particular with their algorithmic properties and cryptographic applications. I then describe some applications of combinatorial algebra in fully homomorphic encryption, and in particular homomorphic machine learning. In the end I will discuss several open problems in this direction. See [1, 2].

References

[1] D. Kahrobaei, R. Flores, M. Noce, *Group-based Cryptography in the Quantum Era*, The Notices of the American Mathematical Society, https://arxiv.org/abs/2202.05917, accepted, 1–15 (2022)

[2] D. Kahrobaei, R. Flores, M. Noce, M. Habeeb, *Book*: *Applications of Group Theory in Cryptography*, the Mathematical Surveys and Monographs series of the American Mathematical Society. 1–200, Under consideration (2022)

A GENERALIZATION OF THE CHERMAK-DELGADO LATTICE TO WORDS IN TWO VARIABLES

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The Chermak-Delgado measure of a subgroup H of a finite group G is defined as the product of the order of H with the order of the centralizer of H in G, i.e. $m_G(H) = |H|\,|C_G(H)|$, and the set of all subgroups with maximal Chermak-Delgado measure forms a lattice in G.

Let $f(x, y)$ be a word in the alphabet $\{x, y, x^{-1}, y^{-1}\}$ and H a subgroup of a group G. The following sets are subgroups of G:

$$\begin{aligned} F_1^\ell(G,H) &= \{a \in G \mid f(ag,h) = f(g,h)\ \forall g \in G, \forall h \in H\} \\ F_1^r(G,H) &= \{a \in G \mid f(ga,h) = f(g,h)\ \forall g \in G, \forall h \in H\} \\ F_2^\ell(G,H) &= \{a \in G \mid f(h,ag) = f(g,h)\ \forall g \in G, \forall h \in H\} \\ F_2^r(G,H) &= \{a \in G \mid f(h,ga) = f(h,g)\ \forall g \in G, \forall h \in H\}. \end{aligned}$$

For the commutator word $f(x, y) = [x, y]$, we have

$$F_1^\ell(H) = F_2^\ell(H) = C_G(H) \text{ and } F_1^r(H) = F_2^r(H) = C_G(H^G).$$

The Chermak-Delgado measure associated with the subgroup $F_i^t(G,H)$, with $i = 1, 2$ and $t = \ell, r$, is $m_G(H) = |H|\,|F_i^t(G,H)|$. The question arises for which words $f(x, y)$ the subgroups having maximal Chermak-Delgado measure form a lattice. We discuss the obstacles that the generalization encounters, and some situations in which they can be surmounted.

FINITE AND PROFINITE GROUPS WITH AUTOMORPHISMS WHOSE FIXED POINTS SATISFY ENGEL-TYPE CONDITIONS

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A left Engel sink of an element g of a group G is a set $\mathscr{E}(g)$ such that for every $x \in G$ all sufficiently long commutators $[...[[x, g], g], \ldots, g]$ belong to $\mathscr{E}(g)$. (Thus, g is a left Engel element precisely when we can choose $\mathscr{E}(g) = \{1\}$.) Earlier we proved that if all elements of a profinite (or compact) group have finite or countable left Engel sinks, then the group is finite-by-(locally nilpotent).

We prove a similar structure theorem for profinite groups admitting a coprime non-cyclic elementary group of automorphisms each of whose non-trivial elements fixes only elements with finite or countable left Engel sinks. In the case where a profinite group G admits a coprime automorphism of prime order whose fixed points have finite left Engel sinks, we prove that G has an open pronilpotent-by-nilpotent subgroup.

A right Engel sink of an element g of a group G is a set $\mathscr{R}(g)$ such that for every $x \in G$ all sufficiently long commutators $[...[[g, x], x], \ldots, x]$ belong to $\mathscr{R}(g)$. If a profinite group G admits a coprime automorphism of prime order whose fixed points have finite right Engel sinks, we prove that G has an open locally nilpotent subgroup.

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*Joint work with Pavel Shumyatsky.

COINCIDENCE REIDEMEISTER ZETA FUNCTIONS FOR NILPOTENT GROUPS OF FINITE RANK

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In classical topological fixed point theory, Reidemeister numbers arise as homotopy invariants associated to iterates of a continuous self-map of a connected compact polyhedron.

In a recent joint paper with A. Fel'shtyn [1], we take a group-theoretic point of view and consider coincidence Reidemeister zeta functions for tame endomorphism pairs of nilpotent groups of finite rank. Our main tools are profinite completion techniques. In particular, we provide a closed formula for coincidence Reidemeister numbers, based on a weak commutativity condition, which derives from simultaneous triangularisability on abelian sections. Moreover, we arrive at results in support of a Pólya–Carlson dichotomy between rationality and a natural boundary for the analytic behaviour of the zeta functions in question.

During the first half of my talk, my aim is to provide an introduction to the subject. In the second half I want to zoom in on (a simplified version of) our main theorem and highlight some of the open questions that this all leads to.

References

[1] A. Fel'shtyn and B. Klopsch, *Pólya–Carlson dichotomy for coincidence Reidemeister zeta functions via profinite completions*, to appear in Indag. Math. (N.S.); in-press version: https://doi.org/10.1016/j.indag.2022.02.004; arXiv preprint version: https://arxiv.org/abs/2102.10900

*Joint work with Alexander Fel'shtyn.

BOUNDING THE NUMBER OF CONJUGACY CLASSES IN TERMS OF A PRIME

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Finding upper and lower bounds for the number of conjugacy classes of a finite group G has been a topic of interest for a long time. In recent years some effort has been made to obtain good lower bounds in terms of the largest prime divisor of the order of the group. It is known that the best possible general such lower bound is $2\sqrt{p-1}$. The examples realizing this bound, however, have a very special structure, so by imposing hypotheses avoiding this structure one can expect to improve the bound. We discuss one way to do so.

* Joint work with A. Moretó.

NORMAL CLOSURES OF SUBGROUPS AND RELATED TYPES OF SUBGROUPS

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If G is a group and H be a subgroup of G, then H^G denotes a minimal normal subgroup of G including H. This normal subgroup is called the *normal closure* of H in G.

A subgroup H of G is called *contranormal* in G if $G = H^G$.

As we can see by the definition, contranormal subgroups are antipodes to normal and subnormal subgroups: a contranormal subgroup H of a group G is normal (respectively subnormal) if and only if $H = G$.

A subgroup H of a group G is called *conormal* in G if H is contranormal in H^G.

The property "to be a conormal subgroup" combines the properties "to be a contranormal subgroup" and "to be a normal subgroup": indeed every contranormal subgroup is conormal, and every normal subgroup is conormal (if H is normal in G, then $H = H^G$ so that $H = H^H$).

A subgroup S of a group G is said to be *polynormal* in G if for every subgroup H including S is contranormal in S^H. In other words, S is a polynormal subgroup if C is conormal in each subgroup including S.

For the finite nilpotent group G we have the following characterizations

- *A finite group G is nilpotent if and only if G does not include proper contranormal subgroups.*
- *A finite group G is nilpotent if and only if every conormal subgroup of G is normal.*
- *A finite group G is nilpotent if and only if every polynormal subgroup of G is normal.*

We will show some results about the structure of infinite group G such that
G does not include proper contranormal subgroups,
or
whose conormal subgroup of G are normal,
or
whose polynormal subgroup of G are normal.

κ-EXISTENTIALLY CLOSED GROUPS AND AUTOMORPHISMS

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Let κ be an infinite cardinal. A group G with $|G| \geq \kappa$ is called **κ-existentially closed** if every system of less than κ-many equations and in-equations with coefficients in G which has a solution in some overgroup $H \geqslant G$ already has a solution in G. $\aleph_0$-existentially closed groups were introduced by W. R. Scott in 1951, see [4]. The motivation for the study of existentially closed groups (algebraically closed groups) comes from algebraically closed fields.

Not much known about the structure of the automorphism group of κ-existentially closed groups. It was proved by Macintyre in [3, Page 56] that every countable, $\aleph_0$-existentially closed group has $2^{\aleph_0}$ automorphisms.

Question What can we say about the cardinality of automorphism groups of κ-existentially closed groups of cardinality $\lambda \geq \kappa$?

We prove the following:

Corollary. [1] *Let κ be inaccessible and let G be the unique κ-existentially closed group of cardinality κ. Then $|Aut(G)| = 2^{\kappa}$.*

We also prove that for a κ-existentially closed group of cardinality κ the $|Aut(G)| = 2^{\kappa}$, see [2].

Open Question. Let G be a κ-existentially closed group of cardinality $\lambda \geq \kappa$ where κ is a regular cardinal. Determine the structure of the $Aut(G)$.

References

[1] B. Kaya, and M. Kuzucuoğlu, Automorphisms of κ-existentially closed groups. To appear in Monatshefte Mathematik; see also https://arxiv.org/abs/2012.15167.

[2] B. Kaya, and M. Kuzucuoğlu, Automorphisms of κ-existentially closed groups of cardinality κ. In preparation.

[3] A. Macintyre; On algebraically closed groups, Annals of Math. **96**, (1972) 53–97.

[4] W. R. Scott, *Algebraically closed groups*, Proc. Amer. Math. Soc. **2** 118–121 (1951).

*Joint work with Burak Kaya and Otto H. Kegel.

GROUPS HAVING ALL ELEMENTS OFF A NORMAL SUBGROUP WITH PRIME POWER ORDER

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We consider a finite group G with a normal subgroup N so that all elements of $G \setminus N$ have prime power order. We prove that if there is a prime p so that all the elements in $G \setminus N$ have p-power order, then either G is a p-group or $G = PN$ where P is a Sylow p-subgroup and $(G, P, P \cap N)$ is a Frobenius-Wielandt triple. We also prove that if all the elements of $G \setminus N$ have prime power orders and the orders are divisible by two primes p and q, then G is a $\{p, q\}$-group and G/N is either a Frobenius group or a 2-Frobenius group. This builds on the classification by Higman of solvable groups where all elements have prime power order. If all the elements of $G \setminus N$ have prime power orders and the orders are divisible by at least three primes, then all elements of G have prime power order and G/N is nonsolvable. The nonsolvable groups where all elements have prime power order have been classified by Brandl extending the work of Suzuki who determined the simple groups with all elements having prime power order.

THE SOLUBLE GRAPH AND THE ENGEL GRAPH OF A FINITE GROUP

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In the commuting graph of a finite group G there is an edge between two vertices g_1 and g_2 if $\langle g_1, g_2\rangle$ is abelian. We may generalize this definition in two different directions. Given a class $\mathfrak{F}$ of finite groups, the $\mathfrak{F}$-graph of G is the undirected graph in which there is an edge $g_1 \leftrightarrow g_2$ if $\langle g_1, g_2\rangle \in \mathfrak{F}$. Given a word $w \in F_2$, the w-graph of G is the direct graph in which there is an edge $g_1 \to g_2$ if $w(g_1, g_2) = 1$. In the first part of the talk we will discuss the connectivity properties of the $\mathfrak{F}$-graphs, with particular attention to the case when $\mathfrak{F}$ is the class of the finite soluble groups, describing some results obtained in a joint paper with Tim Burness and Daniele Nemmi. In the second part of the talk we will consider the n-Engel graph, i.e. the w-graph where $w = [x,_n y]$ is the n-Engel word and we will define the Engel graph of G as the graph in which the vertices are the elements of G which does not belong to the hypercenter $Z_\infty(G)$ of G, and in which there is an edge $g_1 \to g_2$ if $[g_1,_n g_2] = 1$ for some $n \in \mathbb{N}$. In a joint paper with Eloisa Detomi and Daniele Nammi, we proved that if $G/Z_\infty(G)$ is neither an almost simple group nor a Frobenius group, then the Engel graph of G is strongly connected. The connectivity properties of the Engel graphs of the almost simple groups are investigated in a very recent joint work with Pablo Spiga.

*Joint work with Tim Burness, Eloisa Detomi, Daniele Nemmi and Pablo Spiga.

RINGS GENERATED BY CHARACTER VALUES OF REPRESENTATIONS OF FINITE GROUPS

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We study realization fields and integrality of characters of finite subgroups of $GL_n(\mathbf{C})$ and related lattices with a focus on on the integrality of characters of finite groups G. We are interested in the arithmetic aspects of the integral realizability of representations of finite groups, order generated by the character values, the number of minimal realization splitting fields, and, in particular, consider the conditions of realizability in the terms of Hilbert symbols and quaternion algebras and some orders generated by character values over the rings of rational and algebraic integers.

Theory of characters of finite and infinite groups plays the central role in the theory of representations of finite groups and associative algebras. The classical results are related to some arithmetic problems: the description of integral representations are essential for finite groups over rings of integers in number fields, local fields, or, more generally, for Dedekind rings. In this talk we consider the integrality of characters of finite subgroups of $GL_n(\mathbf{C})$ and related lattices. W. Burnside, I. Schur, later W. Feit, J.-P. Serre considered the question: whether every representation $\rho : G \to GL_n(K)$ of finite group G over a number field K is conjugate in $GL_n(K)$ to a representation $\rho' : G \to GL_n(O_K)$ over the ring of integers O_K?

This question is closely related to globally irreducible representations; the concept introduced by J.G. Thompson and B. Gross, was developed and generalized by Pham Huu Tiep, F. Van Oystaeyen and A.E. Zalesskii, and there are still many open questions. We are interested in the arithmetic aspects of the integral realizability of representations of finite groups, and, in particular, prove the existence of infinite number of splitting fields where the representations are not realizable.

For $\chi \in Irr(G)$ and $G \subset GL_n(\mathbf{C})$ let $K_G = \mathbf{Q}(\chi(G)) = \mathbf{Q}(\{\chi(g), g \in G\})$ be the field generated by all traces of matrices in the representation of G over $\mathbf{Q}$.

We define the order generated by the character values of $\chi(G)$ over $\mathbf{Z}$ for the fixed character χ : this order $\mathbf{Z}[G]$ is contained in O_{K_G}. (Note that $\mathbf{Z}[G]$ is neither the group algebra nor the ring of generalized characters.) The deviation of $O_{K_G}[G]$ from $\mathbf{Z}[G]$ can be measured by the structure of the finite abelian group $O_{K_G}/\mathbf{Z}[G]$.

The character values of the character $\chi(G)$ of G for the fixed character χ also generate over the ring O_K an order $O_K[G]$ contained in O_K . The deviation of $O_K[G]$ from O_K can be measured by the structure of the finite abelian group $O_K/O_K[G]$.

In this talk we consider the order generated by character values over $\mathbf{Z}$: this order order $\mathbf{Z}[G]$ is contained in O_K for fixed characters of a globally irreducible representation. We also consider the orders generated by the character values of $\chi(G)$ over $\mathbf{Z}$ for the fixed character χ of degree 2 for globally irreducible representations of $G \subset GL_2(\mathbf{C})$ and prove, in particular, that $O_{K_G} = \mathbf{Z}[G]$. In general, the group $O_{K_G}/\mathbf{Z}[G]$ is not trivial, and there are some restrictions for the exponent of this group.

ON CHARACTERIZATION OF A FINITE GROUP BY ITS GRUENBERG-KEGEL GRAPH

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The *Gruenberg–Kegel graph* or the *prime graph* $\Gamma(G)$ of a finite group G is the simple graph whose vertices are the prime divisors of $|G|$, with p and q adjacent in $\Gamma(G)$ if and only if G contains an element of order pq.

The question of characterization of a finite group by its Gruenberg–Kegel graph is actively investigating, and some known results in this area are surveyed in [1]. In this talk we discuss a recent progress in characterization of a finite group by its Gruenberg–Kegel graph.

References

[1] P. J. Cameron and N. V. Maslova, *Criterion of unrecognizability of a finite group by its Gruenberg-Kegel graph*, J. Algebra, doi: https://doi.org/10.1016/j.jalgebra.2021.12.005.

*Based on joint works with P. Cameron, A. P. Khramova, A. S. Kondrat'ev, V. V. Panshin, and A. M. Staroletov.

COMMUTATORS, CENTRALIZERS AND STRONG CONCISENESS IN PROFINITE GROUPS

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A group G is said to have restricted centralizers if for each g in G the centralizer $C_G(g)$ either is finite or has finite index in G. Shalev showed that a profinite group with restricted centralizers is virtually abelian. We take interest in profinite groups with restricted centralizers of uniform k-step commutators, that is, elements of the form $[x_1, \ldots, x_k]$, where $\pi(x_1) = \pi(x_2) = \cdots = \pi(x_k)$. Here $\pi(x)$ denotes the set of prime divisors of the element $x \in G$. It turns out that such a group necessarily has an open nilpotent subgroup. Moreover, $\gamma_k(G)$ is finite if and only if the cardinality of the set of uniform k-step commutators in G is less than $2^{\aleph_0}$. Some results about finite groups where centralizers of uniform 2-step commutators have index at most n (order at most n, respectively) will be also discussed.

*Joint work with Eloisa Detomi and Pavel Shumyatsky.

CONSTRUCTING THE AUTOMORPHISM GROUP OF A FINITE GROUP

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Constructing the automorphism group of a finite group remains challenging. The critical hard case is that of a finite p-group P where much effort has been invested over the past 20 years in developing recursive algorithms which work down a central series for P. If we can locate characteristic structure in P, then we can often readily solve the problem. The real challenge remains class 2 p-groups of exponent p. In this lecture we will outline algorithmic approaches and report on recent joint work which offer new hope of progress on this intractable problem.

ON THE DISTRIBUTION OF ELEMENT ORDERS IN FINITE GROUPS

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Some recent papers by Herzog, Longobardi, and Maj, as well as by numerous other authors deal with the sum of the orders of all elements in a finite group G, usually denoted by $\Psi(G)$. Now $\Psi(G)/|G|$ is the average order of group element, or—in the language of probability theory—the expected value of the order of a randomly chosen element. Instead of taking the sum, some authors studied the product of element orders. I will argue that this is a more natural approach, since the element order has rather multiplicative features than additive ones. Therefore, the random variable one has to study is the logarithm of the order of a randomly chosen element $g \in G$. A classic result by Erdős and Turán [2] states that for the symmetric groups the distribution of $\log o(g)$ is asymptotically Gaussian, namely,

$$\lim_{n\to\infty} \operatorname*{Prob}_{g\in S_n} \left(\log o(g) < \frac{1}{2}\log^2 n + x \left(\frac{1}{3}\log^3 n \right)^{1/2} \right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt.$$

For a fixed prime p let P_n be the Sylow p-subgroup of the symmetric group of degree p^n (i.e., the iterated wreath product of n copies of the cyclic group of order p). The variance of the distribution of the logarithm (to the base p) of the order of a randomly chosen element $g \in P_n$ is bounded and its expected value is asymptotically cn, where $0 < c = c(p) < 1$ satisfies the equation

$$\frac{1-c}{c}\log(1-c) + \log c = \log\left(1 - \frac{1}{p}\right),$$

(see [3], [1]). We are looking for analogs of results proven for the average of element orders (that is, for their arithmetic mean) in the setting of the average of the logarithms of the element orders (that is, essentially, for their geometric mean).

References

[1] M. Abért and B. Virág, Dimension and randomness in groups acting on rooted trees. J. Am. Math. Soc. 18 (2005), 157–192.

[2] P. Erdős and P. Turán, On some problems of a statistical group-theory, III. Acta Math. Acad. Sci. Hung. 18 (1967), 309–320.

[3] P. P. Pálfy and M. Szalay, On a problem of P. Turán concerning Sylow subgroups, Studies in Pure Mathematics, Birkhäuser, Basel, 1983, pp. 531–542.

COMMUTATOR WIDTH OF CHEVALLEY GROUPS AND MODEL THEORETIC APPLICATIONS

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We will formulate our new results on bounded generation for Chevalley and Kac-Moody groups. In particular, Chevalley groups of rank greater than 1 over polynomial rings and Chevalley groups of arbitrary rank over Laurent polynomial rings (in both cases the coefficients are taken from a finite field) are boundedly elementarily generated. We also state several conjectures and applications. The main objective of this short talk is to describe model theoretic consequences of these results.

*Joint work with B.Kunyavskii, N.Vavilov

HUNTING CYCLES IN PERMUTATION GROUPS

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In trying to decide what to talk about, I chose a topic that I've found a lot of fun with, and which holds lots of memories for me. Cycles in permutation groups, or more specifically in symmetric groups. How many are there, how to find them, and how to use them. The results I'll talk about appeared in print between 1873 and 2021.

References

[1] John Bamberg, Stephen Glasby, Scott Harper, and Cheryl E. Praeger, Permutations with orders coprime to a given integer, *Electronic J. Combin.* 27 (2020), P1.6, doi:10.37236/8678, ArXiv: 1807.10450.

[2] Stephen Glasby, Cheryl E. Praeger and William R. Unger, Most permutations power to a cycle of small prime length, *Proc. Edinburgh Math. Soc.* 64 (2021), 234-246. doi: 10.1017/S0013091521000110 Arxiv: 1911.12613.

[3] C. Jordan, Sur la limite du degré des groupes primitifs qui contiennent une substitution donée, *J. Reine Angew. Math.* 79 (1875), 248–258.

[4] Cheryl E. Praeger, On elements of prime order in primitive permutation groups, *J. Algebra* 60 (1979), 126–157.

*Some is joint work with Stephen Glasby and William Unger.

FINITELY GENERATED METABELIAN GROUPS ARISING FROM INTEGER POLYNOMIALS

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We associate with every integer polynomial f a finitely generated metabelian group G_f of finite torsion-free rank. We show how properties of the group G_f can be read off from the polynomial f.

These include the torsion subgroup, residual properties, finite presentability, the Schur multiplier. One can also determine the centre, Fitting and Frattini subgroups and solve the isomorphism problem for the G_f.

COMMUTING PROBABILITY FOR SUBGROUPS OF A FINITE GROUP

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If K is a subgroup of a finite group G, the probability that an element of G commutes with an element of K is denoted by $Pr(K, G)$. The probability that two randomly chosen elements of G commute is denoted by $Pr(G)$. A well known theorem, due to P. M. Neumann, says that if G is a finite group such that $Pr(G) \geq \epsilon$, then G has a nilpotent normal subgroup T of class at most 2 such that both the index $[G : T]$ and the order $|[T, T]|$ are ϵ-bounded.

In the talk we will discuss a stronger version of Neumann's theorem: if K is a subgroup of G such that $Pr(K, G) \geq \epsilon$, then there is a normal subgroup $T \leq G$ and a subgroup $B \leq K$ such that the indexes $[G : T]$ and $[K : B]$ and the order of the commutator subgroup $[T, B]$ are ϵ-bounded.

We will also discuss a number of corollaries of this result. A typical application is that if in the above theorem K is the generalized Fitting subgroup $F^*(G)$, then G has a class-2-nilpotent normal subgroup R such that both the index $[G : R]$ and the order of the commutator subgroup $[R, R]$ are ϵ-bounded.

This is a joint work with Eloisa Detomi (University of Padova).

A CHARACTERIZATION OF THE QUATERNIONS USING COMMUTATORS

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We will prove the following theorem.

Theorem. *Let R be an associative ring with* **1** *which is not commutative such that*

(*i*) *A non-zero commutator in R is not a divisor of zero in R;*

(*ii*) $(x, y)^2 \in C$, *for all $x, y \in R$, where C is the center of R.*

Then

1. *R contains no divisors of zero.*
2. *If, in addition, the characteristic of R is not 2, then the localization of R at C is a quaternion division algebra, whose center is the fraction field of C.*

We note that if $x, y \in R$ are non-zero elements such that $xy = 0$, then we say that both x and y are zero divisors in R.

References

[KS1] E. Kleinfeld, Y. Segev, *A short characterization of the octonions*, Comm. Algebra **49** (2021), no. 12, 5347–5353.

[KS2] E. Kleinfeld, Y. Segev, *Alternative rings whose associators are not zero-divisors*, Arch. Math. (Basel) **117** (2021), no. 6, 613–616.

[KS3] E. Kleinfeld, Y. Segev, *A characterization of the quaternions using commutators* , to appear in Math. Proc. R. Ir. Acad.

[KS4] E. Kleinfeld, Y. Segev, *A uniform characterization of the octonions and the quaternions using commutators*, submitted.

[*]Joint work with Erwin Kleinfeld.

SKEW BRACES THAT DO NOT COME FROM ROTA–BAXTER OPERATORS

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The study of skew braces, recently introduced by L. Guarnieri and L. Vendramin, has been of interest in the last few years, because these objects present connections with several distinct topics, such as radical rings, regular subgroups of the holomorph, Hopf–Galois structures, and solutions of the Yang–Baxter equation.

In 2022, V. G. Bardakov and V. Gubarev presented a way to obtain skew braces from Rota–Baxter operators on groups, defined one year before by L. Guo, H. Lang, and Y. Sheng.

Skew braces on a group can be characterised in terms of certain functions from the group to its automorphism group, called gamma functions. For the skew braces obtained from a Rota–Baxter operator, the corresponding gamma functions take values in the inner automorphism group.

In this talk, we present a characterisation of the gamma functions on a group, with values in inner automorphism group, which come from a Rota–Baxter operator, in terms of the vanishing of a certain element in a suitable second cohomology group.

Exploiting this characterisation, we can exhibit examples of skew braces whose corresponding gamma functions take values in the inner automorphism group, but cannot be obtained from a Rota–Baxter operator.

*Joint work with Andrea Caranti.

LEFT 3-ENGEL ELEMENTS IN GROUPS

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An element x in a group G is a left 3-Engel element if $[[[g,x],x],x] = 1$ for all $g \in G$. In this talk we will give an overview of these, focusing on advances in recent years.

PROFINITE GROUPS WITH FEW CONJUGACY CLASSES

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It is well known that the cardinality of an infinite profinite group cannot be less than $2^{\aleph_0}$, the cardinality of the continuum. In 2019, Jaikin-Zapirain and Nikolov [1] proved moreover that each infinite profinite group has at least $2^{\aleph_0}$ conjugacy classes. We discuss the consequences of restricting the number of conjugacy classes of elements of various types (such as p-elements or elements of infinite order). In particular, every finitely generated profinite group with fewer than $2^{\aleph_0}$ conjugacy classes of elements of infinite order is finite.

References

[1] An infinite profinite group has uncountably many conjugacy classes. *Proc. Amer. Math. Soc.* **147** (2019), 4083–4089.

AN EIGENVALUE 1 PROBLEM IN REPRESENTATIONS OF FINITE GROUPS OF LIE TYPE

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We consider special cases of the following general problem. Given a finite group G and a field P determine the irreducible P-representations of G in which every group element has eigenvalue 1. We focus on the case where G is a finite simple group of Lie type in defining characteristic $p > 0$ and P is an algebraically closed field of characteristic p.

AUTOMORPHISM GROUPS AND LIE ALGEBRAS OF VECTOR FIELDS ON AFFINE VARIETIES

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Let V be an affine algebraic variety over a commutative ring K and let A be the K-algebra of regular (polynomial) functions on V.

The group of automorphisms of V, namely $\mathrm{Aut}(A)$, is, generally speaking, not linear. We will discuss the following two questions: which properties of linear groups extend to $\mathrm{Aut}(A)$, and which properties of finite dimensional Lie algebras extend to the Lie algebra $\mathrm{Der}(A)$ of vector fields on V?

In particular, we will focus on analogs of classical theorems of Selberg, Burnside, and Schur for $\mathrm{Aut}(A)$ and an analog of the Engel theorem for $\mathrm{Der}(A)$. In order to achieve natural degree of generality and to include some interesting noncommutative cases we prove the theorems for PI-algebras.

*Joint work with O. Bezushchak and A. Petravchuk.