

CONTENTS

CONTENTS	VII
PREFACE	XI
CHAPTER 1. Abstract logic in a lattice	
1 Introduction	1
2 Lattices, Boolean algebras, triangular norms	1
3 Closure operators and closure systems	4
4 A Galois connection between operators and classes	6
5 Abstract logic in a lattice	8
6 Continuity for abstract logics	10
7 Step-by-step deduction systems	12
8 Logical compactness	13
9 Product of two abstract deduction systems	15
10 Duality principle for ordered sets	16
CHAPTER 2. Abstract fuzzy logic	
1 Fuzzy subsets for vagueness	19
2 Basic notions	21
3 Closed and open cuts	24
4 Fuzzy subsets and continuous chains	26
5 Abstract fuzzy logic	28
6 Compactness and continuity	30
7 Logical compactness	32
8 Ultraproduct of a family of fuzzy models	33
9 Fuzzy logic is not monotone	36
10 Abstract similarity logic	38
11 Any fuzzy logic is equivalent to a crisp logic	40
CHAPTER 3. Extending an abstract crisp logic	
1 An extension principle for closure operators	45
2 An extension principle for closure systems	48
3 Canonical extensions and continuous deformations	50
4 Dualizing the extension principle	52
5 Extension of a compact closure operator	53
6 Extension of a crisp logic	56
7 Characterizations of the canonical extensions	58
8 Degree of inconsistency of a canonical extension	61
9 Canonical similarity logic	64
10 Fuzzy metalogic, facts and preferences	66
CHAPTER 4. Approximate reasoning	
1 The heap paradox	69

2 Fuzzy inference rules	71
3 Fuzzy Hilbert logic and homomorphisms	76
4 Degree of consistency and non-monotonicity	78
5 Step-by-step deduction and continuity	79
6 Building up fuzzy Hilbert systems by inequalities	81
7 Any fuzzy Hilbert system is equivalent to a crisp system	83
8 Bald men, Lukasiewicz conjunction and induction principle	86
 CHAPTER 5. Logic as management of constraints on the truth values	
1 Heap paradox by negative information	89
2 Constraints on the truth values	91
3 Examples: Zadeh logic, Boolean logic, Probability logic	94
4 Hilbert systems for constraints	96
5 Fuzzy logics with a negation	97
6 Refutation procedures	100
7 Equivalence to a crisp logic	102
8 Tableaux method	104
 CHAPTER 6. Canonical extension of a crisp Hilbert logic	
1 Extending a crisp deduction Hilbert system	109
2 Controlling the inconsistency	111
3 Necessity logic	113
4 A simple example of non-monotone fuzzy logic	116
5 Fuzzy filters and fuzzy subalgebras	118
6 Necessity measures as fuzzy theories	119
7 Fuzzy Hilbert systems and fuzzy subalgebras	121
8 Extensions by continuous triangular norms	124
 CHAPTER 7. Graded consequence relations	
1 Graded information with graded deductive tools	129
2 Stratified fuzzy closure operators	129
3 Stratified fuzzy closure systems	132
4 A characterization of stratified closure systems	135
5 A characterization of stratified operators	138
6 Stratified deduction systems	140
7 Sequents and consequence relations	142
8 Graded consequences and sequent calculus	144
9 Finite sequent calculus and compact graded consequences	146
10 Graded consequences and stratified operators	147
 CHAPTER 8. Truth-functional logic and fuzzy logic	
1 Truth-functional fuzzy semantics	151
2 The main properties	152
3 Two discontinuous truth-functional semantics	156
4 Any continuous truth-functional semantics is axiomatizable	158
5 Any axiomatizable truth-functional semantics is continuous	160
6 Zadeh (continuous) logic	163

7 Łukasiewicz (continuous) logic	165
8 Comparing truth-functional logic with fuzzy logic	168
 CHAPTER 9. Probabilistic fuzzy logics	
1 Vagueness and uncertainty	171
2 Logic of super-additive measures	172
3 Completeness theorem	176
4 Logic of upper-lower probabilities	177
5 Probability logic: semantics	180
6 Probability logic: Hilbert system	184
7 Completeness theorem	187
8 Refutations in probability logic	189
9 Two remarks: probability of formulas, subjective probability	191
10 Belief logic and Boolean logic	194
11 Qualitative probability logics	196
 CHAPTER 10. Fuzzy control and approximate reasoning	
1 Information by words versus information by numbers	199
2 Control by triangular norms	200
3 Programs and Herbrand models	204
4 Fuzzy programs and fuzzy Herbrand models	205
5 Logic approach to fuzzy control	206
6 The logical interpretation suggests new tools	208
7 Control by implication and negative information	210
8 Control by similarity and prototypes	214
9 Logic interpretation of defuzzification: an open question	216
10 The predicate <i>MAMD</i> and some observations	219
 CHAPTER 11. Effectiveness in fuzzy logic	
1 Introduction	221
2 Recursively enumerable fuzzy sets	221
3 Decidability and fuzzy computability	225
4 Enumerability by discrete topology	228
5 Kleene hierarchy	231
6 Gödel numbering and Church Thesis	233
7 Reducibility and Universal Machines	236
8 Effective abstract fuzzy logic	238
9 Fuzzy logic = enumeration fuzzy closure operator	240
10 Creative fuzzy sets and Gödel theorems	245
11 Sharpened and shaded versions: limitative theorems	248
 REFERENCES	
INDEX	261
LIST OF SYMBOLS	267

