Testing heteroskedasticity in diffusion processes

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1. Abstract

Diffusion processes derived from deterministic growth equations are widely employed in modeling population dynamics and related socio–economic phenomena. While classical deterministic models capture the average trajectory of the system, stochastic extensions provide a more realistic description of the observed dynamics. In particular, when the variance of the stochastic component depends on the state of the process or evolves over time, the system exhibits heteroskedasticity. Traditional heteroskedasticity tests are typically based on regression residuals and assume a fixed discrete–time model structure; however, such approaches are not directly applicable in the context of continuous–time diffusion processes.

We propose a heteroskedasticity test for diffusion processes arising from stochastic versions of growth equations. The test is formulated as a distributional specification test and relies on the Jensen–Shannon divergence to measure discrepancies between the empirical distribution of standardized diffusion increments and the theoretical normal distribution implied by a homoskedastic diffusion model. Since the finite–sample distribution of the test statistic is not available in closed form and depends on parameter estimation and discrete observation schemes, critical values are obtained via Monte Carlo simulation under the null hypothesis of homoskedasticity. The Monte Carlo calibration explicitly accounts for estimation uncertainty and discretization effects by replicating the entire testing procedure on simulated paths generated under the null model.

A comprehensive Monte Carlo simulation study is conducted to assess the finite–sample performance of the proposed test. The results show that the Monte Carlo calibrated procedure achieves accurate size control under the homoskedastic null hypothesis and exhibits good power against a broad class of heteroskedastic alternatives, including state–dependent, multiplicative, and time–varying diffusion coefficients.

References

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