

Fractional Pearson diffusions and continuous time random walks

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We define fractional Pearson diffusions [4,5,6] by non-Markovian time change in the corresponding Pearson diffusions [1,2,3]. They are governed by the time-fractional diffusion equations with polynomial coefficients depending on the parameters of the corresponding Pearson distribution. We present the spectral representation of transition densities of fractional Pearson diffusions, which depend heavily on the structure of the spectrum of the infinitesimal generator of the corresponding non-fractional Pearson diffusions (fPDs). Also, we present the strong solutions of the Cauchy problems associated with fractional Pearson diffusions and the correlation structure of these diffusions. Continuous time random walks (CTRW) have random waiting times between particle jumps. We define the correlated CTRWs that converge to fPDs [4,5,6]. The jumps in these CTRWs are obtained from Markov chains through the Bernoulli urn-scheme model, Wright-Fisher model and Ehrenfest-Brillouin-type models. The jumps are correlated so that the limiting processes are not Levy but diffusion processes with non-independent increments. This is a joint work with M. Meerschaert (Michigan State University, USA), I. Papic (University of Osijek, Croatia), N. Suvak (University of Osijek, Croatia) and A. Sikorskii (Michigan State University, USA).

We also briefly discussed the related problems for Stretched non-local Pearson diffusions (based on the joint paper [7] with L.Beghin (La Sapienza University, Rome, Italy), I. Papic (University of Osijek, Croatia) and J. Vaz (UNICAMP. Campinas, Brazil)

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