**Gerla Giangiacomo**
**Fuzzy logic. Mathematical tools for approximate reasoning.** (English)
[B] Trends in Logic--Studia Logica Library. 11. Dordrecht: Kluwer Academic Publishers. xii, 269 p. EUR 95.00; \$ 103.00; \sterling 64.00 (2001). [ISBN 0-7923-6941-6/hbk]

The author chooses an algebraic approach toward logic in the style of {*D. J. Brown*} and {*R. Suszko*} [``Abstract logics", Diss. Math. 102, 9-41 (1973; Zbl 0317.02071)]. However, he intends to give an essentially self-contained treatment, and thus offers all the necessary definitions. That makes the book accessible also, e.g., to readers from computer science and engineering who are not so familiar with algebraic approaches.
In **Chapter 1** the author explains the basic notions: closure operators and closure systems in lattices, and gives their fundamental properties.
**Chapter 2** introduces fuzzy sets, represents them by continuous sequences of cuts, and considers fuzzy logics as abstract logics in lattices of fuzzy (sub-)sets. Via signed formulas fuzzy logics become representable by crisp ones.
The converse way from crisp to fuzzy logics is discussed in **Chapter 3**, essentially by defining the fuzzy closure of a fuzzy set of formulas ``cutwise" related to a given crisp closure operation.
**Chapter 4** on ``Approximate Reasoning" introduces and discusses Pavelka style graded inference operators, and explains again their equivalence to crisp ones via signed formulas.
**Chapter 5** considers the more general situation that formulas come with grades which are intervals instead of numbers, read as constraints on the (allowed) truth degrees of these formulas. This chapter reaches up to the consideration of tableau methods.
**Chapter 6** is devoted to a problem similar to that of Chapter 3: now a fuzzy closure operation, represented by some chain of crisp closures, is applied to crisp as well as to fuzzy sets of formulas.
**Chapter 7** continues this topic and proves that very general chains of crisp closure operators determine fuzzy logics. In **Chapter 8** the author comes from the general algebraic context down to a first application: he considers standard truth functional many-valued logics and proves a general connection between the axiomatizability of the entailment operator and the continuity of the primitive logical connectives.
A second type of applications follows in **Chapter 9**: to probability logics. Here it is shown that the classes (i) of all constant-sum super-additive measures, (ii) of all upper-lower probabilities, and (iii) of all finitely additive probabilities form fuzzy semantics in the abstract sense. And in all these cases these semantics are completed with closure systems to give fuzzy logics. Furthermore it is, e.g., shown that there does not exist a fuzzy logic whose class of theories coincides with the class of all belief measures.
**Chapter 10** applies the author's approach to fuzzy control. Based upon a certain version of fuzzified logical programming, viz. Herbrand models for fuzzy logics, the author interprets lists of fuzzy control rules by fuzzy logic programs. With some additional fuzzy predicates, and with suitable interpretations of the fuzzy rules, he is able to model the Mamdani approach, as well as approaches which read the fuzzy rules as generalized implications.
The final **Chapter 11** returns to a theoretical topic: it discusses effectivity problems for fuzzy logics. The notions of recursivity and recursive enumerability are generalized to fuzzy sets. On this basis, recursion theoretic notions up to the Kleene hierarchy and up to productive and creative sets are generalized to the fuzzy setting too.
This is a very interesting book which, also because of its algebraic style, nicely fits into and completes the series of books of the last years in mathematical fuzzy logic. It has a subject index and a list of symbols.

S. J. Gottwald