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BOOK REVIEW

**FUZZY LOGIC: Mathematical Tools for Approximate Reasoning,** by Giangiacomo Gerla. Kluwer Academic Publishers (Trends in Logic, Studia Logica Library, Vol. 11), Dordrecht, 2001, 288 pages, ISBN 0-7923-6941-6.

The present book is another book from the Kluwer's series Trends in Logic which is devoted to fuzzy logic. The book deals with several general aspects of fuzzy logic in narrow sense and foundational aspects of fuzzy reasoning. The book is based on the author's contributions that were published during the last 20 years. The book is quite unique in that unlike other books on fuzzy logic in narrow sense, this one deals with an abstract approach to fuzzy logic. By abstract approach we mean here that rather than particular logical calculi (like Lukasiewicz logic, BL-logic etc.), one studies the various structures and problems related to logical calculi like inference rules, closure and consequence operators, axiomatizability, effectiveness etc. The book is much in the spirit of the approach due to Tarski by which a logic is a set of formulas and a suitable deduction operator. This point of view is elaborated in detail for the case of fuzzy logic, i.e. logic that deals with reasoning in the presence of vagueness. In the following, I will go through the book chapter by chapter.

Chapter 1 introduces basic notions (lattices, closure operators and systems in lattices, abstract logic, and related notions). Under the abstract approach, an abstract logic consists of a complete lattice, a closure operator on this lattice and an abstract semantics which is a subset of the lattice not containing the largest element. A prototypical example of an abstract logic has the power set of a set of all logical formulas of a particular logical calculus as its complete lattice, the naturally induced deduction operator as the closure operator, and the set of all complete theories as the abstract semantics.

Fundamental concepts of abstract fuzzy logic are introduced in Chapter 2. One should point out that the author uses the real interval [0,1] as the standard structure of truth values with min and max as the standard conjunction and disjunction operations, and $1-x$ as negation operation. Since these operations have relatively strong properties (for example, both min and max are idempotent and $1-x$ is involutive), one cannot expect that the results can be easily generalized to other structures of truth values where one uses other logical operations. Thus, while considering arbitrary fuzzy logics fitting the definition of an abstract fuzzy logic makes the approach very general, the particular choice of structure of truth values makes it in a sense specific. Note, however, that additional operations on [0,1] are considered in later chapters. Chapter 2 starts with basic notions of fuzzy sets and cuts (alpha cuts) of fuzzy sets (cuts of fuzzy sets are heavily used in the book as a means for extending crisp notions to fuzzy ones). Then, fuzzy abstract logic is introduced by a natural generalization of the crisp approach. Other issues treated are the ultraproduct construction, nonmonotonicity of a normalized deduction operator, abstract similarity logic (a particular example of abstract fuzzy logic), and a result showing that, in principle, every abstract fuzzy logic is equivalent to an abstract (crisp) logic.

Chapter 3 deals with extensions of an abstract logic to an abstract fuzzy logic. The basic idea is that of extending a closure operator to a fuzzy closure operator in a cut-like manner: One takes a fuzzy set, decomposes it into cuts, applies the original closure operator to the cuts and the resulting system of sets represents the closure of the fuzzy set under the fuzzy closure operator. Several results are obtained for this straightforward construction and its application to extending crisp logic to fuzzy logic.

Chapter 4 is devoted to abstract fuzzy logics with deduction operator obtained using elementary fuzzy inference rules. Such logics are called Hilbert logics. The idea of a fuzzy inference rule goes back to Pavelka and is elaborated on the abstract level here. Note that, in addition to a syntactic part which is the only one constituting a crisp inference rule, a fuzzy inference rule has its semantical part which, given the truth degrees of formulas that input the inference, gives the lower estimation of the truth degree of the inferred formula. This is a very general approach to making inferences under various types of indeterminacy, not only vagueness (as shown in Chapter 9).

A natural extension of fuzzy inference rules are rules that allow for reasoning over statements like "truth degree of a formula is between 0.3 and 0.5". This problem is addressed in Chapter 5.

In Chapter 6, extension of crisp Hilbert logics to Hilbert fuzzy logics is studied. After presenting the general case, several interesting examples are presented. These include necessity logic which is related to the so-called possibility logic, and furthermore, various connections to important notions of fuzzy set theory (like necessity measures and fuzzy subalgebras).

Chapter 7 considers graded consequence relations. The principle enabling us to extend a closure operator to a fuzzy closure operator (formulated in Chapter 3) is generalized here so that one can start from a family of closure operators indexed by [0,1] and obtain a fuzzy closure operator. Such fuzzy closure operators are called stratified. This construction is studied in detail. After that, consequence relations are generalized into fuzzy setting and their connection to stratified fuzzy closure operators is investigated.

Truth-functional fuzzy semantics is studied in the context of previous investigations in Chapter 8. Truth-functionality means that the truth value of a compound proposition (logical formula) is determined by the truth values of the constitutive proposition (subformulas). One of the main problems investigated in this chapter is the axiomatizability of a fuzzy semantics: given a truth-functional fuzzy semantics, is it axiomatizable by some deduction operator? Following the fundamental discovery of Pavelka, it is shown that this question is strongly related to continuity of operations on [0,1] that interpret logical connectives (it is shown that any continuous truth-functional fuzzy semantics is, indeed, axiomatizable and vice versa).

Chapter 9 shows that the interpretability of the calculi with fuzzy inference rules described in the book goes far beyond the idea that degrees from [0,1] are truth degrees. In Chapter 10, these degrees are interpreted in a probabilistic manner. This shows a surprising application of the approach developed: Presented are logics for inferences over probabilities of statements and their completeness results.

Chapter 10 presents a logical approach to fuzzy control. Fuzzy control is the most commercially successful area of fuzzy modeling and this chapter contributes to logical investigation of this area. In particular, fuzzy control is approach via fuzzy logic programming.

The last chapter, Chapter 11, deals with selected computability aspects of fuzzy logic. The central theme is the search for proper generalization of concepts like enumerable set to the framework of fuzzy sets. This chapter, like all the previous, is based on author's active research in the field.

The book contains a representative list of references which is heavily used in the book (there are many citations which keeps the reader well-informed about the development of the presented ideas), an index (I did not, for example, find the term fuzzy closure operator in the index, but this seems to be an exception), and a list of symbols (which is very useful because the notation is not commonly known).

Gerla's book is written in a lucid style. Except for some very minor points (e.g., I find the term fuzzy class a bit misleading: I would expect that a fuzzy class is a fuzzified notion of a class but this is not so and a fuzzy class as introduced in the book is a crisp subset of the set of all fuzzy sets) I cannot imagine objections with respect to the presentation and arguments motivating the study of topics covered. The book shows an enormously rich world of fuzzy logic from the perspective that has not been presented so far. Systematic treatment of fuzzy approach to metalogical notions makes the book unique. The book provides a coherent view on several important problems connected to the abstract approach to fuzzy logic. On the other hand, taking into account the recent investigations of various particular fuzzy logical calculi given by selecting a t-norm (or t-norms) as pursued e.g. by P. Hajek, it is fairly obvious that a lot of interesting research is still needed to advance our understanding of reasoning under vagueness. The book by Prof. Gerla provides us with both deep and broad fundamentals for the future research. The book is a research monograph and will probably be used mainly by researchers in the field and in graduate courses on fuzzy logic in narrow sense.

Radim Belohlavek, Palacky University, Olomouc
radim.belohlavek@upol.cz