# Parameter sensitivity analysis in a stochastic birth–death model for B-cells dynamics

## Workshop SMOD 2026

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The aim of this talk is to present a stochastic model, based on a two-dimensional birth-death process, to represent the dynamics of antigen processing in B cells, a fundamental component of the adaptive immune system. Such cells display protein receptors on their membrane, which bind with foreign antigens and process them. Within this framework, we introduce the process $\mathcal{X}(t) := \{(X(t), Y(t)),\ t \geq 0\}$, where $X(t)$ and $Y(t)$ represent the number of free and occupied receptors, respectively. After deriving the partial differential equation satisfied by the probability generating function of the process, we obtain closed and tractable expressions for the main moments and analyze both their transient and long-term behavior. Particular interest is given to the role of the model parameters in the system dynamics from a biological perspective. To this aim, we conduct a sensitivity analysis to assess how variations in the model parameters affect the first-order moments. Specifically, we combine global variance-based sensitivity analysis, through the computation of Sobol' indices, with a local analysis based on small perturbations around fixed points in the parameter space, carried out using elasticity functions. This twofold approach allows us to quantify the individual and joint contributions of the parameters to the output variability, as well as to assess the system's responsiveness to localized changes. The results highlight that even minimal variations in parameters associated with antigen–receptor binding, especially at early stages of the process, can lead to significant changes in the expected number of occupied receptors.
The results presented in this talk are based on a joint work with V. Casolaro and A. Di Crescenzo, currently in press in the Journal of Theoretical Biology.

### References

1. Mustaro V., Casolaro V. and Di Crescenzo A. (2026). On the dynamics of antigen receptors on the B-cell membrane through a two-dimensional stochastic process. *Journal of Theoretical Biology*, accepted for publication.