A Stochastic Growth Model with Random Catastrophes Applied to Population Dynamics

A. Di Crescenzo [a], S. Musto[a], P. Paraggio[a] and F. Torres-Ruiz[b,c]

[a]*Dipartimento di Matematica, Università degli studi di Salerno, Salerno, Italy*
[b]*Departamento de Estadística e I.O., Universidad de Granada, Granada, Spain*
[c]*Institute of Mathematics of the University of Granada (IMAG), Granada, Spain*

Stochastic growth models and sigmoidal dynamics play a fundamental role in capturing patterns frequently encountered in natural systems. They are widely employed in biology and ecology to describe processes such as population evolution, disease transmission, and adaptive responses to environmental variability. In this work, we investigate a lognormal diffusion process subject to random catastrophic events, modeled as sudden jumps that reset the system to a new random state. The novelty of the model lies in the assumption that the post-catastrophe restart level follows a binomial distribution. As a consequence, after each catastrophic event the population does not simply revert to its initial size, but instead to a random level reflecting the survival probability of individual members.
The proposed framework is applied to the dynamics of a wolf population subject to external disturbances. The results indicate that the model is able to realistically reproduce complex population trajectories, characterized by phases of gradual growth alternated with sudden declines driven by random external events.

References

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