The time-nonlocal harmonic oscillator and the time-nonlocal Ornstein-Uhlenbeck process

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While in recent years, time-nonlocal Kolmogorov equations have been used to describe several physical models, in particular anomalous diffusions, there are not so many results concerning time-nonlocal Schrödinger-type equations. Time-fractional Schrödinger equations were first considered in [1], where the author already noticed the loss of several fundamental properties of the solutions, that implied the non-coherence with the classical quantum mechanics. Nevertheless, these equations have been shown to be of particular interest in quantum information theory in [2]. In this talks, we will consider a time-nonlocal Schrödinger equation, both in the imaginary and the real time axis, with the quantum harmonic oscillator potential. In particular, we will exhibit both strong and distributional solutions in negative order Hermite-Sobolev spaces and we will provide a Mehler-type formula for the integral kernel of the time-changed semigroup. Furthermore, in the fractional setting, we will also consider the equation coming from the application of a Wick rotation, as in [1], proving that solutions of such an equation converge towards a non-constant steady state. All these results are obtained by means of an intertwining relation between the Hermitian with the quadratic potential and the generator of an Ornstein-Uhlenbeck process. This is part of an ongoing joint work with Nikolai Leonenko.

References

1. Naber M. (2004) Time fractional Schrödinger equation *J. Math. Phys.* **45.8**, 3339–3352

2. Zu C., Gao Y. and Yu X. (2021) Time fractional evolution of a single quantum state and entangled state *Chaos, Solitons & Fractals* **147**, 110930